$$\tau \sim \frac{a}{v_{th_i}} \qquad v_{th_i} \ll c \quad \Rightarrow \quad \frac{\varepsilon_0\mu_0 \partial \underline{E}/\partial t}{\|\underline{\nabla} \times \underline{B}\|} \ll 1 \qquad n_e \approx Z n_i$$

$$m_e \approx 0$$

$$\frac{\partial n_i}{\partial t} + \underline{\nabla}\cdot(n_i \underline{u}_i) = 0 \qquad 0 \approx -e n_e(\underline{E} + \underline{u}_e \times \underline{B}) - \underline{\nabla} p_e - \cancel{m_e n_e \nu_{ei}}(\underline{u}_e - \underline{u}_i) \qquad \text{(elettroni)}$$

$$\frac{\partial n_e}{\partial t} + \underline{\nabla}\cdot(n_e \underline{u}_e) = 0$$

$$m_i n_i \left[\frac{\partial \underline{u}_i}{\partial t} + (\underline{u}_i \cdot \underline{\nabla})\underline{u}_i\right] = \qquad \text{(ioni)}$$

$$\underline{\nabla} \times \underline{E} = -\partial \underline{B}/\partial t \qquad = e n_i(\underline{E} + \underline{u}_i \times \underline{B}) - \underline{\nabla} p_i - \cancel{m_e n_e \nu_{ei}}(\underline{u}_i - \underline{u}_e)$$

$$\underline{\nabla} \times \underline{B} = \mu_0 e n_e(\underline{u}_i - \underline{u}_e) \qquad Z n_i \approx n_e \qquad \underline{\nabla}\cdot\underline{B} = 0$$

$$\rho = m_i n_i + m_e n_e \approx m_i n_i$$

$$\underline{u} \sim \underline{u}_e \sim \underline{u}_i$$

$$\underline{u} = \frac{\underline{E} \times \underline{B}}{B^2} \cancel{- \frac{\underline{\nabla} p \times \underline{B}}{q n B^2}}$$

$$v_{th}^2 \sim T/m$$

$$v_{th} \sim L/\tau$$

$$\frac{u_{dia}}{u_{E\times B}} \approx \frac{\frac{p}{L} \cdot \cancel{B}}{q n \cancel{B^2}} \cdot \frac{\cancel{B^2}}{E \cancel{B}} \sim \frac{\cancel{n} T}{q \cancel{n} L E} \sim \frac{T}{q L \, B L/\tau} = \frac{m \, T \, \tau}{m \, q B L^2} \sim \frac{v_{th}^2 \, T_{Larm} \, \tau}{L^2} \sim \frac{\cancel{v_{th}} \, T_{Larm}}{\cancel{v_{th}} \, L} \sim \frac{T_{Larm}}{\tau} \ll 1$$

legge di Faraday

$$\underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$E/L \sim \frac{B}{\tau} \; ; \; E \sim \frac{BL}{\tau}$$

$$m_e \underline{u}_e + m_i \underline{u}_i \simeq m_i \underline{u}_i$$ (mom. ~~totale~~ lineare)

$\rho \quad \underline{u}$

$$\underline{u} = \frac{m_e \underline{u}_e + m_i \underline{u}_i}{m_e + m_i} \simeq \frac{m_i \underline{u}_i}{m_i} \simeq \underline{u}_i$$

$$p = p_e + p_i$$

$$\underline{j} = ne(\underline{u}_i - \underline{u}_e) \qquad \underline{u}_e \simeq \underline{u}_i - \underline{j}/en_e \simeq \underline{u} - \underline{j}/en_e$$

$$\rho, \underline{u}, p, \underline{j}$$

$$m_i \times \left[\frac{\partial n_i}{\partial t} + \underline{\nabla}\cdot(n_i \underline{u}_i)\right] = 0$$

$+$

$$m_e \times \left[\frac{\partial n_e}{\partial t} + \underline{\nabla}\cdot(n_e \underline{u}_e)\right] = 0$$

$$\frac{\partial}{\partial t}\underbrace{\left[m_i n_i + m_e n_e\right]}_{\rho} + \underline{\nabla}\cdot\underbrace{\left[m_i n_i \underline{u}_i + m_e n_e \underline{u}_e\right]}_{\shortparallel} = 0$$

$$\underbrace{m_i n_i}_{\shortparallel\,\rho} \underline{u}_i \simeq \rho \underline{u}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\underbrace{\frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla)\rho} + \rho \nabla \cdot \mathbf{u} = 0$$

$$\frac{d}{dt}\rho(\mathbf{x}, t) + \rho \nabla \cdot \mathbf{u} = 0$$

$$n_e \approx n_i$$

[cont. i] − [cont. elettr] $\Rightarrow$ $\cancel{\frac{\partial n_e}{\partial t}} + \nabla \cdot (n_e \mathbf{u}_e) - \cancel{\frac{\partial n_i}{\partial t}} - \nabla (n_i \mathbf{u}_i) = 0$

$$\nabla \cdot \mathbf{j} = 0 \qquad \mathbf{j} = e n_e (\mathbf{u}_i - \mathbf{u}_e)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \qquad \nabla \cdot (\nabla \times \mathbf{B}) = 0 = \nabla \cdot \mathbf{j}$$

$$0 \simeq -en_e\left[\underline{E}+\underline{u}_e\times\underline{B}\right] - \nabla p_e - m_e n_e \bar{\nu}_{ei}(\underline{u}_e-\underline{u}_i)$$

$$+$$

$$m_i n_i\left[\underbrace{\frac{\partial}{\partial t}\underline{u}_i + (\underline{u}_i\cdot\underline{\nabla})\underline{u}_i}\right] \simeq e n_i\left[\underline{E}+\underline{u}_i\times\underline{B}\right] - \underline{\nabla} p_i - m_e n_e \bar{\nu}_{ei}(\underline{u}_i-\underline{u}_e)$$

$$\xleftarrow{\underline{\nabla} p}$$

$$\rho\frac{d\underline{u}}{dt} \simeq \underline{j}\times\underline{B} - \underline{\nabla} p$$

più pressione $\longrightarrow$ meno pressione

moto fluido

$-\underline{\nabla} p$

$$\underline{F} = I\,\underline{l}\times\underline{B}$$

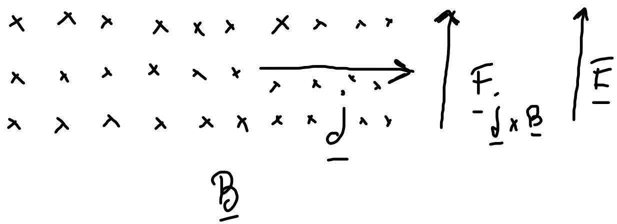
Caso stazionario $\quad \frac{d\underline{u}}{dt}=0 \qquad \underline{j}\times\underline{B} = \underline{\nabla} p \quad$ equilibrio

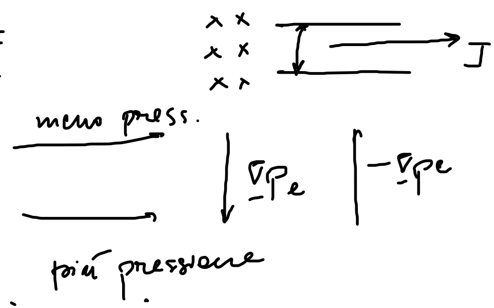
$$jB \simeq \frac{p}{L} \Rightarrow j \sim \frac{p}{LB} \sim \frac{nT}{LB} \sim \frac{n m v_{th}^2}{LB} \qquad \frac{j}{j_i} \sim \frac{\cancel{n} m v_{th}^{\cancel{2}}}{LB\,\underbrace{e\cancel{n}\cancel{v_{th}}}_{j_i}} \sim \frac{r_{Li}}{L} \ll 1$$

Cons. q.tà moto per elettroni; isolo $\underline{E} + \underline{u}_e \times \underline{B} \approx \underline{E} + \underline{u} \times \underline{B}$

$$\underbrace{\underline{E} + \underline{u} \times \underline{B}}_{\underline{E}\ \text{plasma}} \approx \frac{1}{e n_e}\Big[\underbrace{\underline{j} \times \underline{B}}_{\text{effetto Hall}} - \underbrace{\cancel{\underline{\nabla} p_e}}_{\text{gradiente pressione}}\Big] + \underbrace{\eta \underline{j}}_{\text{collisioni}}$$

$\eta = \dfrac{m_e \nu_{ei}}{n_e e^2}$ resistività





Termine Diamagnetico

$$\frac{\nabla p}{e n e\, v B} \sim \frac{p}{e n k\, v B} \sim \frac{j}{e n v} \ll 1$$

Effetto Hall

$$\frac{j\cancel{B}}{\underbrace{e n_e v}_{j_i}\cancel{B}} \ll 1$$

$$\underline{E} + \underline{u} \times \underline{B} \approx \eta \underline{j}$$

Se $\eta \approx 0$ (MHD ideale) $\underline{E} + \underline{u} \times \underline{B} = 0$

$$\Rightarrow E_{\parallel} = 0$$

$$\underline{u} \times \underline{B} \perp \underline{B}$$

Se $\eta \neq 0$ $\underline{E} + \underline{u} \times \underline{B} = \eta \underline{j}$ (MHD resistiva)

$$\frac{dP}{dV} = \underline{j} \cdot \left[\underline{E} + \cancel{(\underline{u} \times \underline{B})} \right] = \underline{j} \cdot \underline{E}$$

$$\underline{F} = q(\underline{E} + \underline{u} \times \underline{B})$$

$$dP = \frac{\delta L}{dt} = \underline{F} \cdot \frac{d\underline{s}}{dt} = \underline{F} \cdot \underline{u} = q\underline{u}(\underline{E} + \underline{u} \times \underline{B})$$

$$\frac{d\rho}{dt} + \rho\,\underline{\nabla}\cdot\underline{u} = 0$$ continuità

$$\rho\frac{d\underline{u}}{dt} = \underline{j}\times\underline{B} - \underline{\nabla}p$$ cons. q.tà moto

$$\underline{E} + \underline{u}\times\underline{B} = 0$$ legge di Ohm

$$\underline{E} + \underline{u}\times\underline{B} = \eta\,\underline{j}$$

$p,\ \rho,\ \underline{u},\ \underline{j},\ \underline{E},\ \underline{B}$

$$\underline{\nabla}\times\underline{E} = -\frac{\partial\underline{B}}{\partial t} \qquad \underline{\nabla}\times\underline{B} = \mu_0\underline{j}$$ eq. Maxwell

$$\underline{\nabla}\cdot\underline{B} = 0 \qquad n_e = n_i$$

$$\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0$$ eq. stato

$\tau \sim \frac{L}{v_{th,i}}$

eg Plasma da fusione $T \sim keV$

$v_{th} \sim 10^5\ m/s$

$L \sim$ diversi m

$\tau \sim$ qualche s — qualche h

$\tau_{conf} \sim s$

$\tau_{MHD} \sim$ qualche μs

$\omega_c \sim ns$

Plasma freddo $T \sim eV$ $\tau_{MHD} \sim 10^{-4}\ s$ $\tau_{ap} \sim$ minuti/h