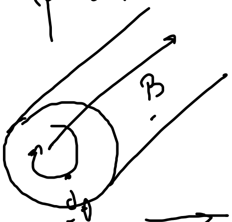
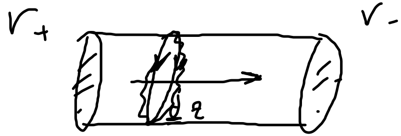
$$\nabla p = \underline{j} \times \underline{B} \quad \Rightarrow \quad \nabla_\perp \left(p + \frac{B^2}{2\mu_0} \right) - \frac{B^2}{\mu_0}\hat{\underline{k}} = 0$$

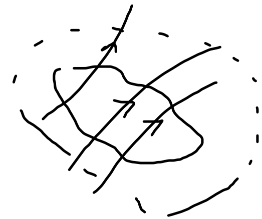
θ pinch



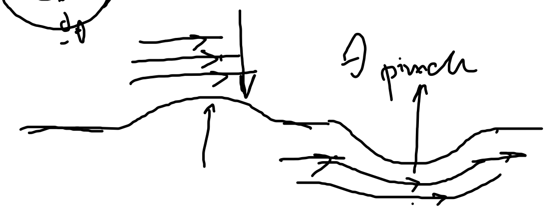
z pinch



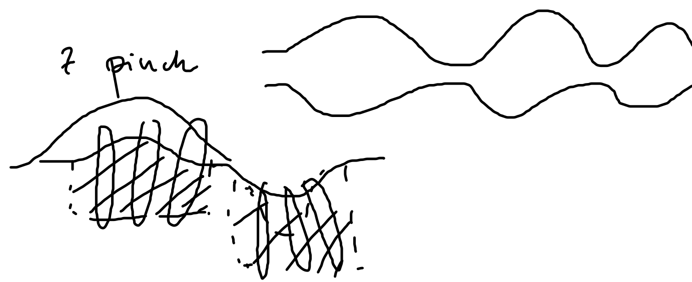
$\phi = \text{const}$ (legge di Gelo)



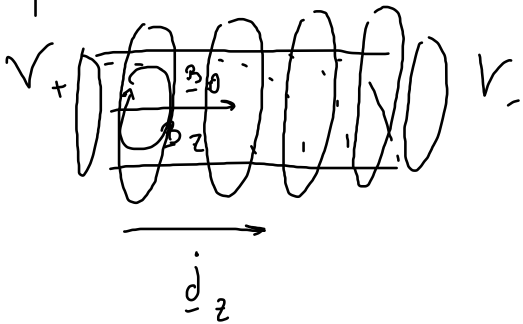
θ pinch



z pinch



Screw pinch



$$\underline{\nabla}_\perp \left(p + \frac{B^2}{2\mu_0} \right) - \frac{B^2}{\mu_0} \hat{\underline{\kappa}} = 0 \qquad \hat{\underline{\kappa}} = (\hat{\underline{b}} \cdot \underline{\nabla}) \hat{\underline{b}}$$

$$\underline{B} = B_\theta(r)\, \hat{\underline{e}}_\theta + B_z(r)\, \hat{\underline{e}}_z$$

$$B^2 = B_\theta^2(r) + B_z^2(r)$$

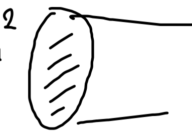
$$\hat{\underline{\kappa}} = \left[\underbrace{\frac{1}{B} \left(B_\theta(r)\, \hat{\underline{e}}_\theta + B_z(r)\, \hat{\underline{e}}_z \right)}_{\hat{\underline{b}}} \right] \cdot \underline{\nabla} \left[\frac{1}{B} \left(B_\theta(r)\, \hat{\underline{e}}_\theta + B_z(r)\, \hat{\underline{e}}_z \right) \right]$$

$$= \left[\frac{B_\theta}{B} \frac{1}{r} \frac{\partial}{\partial \theta} + \frac{B_z}{B} \frac{\partial}{\partial z} \right] \left[\frac{1}{B} \left(B_\theta(r)\, \hat{\underline{e}}_\theta + B_z(r)\, \hat{\underline{e}}_z \right) \right]$$

$$= \frac{1}{B^2}\left[\frac{B_\theta^2(r)}{r}\underbrace{\frac{\partial}{\partial\theta}\hat{e}_\theta}_{-\hat{e}_r}\right] = -\frac{B_\theta^2}{B^2}\frac{1}{r}\hat{e}_r$$

$$\underbrace{\hat{e}_r\frac{\partial}{\partial r}}_{\nabla_\perp}\left[p + \frac{B_\theta^2}{2\mu_0} + \frac{B_z^2}{2\mu_0}\right] + \cancel{\left[\frac{B_\theta^2 + B_z^2}{\mu_0}\right]}\cdot\frac{B_\theta^2}{\cancel{B^2}}\frac{1}{r}\hat{e}_r = 0$$

πa^2



$$\frac{\partial}{\partial r}\left[p + \frac{B_\theta^2}{2\mu_0} + \frac{B_z^2}{2\mu_0}\right] + \frac{B_\theta^2}{\mu_0 r} = 0$$

$$\beta = \frac{2\mu_0 p}{B^2}$$

θ pinch: $0 < \beta < 1$

z pinch: $\beta = 2$

$$\frac{1}{r^2}\int_0^a dr\cdot r^2\left[\frac{\partial}{\partial r}\left[p+\frac{B_\theta^2}{\cancel{2\mu_0}}+\frac{B_z^2}{\cancel{2\mu_0}}\right]+\frac{B_\theta^2}{\cancel{\mu_0} r}\right]=0$$

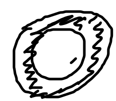
$$\int_0^a dr\, r^2\,\frac{dp}{dr} = \cancel{r^2 p(r)}\Big|_0^a \underbrace{-2\int_0^a dr\, r\, p(r)}_{-a^2\langle p\rangle} = -a^2\langle p\rangle$$

inf.

f $\qquad g'$ per parti

$$\langle p\rangle = \int_0^a \frac{dr\; p(r)\, r\, 2\cancel{\pi}}{\cancel{\pi} a^2} = \frac{2}{a^2}\int_0^a dr\, p(r)\, r$$

$$\int \frac{p(r)\cdot dS}{\pi a^2}$$



$p(r)$ è costante su anelli

$$dS = \pi\left[r+dr\right]^2 - \pi r^2 = 2\pi r\, dr + O(dr^2)$$

$$-\langle p\rangle - \frac{1}{2\mu_0}\left(\langle B_z^2\rangle - B_z^2(a)\right) + \frac{B_\theta^2(a)}{2\mu_0} = 0$$

$$\langle p\rangle = \frac{1}{2\mu_0}\left[B_z^2(a) - \langle B_z^2\rangle\right] + \frac{B_\theta^2(a)}{2\mu_0}$$

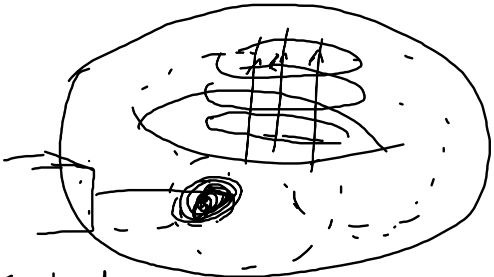
$$\beta = \frac{\langle p\rangle}{\frac{B^2(a)}{2\mu_0}} = \frac{2\mu_0\langle p\rangle}{B_z^2(a) + B_\theta^2(a)}$$

$$\frac{1}{\beta} = \frac{1}{\beta_t} + \frac{1}{\beta_p}$$

$$\frac{1}{\beta_t} = \left(\frac{2\mu_0\langle p\rangle}{B_z^2(a)}\right)^{-1}, \quad \frac{1}{\beta_p} = \left(\frac{2\mu_0\langle p\rangle}{B_\theta^2(a)}\right)^{-1}$$

$$\beta = \frac{B_z^2(a) + B_\theta^2(a)}{B_z^2(a) + B_\theta^2(a)} - \frac{\langle B_z^2\rangle}{B_z^2(a) + B_\theta^2(a)} = 1 - \frac{\langle B_z^2\rangle}{B_z^2(a) + B_\theta^2(a)} \leq 1$$

$$0 < \beta < 1$$



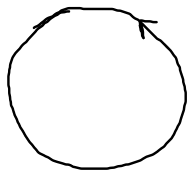
Neutral beam injection $\simeq 100$ keV
$P \simeq 30$ MW

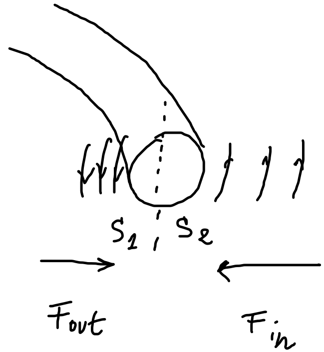
$P \simeq 50$ MW
$E \sim 1$ MeV

$\left\{ \begin{array}{l} \text{SPIDER} \\ \text{MITICA} \end{array} \right.$ Padova

$B_{TOR} \propto \frac{1}{R}$

Hoop force



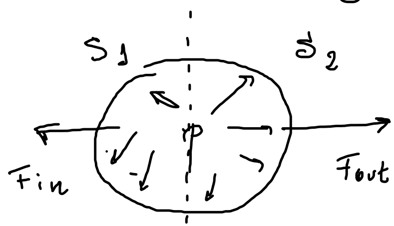


$$F_{out} \simeq \frac{B_{out}^2}{2\mu_0} \cdot S_1$$

$$F_{in} \simeq \frac{B_{in}^2}{2\mu_0} S_2$$

$F_{out} > F_{in} \Rightarrow$ allungamento vs ext

Forza da tubo pneumatico



$F_{in} = p\, S_1$

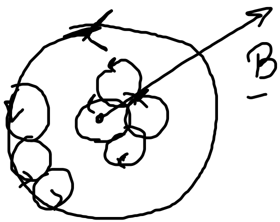
$F_{out} = p\, S_2$

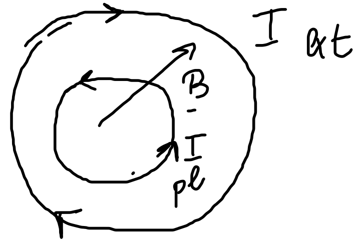
Per un toro $S_1 < S_2$

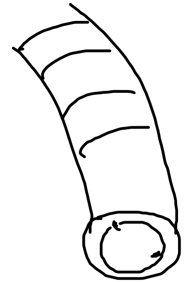
$F_{out} > F_{in}$

Forza $\frac{1}{R}$

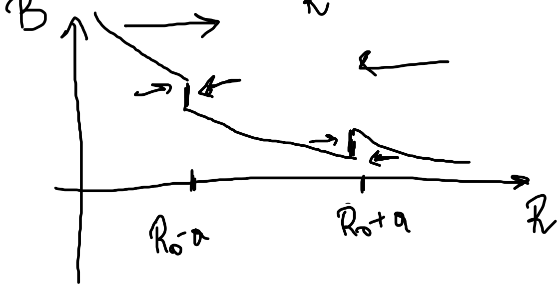
$B_{tor} \propto \frac{1}{R}$ $\qquad B(R) = \frac{B_0 R_0}{R}$





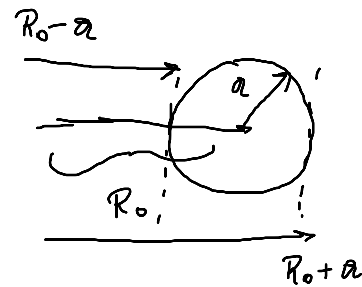


$B \propto \frac{1}{R}$



$B_{ut} = \frac{\mu_0 I_{bobine} N}{2\pi R}$ (th. ampere)

$B_{pl} = \frac{\mu_0}{2\pi R} I_{pl}$

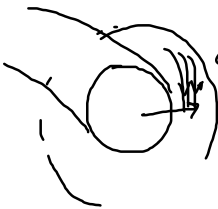


$$F_{out} = \left[\frac{B^2}{2\mu_0}(R_0 - r) - \frac{B^2(pl)}{2\mu_0} \right] S_1$$

$$F_{in} = \left[\frac{B^2}{2\mu_0}(R_0 + r) - \frac{B^2_{pl}}{2\mu_0} \right] S_2$$

$$F_{out} > F_{in}$$

Spinta verso la parete del plasma



Parete perfettamente concentrica

$$\frac{d\phi}{dt} = \int_C \left[\cancel{(\underline{u}_\perp \times \underline{B})} \cdot d\underline{l} + \cancel{\underline{B}(t) \cdot (\underline{u}_\perp \times d\underline{l})} \right]$$

$$= 0$$