Principio di indet.

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk\, \tilde{u}(k)\, e^{i(kx - \underline{\omega(k)}t)}$$

Al tempo $t=0$: $u(x)$ descrive il profilo dell'onda

$$u(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk\, \tilde{u}(k)\, e^{ikx}$$

$$\tilde{u}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx\, u(x)\, e^{-ikx}$$

Varianza di x

$$\sigma_x^2 = \langle x^2 \rangle - (\langle x \rangle)^2$$
$$= \langle (x - \langle x \rangle)^2 \rangle$$

Scegliendo opport. il S.R: $\langle x \rangle = 0$

$$\sigma_x^2 = \langle x^2 \rangle$$

allarg. $\Delta x = \sqrt{\sigma_x^2}$
$= \sqrt{\langle x^2 \rangle}$

$$\langle x^2 \rangle = \frac{\int_{-\infty}^{+\infty} dx\, x^2 u^2(x)}{\int_{-\infty}^{+\infty} dx\, u^2(x)}$$

$$\sigma_x = \sqrt{\frac{\int_{-\infty}^{+\infty} dx\, x^2 u^2(x)}{\int_{-\infty}^{+\infty} dx\, u^2(x)}} = \sqrt{\frac{\|g\|^2}{\|u\|^2}}$$

$$u(x) \longrightarrow \tilde{u}(k)$$

$$\sigma_k = \sqrt{\frac{\int_{-\infty}^{+\infty} dk\, k^2 \tilde{u}^2(k)}{\int_{-\infty}^{+\infty} dk\, \tilde{u}^2(k)}} = \sqrt{\frac{\|\tilde{g}\|^2}{\|\tilde{u}\|^2}}$$

$$g(x) = x \cdot u(x)$$

$$\tilde{g}(k) = k\, \tilde{u}(k)$$

$$\langle g | g \rangle = \int_{-\infty}^{+\infty} dx\, g(x)\, g^*(x)$$

$$\|g\| = \langle g | g \rangle$$

1) Teorema di Parseval

$$\langle f | f \rangle = \langle \tilde{f} | \tilde{f} \rangle$$

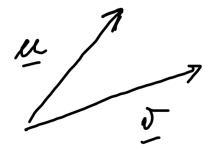
$$\int dx\, \underline{f^2(x)} = \int dk\, \tilde{f}^2(k)$$

$\propto$ energia contenuta nel pacchetto $\quad \frac{1}{2}\varepsilon_0 \int dV\, E^2(\underline{r})$

2) Disuguaglianza di Schwartz

$$|\langle f | g \rangle| \le \|f\| \, \|g\|$$

es



$$\langle \underline{u} | \underline{v} \rangle = uv \cos\theta$$

$$\le uv$$

$$\sigma_x^2 \sigma_k^2 = \frac{\langle g|g\rangle}{\|u\|^2} \cdot \frac{\langle \tilde{g}|\tilde{g}\rangle}{\|\tilde{u}\|^2} = \frac{\langle g|g\rangle \langle g|g\rangle}{\|u\|^2 \cdot \|u\|^2} = \frac{\|g\|^2 \cdot \|g\|^2}{\|u\|^2 \cdot \|u\|^2} \geq \frac{\|\langle g|g\rangle\|^2}{\|u\|^2 \cdot \|u\|^2}$$

Th Parseval

dis. Schwartz

Trovare l'anti trasf. di $\tilde{g}(k) = k \cdot \tilde{u}(k)$

$$g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk\, k\, \tilde{u}(k)\, e^{ikx}$$

$$\frac{1}{-ik} \frac{d}{d\xi} \left(e^{-ik\xi} \right)$$

$$\tilde{u}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\xi\, u(\xi)\, e^{-ik\xi} = \cancel{\frac{-1}{ik} u(\xi) e^{-ik\xi} \Big|_{-\infty}^{+\infty}} + \frac{1}{ik \sqrt{2\pi}} \int_{-\infty}^{+\infty} d\xi \frac{du}{d\xi} e^{-ik\xi}$$

int. per parti

$$= \frac{1}{ik} \widetilde{\frac{du}{dx}}$$

$$g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk\, \cancel{k} \frac{1}{i\cancel{k}} \widetilde{\frac{du}{dx}}\, e^{ikx} = -i \frac{du}{dx}$$

$$\langle f|g\rangle = i \int_{-\infty}^{+\infty} dx\; x \underbrace{u(x) \frac{du}{dx}}_{\frac{1}{2}\frac{du^2}{dx}} = \frac{i}{2} \int_{-\infty}^{+\infty} dx\; x \frac{du^2}{dx} = \frac{i}{2} \left[\cancel{x u^2 \Big|_{-\infty}^{+\infty}} - \int_{-\infty}^{+\infty} dx\, u^2 \right]$$

per parti

$$= -\frac{i}{2} \langle u^2 \rangle = -\frac{i}{2} \|u\|^2$$

$$\sigma_x^2 \sigma_k^2 \geq \frac{\|\langle f|g\rangle\|^2}{\|u\|^2 \|u\|^2} = \frac{1}{4} \frac{\cancel{\|u\|^2} \cancel{\|u\|^2}}{\cancel{\|u\|^2} \cancel{\|u\|^2}} \;;\quad \boxed{\sigma_x \sigma_k \geq \frac{1}{2}}$$

Se : De Broglie

$$p = \hbar k \Rightarrow \sigma_p = \hbar \sigma_k$$

$$\boxed{\sigma_x \sigma_p \geq \frac{\hbar}{2}}$$

$\omega = \omega(k)$

1) $\exists$ uno stato di eq. per un plasma uniforme e infinitamente esteso

2) La rappr. per onde monocromatiche è corretta

3) Onda è una piccola pert. dell'equilibrio (teoria lineare)

$$Q(x,t) = \underbrace{Q_0(x,t)}_{\text{Valore all'eq.}} + \underbrace{Q_1(x,t)}_{\text{Perturbazione dovuta all'onda}} \qquad Q_1 \ll Q_0$$

a) Onde in modello a 2 fluidi se $B = 0$

b) Onde in MHD se $B \neq 0$

a) Risolvere sistema accoppiato $\rightarrow$ eq. Maxwell
$\rightarrow$ eq. fluide
(modello di plasma)

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\varepsilon_0} \qquad \underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad \underline{\nabla} \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

Equilibrio:

$$\underline{B}_0 = \underline{0} \qquad \underline{E}_0 = \underline{0}$$

$$\rho_0 = 0 \qquad \underline{j}_0 = \underline{0}$$

$$\nabla \cdot \mathbf{E}_1 = \rho_1/\varepsilon_0 \qquad \nabla \times \mathbf{E}_1 = -\frac{\partial \mathbf{B}_1}{\partial t}$$

$$\nabla \cdot \mathbf{B}_1 = 0 \qquad \nabla \times \mathbf{B}_1 = \mu_0 \mathbf{j}_1 + \varepsilon_0\mu_0 \frac{\partial \mathbf{E}_1}{\partial t}$$

Modi normali:

$$Q \propto e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} \qquad \nabla Q = i\mathbf{k} Q \qquad \nabla \cdot \vec{Q} = i\vec{k}\cdot\vec{Q}$$

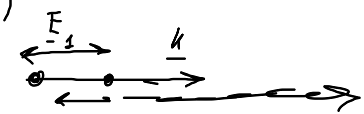
$$\nabla \times \vec{Q} = i\mathbf{k} \times \vec{Q}$$

Nel vuoto: $\mathbf{E} \perp \mathbf{B} \perp \mathbf{k}$

$\Rightarrow \mathbf{k}\cdot\mathbf{E} = 0 \qquad \omega/k = c$

$$i\mathbf{k}\cdot\mathbf{E}_1 = \rho_1/\varepsilon_0$$

- trasversali (e.m.) $\mathbf{k}\cdot\mathbf{E}_1 = 0$ $(\rho_1 = 0)$
- onde long. $\mathbf{k}\cdot\mathbf{E}_1 \neq 0$



Eq. continuité

Eq. n_{0j} uniforme

$$\underline{u}_{0j} = \underline{0}$$

$$\frac{\partial n_j}{\partial t} + \underline{\nabla} \cdot (n_j \underline{u}_j) = 0$$

$$\frac{\partial n_{1j}}{\partial t} + \underline{\nabla} \cdot \left[(n_{0j} + n_{1j}) \cdot (\underline{u}_{1j}) \right] = 0$$

$$\frac{\partial n_{1j}}{\partial t} + n_{0j} \underline{\nabla} \cdot \underline{u}_{1j} = 0$$

eq. cons. momentum

$$m_j n_j \left[\frac{\partial}{\partial t} + (\underline{u}_j \cdot \underline{\nabla}) \right] \underline{u}_j = n_j q_j \left(\underline{E} + \underline{u}_j \times \underline{B} \right) - \underline{\nabla} p_j$$

$$m_j \left[(n_{0j} + \cancel{n_{1j}}) \right] \left[\frac{\partial}{\partial t} + \cancel{(\underline{u}_{0j} + \underline{u}_{1j}) \cdot \underline{\nabla}} \right] (\cancel{\underline{u}_{0j}} + \underline{u}_{1j}) = (n_{0j} + \cancel{n_{1j}}) \left(\underline{E}_1 + \cancel{(\underline{u}_{0j} + \underline{u}_{1j}) \times \underline{B}_1} \right) - \underline{\nabla} (\cancel{p_{0j}} + p_{1j})$$

$$m_j n_{0j} \frac{\partial}{\partial t} \underline{u}_{1j} = n_{0j} q_j \underline{E}_1 - \underline{\nabla} p_{1j}$$ (cons. momento al $\underline{I}$ ordine)