Onde longitudinali

$\rho_1 \neq 0$

$\underline{k} \cdot \underline{E}_1 \neq 0 \qquad \underline{\nabla} \cdot \underline{E}_1 = \rho_1 / \varepsilon_0 \qquad \underline{\nabla} \to i \underline{k}$

$\underline{B}_0 = \underline{0} \qquad i\, \underline{k} \cdot \underline{E}_1 = \rho_1/\varepsilon_0 \neq 0$

Eq. continuità:

$$\frac{\partial n_{1j}}{\partial t} + n_{0j} \underline{\nabla} \cdot \underline{u}_{1j} = 0$$

Eq. cons. momento lineare:

$$m_j n_{0j} \frac{\partial}{\partial t} \underline{u}_{1j} = n_{0j} q_j \underline{E}_1 - \underline{\nabla} P_{1j}$$

Eq. stato

$$\frac{\underline{\nabla} p_j}{p_j} = \gamma \frac{\underline{\nabla} n_j}{n_j}$$

$\gamma = \frac{2+N}{N}$ N : # gradi di libertà (adiabatico) associato al processo

$\gamma = 1$ processo isotermico

$$\frac{\underline{\nabla} p_j}{p_j} = \gamma \frac{\underline{\nabla} n_j}{n_j}$$

$$n_j = n_{0j} + n_{1j}$$

$$p_j = p_{0j} + p_{1j}$$

$$n_{1j}\ \underline{u}_{1j}\ p_{1j}\ \underline{E}_1$$

$$\frac{\underline{\nabla}(\cancel{p_{0j}} + p_{1j})}{p_{0j} + \overline{p_{1j}}} = \gamma \frac{\underline{\nabla}(\cancel{n_{0j}} + n_{1j})}{n_{0j} + \overline{n_{1j}}} \; ; \quad \frac{\underline{\nabla} p_{1j}}{p_{0j}} = \gamma \frac{\underline{\nabla} n_{1j}}{n_{0j}}$$

1^2 0   12 0

$$\frac{\partial n_{1j}}{\partial t} + n_{0j}\,\underline{\nabla}\cdot\underline{u}_{1j} = 0$$

Modi normali

$$-i\omega n_{1j} + n_{0j}\, i\,\underline{k}\cdot\underline{u}_{1j} = 0 \qquad n_{1j} = \frac{n_{0j}\,\underline{k}\cdot\underline{u}_{1j}}{\omega}$$

$$m_j n_{0j} \frac{\partial}{\partial t} \underline{u}_{1j} = n_{0j} q_j \underline{E}_1 - \underline{\nabla} p_{1j}$$

Modi normali

$$\underline{\nabla} p_{1j} = \gamma p_{0j} \frac{\underline{\nabla} n_{1j}}{n_{0j}}$$

$$= i \gamma p_{0j} \frac{\underline{k} n_{1j}}{n_{0j}}$$

modi norm.

$$-i\omega m_j n_{0j} \underline{u}_{1j} = n_{0j} q_j \underline{E}_1 - i\gamma p_{0j} \frac{\underline{k} n_{1j}}{n_{0j}}$$

Moltiplico $\cdot \underline{k}$

$$-i\omega m_j n_{0j} (\underline{k} \cdot \underline{u}_{1j}) = n_{0j} q_j \underline{k} \cdot \underline{E}_1 - i\gamma \frac{p_{0j} k^2 n_{1j}}{n_{0j}}$$

Eq. continuità: $\underline{k} \cdot \underline{u}_{1j} = \dfrac{\omega n_{1j}}{n_{0j}} \Rightarrow -i\omega^2 m_j n_{1j} = n_{0j} q_j \underline{k} \cdot \underline{E}_1 - i\gamma \dfrac{p_{0j} k^2 n_{1j}}{n_{0j}}$

$$n_{1j} = \frac{\underline{k} \cdot \underline{E}_1 n_{0j} q_j}{i} \cdot \frac{1}{\frac{k^2 \gamma p_{0j}}{n_{0j}} - \omega^2 m_j}$$

$$p_0 = n_0 T_0$$

$$\rho_1 = \sum_j n_{1j}\, q_j = \sum_j \frac{n_{0j}\, q_j^2}{i\, m_j} \, \frac{\underline{k}\cdot\underline{E}_1}{\frac{\gamma k^2 T_{0j}}{m_j} - \omega^2}$$

Eq. Poisson:

$$\cancel{i}\,\underline{k}\cdot\underline{E}_1 = \rho_1 / \varepsilon_0 = \sum_j \frac{\cancel{i}\, n_{0j}\, q_j^2}{m_j\, \varepsilon_0} \, \frac{\underline{k}\cdot\underline{E}_1}{\omega^2 - \frac{\gamma k^2 T_{0j}}{m_j}}$$

$$\underline{k}\cdot\underline{E}_1 \left[1 - \sum_j \underbrace{\frac{n_0\, q_j^2}{m_j\, \varepsilon_0}}_{\omega_{pj}^2} \, \frac{1}{\omega^2 - \frac{\gamma k^2 T_{0j}}{m_j}} \right] = 0$$

$$\underbrace{\underline{K}\cdot\underline{\underline{\varepsilon}}_1}_{\neq 0}\ \underbrace{\left[1-\sum_j \frac{\omega^2_{p_j}}{\omega^2-\dfrac{\gamma_j K^2 T_{0j}}{m_j}}\right]}_{=0} = 0$$

$\omega = \omega(k)$

Se $T_{0j} = 0$

$$1-\sum_j \frac{\omega^2_{p_j}}{\omega^2} = 0;$$

Una specie ionica $(q_i > 0)$ + elettroni

$n_e \sim n_i$

$n_e = Z n_i$

$$\omega^2_p = \frac{n_j q^2_j}{m_j \varepsilon_0}$$

$$\omega^2_{p_e} \gg \omega^2_{p_i}$$

$$1-\frac{\omega^2_{p_i}}{\omega^2}-\frac{\omega^2_{p_e}}{\omega^2} = 0;$$

$$\omega^2 = \omega^2_{p_e} + \omega^2_{p_i}$$

$$\Rightarrow \omega \approx \omega_{p_e}$$

Se $T_{0j} \neq 0$

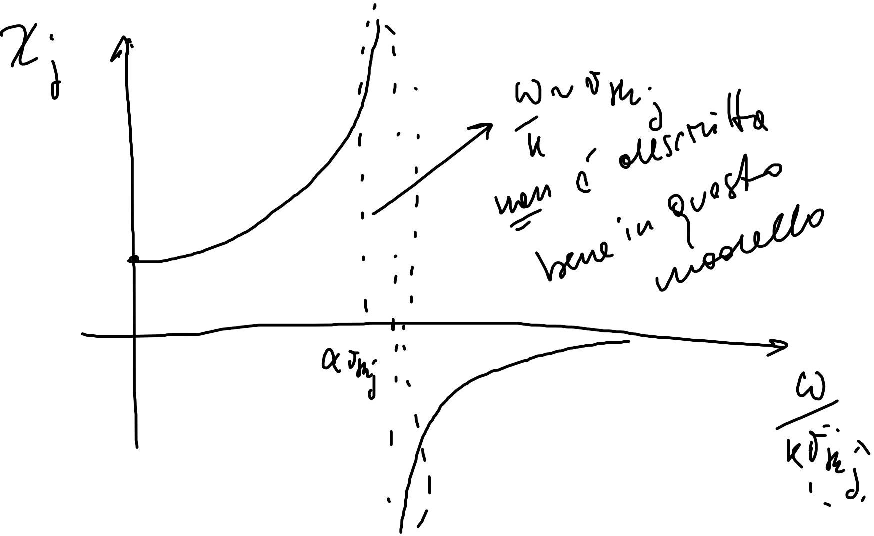
$$\frac{1}{2} m v_{th_j}^2 = T_{0j} \;;\; \frac{T_{0j}}{m_j} = \frac{1}{2} v_{th_j}^2$$

$$\frac{\omega_{p_j}^2}{\omega^2 - \frac{\gamma}{2} v_{th_j}^2 \cdot k^2} = \chi_j \qquad \underline{\text{suscettività}}$$

In dielettrico: $\underline{\nabla} \cdot \underline{D} = 0 \qquad \underline{D} = \varepsilon \underline{E} = \varepsilon_0 \varepsilon_r \underline{E}$

$$i \underline{k} \cdot \underline{D} = 0 \qquad \underline{D} = \varepsilon_0 \underline{E} + \underline{P} = \varepsilon_0 \underline{E} + \varepsilon_0 \underbrace{\chi_r} \underline{E} = \varepsilon_0 \underline{E} (1 + \chi_r)$$

$$i \varepsilon_0 \underline{k} \cdot \underline{E} \underline{(1 + \chi_r)} = 0 \Rightarrow 1 + \chi_r = 0 \qquad \chi_r \stackrel{def}{=} \sum_j \chi_j$$



$\frac{\omega}{k} = v_{fase}$

$\frac{\omega}{k} \ll v_{th_j}$ $\qquad T_e \sim T_i$

$\frac{\omega}{k} \gg v_{th_j}$ $\qquad v_{th_e} \gg v_{th_i}$

a) $\frac{\omega}{k} \ll v_{th,e}, v_{th,i}$

b) $v_{th,i} \ll \frac{\omega}{k} \ll v_{th,e}$

c) $\frac{\omega}{k} \gg v_{th_e}, v_{th_i}$

$$1+\chi_e+\chi_i=0$$

$$\chi_e=\frac{\omega_{pe}^2}{\omega^2-\frac{\gamma_e}{2}v_{the}^2k^2}\qquad \chi_i=\frac{\omega_{pi}^2}{\omega^2-\frac{\gamma_i}{2}k^2v_{the}^2}$$

a) $\frac{\omega}{k}\ll v_{the},v_{thi}$ $\qquad \gamma_e=\gamma_i=1$ (risposta isoterma di i ed e)

$$\chi_e=\frac{-\omega_{pe}^2}{k^2\left[\frac{\omega^2}{k^2}-\frac{v_{the}^2}{2}\right]}\approx\frac{+2\omega_{pe}^2}{k^2v_{the}^2}\approx+\cancel{2}\frac{n_ee^2}{\cancel{m_e}\varepsilon_0}\frac{\cancel{m_e}}{k^2\cancel{2}T_{e0}}\approx+\frac{1}{k^2\lambda_{De}^2}$$

$$\frac{1}{\lambda_{De}^2}=\frac{n_ee^2}{\varepsilon_0T_{e0}}\quad \text{def}\quad \frac{1}{\lambda_D^2}\qquad \left[1+\frac{1}{k^2\lambda_D^2}\right]=0,$$

$$\Phi_1\propto e^{-x/\lambda_D}e^{-i\omega t}\qquad \chi_i\approx+\frac{1}{k^2\lambda_{Di}^2}\qquad 1+\chi_r=0;\quad 1+\frac{1}{k^2}\underbrace{\left(\frac{1}{\lambda_{Di}^2}+\frac{1}{\lambda_{De}^2}\right)}=0,\quad k=-i\lambda_D$$

b) $v_{thi} \ll \frac{\omega}{k} \ll v_{the}$

$$\chi_e \simeq \frac{2\omega_{pe}^2}{k^2 v_{the}^2} \qquad \chi_i \sim \frac{-\omega_{pi}^2}{\omega^2\left(1-\frac{3}{2}v_{thi}^2\frac{k^2}{\omega^2}\right)}$$

$\gamma_e = 1$

$$\gamma_i = \frac{2+N}{N} = 3 \quad N=1$$

risp. adiab.

$$\simeq \frac{-\omega_{pi}^2}{\omega^2}\left(1+\frac{3}{2}v_{thi}^2\frac{k^2}{\omega^2}\right)$$

$1+\chi_n = 0;$

$$1+\frac{1}{k^2\lambda_{De}^2}-\frac{\omega_{pi}^2}{\omega^2}\left(1+\frac{3}{2}v_{thi}^2\frac{k^2}{\omega^2}\right)=0; \qquad \frac{k^2\lambda_{De}^2+1}{k^2\lambda_{De}^2} = \frac{\omega_{pi}^2}{\omega^2}\left(\qquad\right)$$

$$\omega^2 = \frac{k^2\omega_{pi}^2\lambda_{De}^2}{1+k^2\lambda_{De}^2}\left(1+\frac{3}{2}v_{thi}^2\,k^2/\omega^2\right)$$

$$\underbrace{\omega_{pi}^2}_{s^{-2}} \underbrace{\lambda_{De}^2}_{m^2} = \frac{\cancel{n_i e^2}}{m_i \cancel{\varepsilon_0}} \cdot \frac{\cancel{\varepsilon_0} T_e}{\cancel{n_e} \cancel{e^2}} = \frac{T_e}{m_i} \overset{def}{=} c_S^2 \qquad c_S = \sqrt{\frac{\gamma T}{m}}$$

$$\omega^2 = \frac{k^2 c_S^2}{1+k^2\lambda_{De}^2}\left(1 + \frac{3k^2}{\omega^2}\frac{T_{io}}{m_i}\right) ; \quad \omega^2 \approx \frac{k^2 C_S^2}{1+k^2\lambda_{De}^2}$$

12

0

Se $\lambda_{De} >> \lambda \quad \omega^2 \approx \omega_{pi}^2$

Ordine 1: $\frac{\omega^2}{k^2} \approx \frac{C_S^2}{1+k^2\lambda_{De}^2}$

$$\omega^2 \approx \frac{k^2 C_S^2}{1+k^2\lambda_{De}^2} + \frac{3T_{io}}{m_i}k^2 \approx \omega_{pi}^2 + \frac{3k^2 T_{io}}{m_i}$$

Possibile solo se $T_e >> T_i$