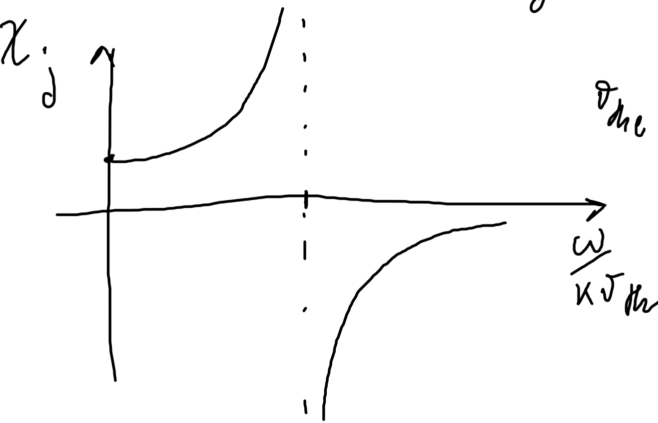
$$1 + \chi_L = 0 \qquad \chi_L = \sum_j \chi_j \qquad \chi_j = \frac{-\omega_{pj}^2}{\omega^2 - \gamma \frac{k^2 T_{0j}}{m_j}}$$



$v_{the} >> v_{thi}$

1) $\frac{\omega}{k} << v_{the}, v_{thi}$: Schermo di Debye

2) $v_{thi} << \frac{\omega}{k} << v_{the}$

elettroni isotermi $\gamma_e = 1$

ioni adiabatici $\gamma_i = 3$ $\quad c_s^2 = \frac{\gamma T}{m}$

$$\omega^2 = \frac{k^2 c_s^2}{1 + k^2 \lambda_{De}^2} \left(1 + \frac{3k^2}{\omega^2} \frac{T_{i0}}{m_i} \right) \qquad c_s^2 = \omega_{pi}^2 \lambda_{De}^2 = \frac{T_e}{m_i}$$

vel. ione-acustica

Ordine 0:

$$\omega^2 \simeq \frac{k^2 c_s^2}{1+k^2\lambda_{De}^2}$$

Se $k^2\lambda_{De}^2 \gg 1$

$\lambda_{De} \gg \lambda$

$$\omega^2 \simeq c_s^2/\lambda_{De}^2 = \omega_{pi}^2$$

Ordine 1:

$$\omega^2 \simeq \frac{k^2 c_s^2}{1+k^2\lambda_{De}^2}\left(1 + \frac{3T_{i0}}{m_i}\frac{1+k^2\lambda_{De}^2}{c_s^2}\right)$$

$\Rightarrow$ $\frac{T_{e0}}{\cancel{m_i}} \gg \frac{T_{i0}}{\cancel{m_i}}$

$$\simeq \frac{k^2 c_s^2}{1+k^2\lambda_{De}^2} + \frac{3T_{i0}}{m_i}k^2$$

$c_s^2 \gg T_{i0}/m_i$

Se __non__ succede

$$\omega^2 \simeq \frac{3k^2 T_{i0}}{m_i} \; ; \; \frac{\omega}{k} \simeq v_{thi}$$

3) $\frac{\omega}{k} \gg v_{th_e} \gg v_{th_i}$ fenomeno rapido sia per e, sia per i

adiabatiche $\gamma_e = \gamma_i = \frac{2+N}{N} = 3$ $\quad N = 1$

$$\chi_j = \frac{-\omega_{p_j}^2}{\omega^2 - 3k^2 \frac{T_{0j}}{m_j}} = \frac{-\omega_{p_j}^2}{\omega^2\left(1 - \frac{3T_{0j}/m_j}{\omega^2/k^2}\right)} = \frac{-\omega_{p_j}^2}{\omega^2}\left(1 + 3T_{0j}/m_j \cdot \frac{k^2}{\omega^2}\right)$$

$$\frac{1}{1-\varepsilon} \simeq 1 + \varepsilon$$

$$= \frac{-\omega_{p_j}^2}{\omega^2}\left(1 + \frac{3}{2} v_{th_j}^2 \frac{k^2}{\omega^2}\right)$$

$1 + \sum_j \chi_j = 0$; $\quad 1 + \chi_e + \chi_i = 0$;

$$1 - \frac{\omega_{pe}^2}{\omega^2}\left(1 + \underbrace{\frac{3}{2} v_{the}^2 \frac{k^2}{\omega^2}}_{\ll 1}\right) - \frac{\omega_{pi}^2}{\omega^2}\left(1 + \underbrace{\frac{3}{2} v_{thi}^2 \frac{k^2}{\omega^2}}_{\ll 1}\right) = 0$$

Ordine 0 $\quad \frac{v_{th\,e,i}\, k}{\omega} \ll 1 \simeq 0$

$$1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \approx 0; \quad \omega^2 = \omega_{pe}^2 + \omega_{pi}^2 \approx \omega_{pe}^2$$

$$\omega_p^2 = \frac{ne^2}{m\varepsilon_0}$$

$$m_i \gg m_e \quad \underline{n_i \sim n_e} \Rightarrow \omega_{pe}^2 \gg \omega_{pi}^2$$

$$\underline{\omega^2 \approx \omega_{pe}^2 + \frac{3}{2} k^2 v_{th\,e}^2}$$

Unde Langmuir

Bohm-Gross

Ordine 1:

$$\omega^2 = \omega_{pe}^2 \left(1 + \frac{3}{2}\frac{v_{th\,e}^2 k^2}{\omega^2}\right) + \omega_{pi}^2 \left(1 + \frac{3}{2}\frac{v_{th\,i}^2 k^2}{\omega^2}\right)$$

$\omega \approx \omega_{pe}$

$$\omega^2 = \underbrace{\omega_{pe}^2 + \omega_{pi}^2}_{\simeq \omega_{pe}^2} + \frac{3}{2} k^2 v_{th\,e}^2 + \overbrace{\frac{\omega_{pi}^2}{\omega_{pe}^2}}^{\simeq 0} \frac{3}{2} k^2 v_{th\,i}^2$$

Onde e.m. (o trasversali)

$$\underline{\nabla} \cdot \underline{E}_1 = \frac{\rho_1}{\varepsilon_0} \; ; \quad i\,\underline{k} \cdot \underline{E}_1 = \rho_1 / \varepsilon_0$$

Se $\rho_1 \neq 0 \Rightarrow$ onde long. $\underline{k} \cdot \underline{E}_1 \neq 0$

$\underline{k} \parallel \underline{E}_1$

Se $\rho_1 = 0 \Rightarrow \underline{k} \cdot \underline{E}_1 = 0$

$$\underline{\nabla} \times \underline{E}_1 = -\frac{\partial \underline{B}_1}{\partial t} \qquad \underline{\nabla} \times \underline{B}_1 = \mu_0 \underline{j}_1 + \varepsilon_0 \mu_0 \frac{\partial \underline{E}_1}{\partial t} \qquad \underline{\nabla} \cdot \underline{B}_1 = 0$$

Ipotesi: $\underline{j}_1 = \sigma \underline{E}_1$

$$\cancel{i}\, \underline{k} \times \underline{E}_1 = \cancel{i}\, \omega \underline{B}_1 \qquad i\,\underline{k} \times \underline{B}_1 = \mu_0 \underline{j}_1 - i \omega \varepsilon_0 \mu_0 \underline{E}_1$$

$$\underline{B}_1 = \frac{\underline{k} \times \underline{E}_1}{\omega} \qquad \frac{i}{\omega} \left[\underline{k} \times (\underline{k} \times \underline{E}_1) \right] = \mu_0 \sigma \underline{E}_1 - i \omega \varepsilon_0 \mu_0 \underline{E}_1$$

$$\underline{\underline{M}} \cdot \underline{E}_1 = 0$$

$$\underline{k} \times (\underline{k} \times \underline{E}_1) = \underline{k} \underbrace{(\underline{k} \cdot \underline{E}_1)}_{\text{Onde trasv.} = 0} - k^2 \underline{E}_1 = -k^2 \underline{E}_1$$

$\omega \cdot$

$$-\frac{\cancel{i}}{\omega} k^2 \underline{E}_1 = \frac{\cancel{\mu_0 \sigma}}{i} \underline{E}_1 - \frac{i\omega}{c^2} \underline{E}_1 \quad \cdot \omega$$

$$\underline{E}_1 \left[k^2 - i \sigma \mu_0 \omega - \frac{\omega^2}{c^2} \right] = 0$$

$$k^2 - i \sigma \mu_0 \omega - \frac{\omega^2}{c^2} = 0 \qquad \text{(Rel. disp.)}$$

Se $\sigma = 0$ (no plasma) $\qquad \frac{\omega}{k} = c$

$\sigma = ?$

$$\underline{j}_1 = \sum_\alpha \underline{j}_{1\alpha} = \sum_\alpha n_{0\alpha} q_\alpha \underline{u}_{1\alpha}$$

eq. cons. momento lin.

$$m_\alpha n_{0\alpha} \frac{\partial \underline{u}_{1\alpha}}{\partial t} = n_{0\alpha} q_\alpha \underline{E}_1 - \underline{\nabla} p_{1\alpha}$$

$$\underline{\nabla} p_{1\alpha} = \gamma T_{0\alpha} \underline{\nabla} n_{1\alpha} = \gamma T_\alpha \, i \underline{k} n_{1\alpha} \quad \alpha \, \underline{k}$$

$\underline{j}_1 \perp \underline{k}$: eq. continuità

moltiplico $\cdot q_\alpha$ e $\sum_\alpha$

$$\frac{\partial n_{1\alpha}}{\partial t} + n_{0\alpha} \underline{\nabla} \cdot \underline{u}_{1\alpha} = 0$$

$$-i\omega n_{1\alpha} + n_{0\alpha} \, i \, \underline{k} \cdot \underline{u}_{1\alpha} = 0$$

$$-i\omega \sum_\alpha q_\alpha n_{1\alpha} + i\underline{k}\cdot \underbrace{\sum_\alpha q_\alpha n_{0\alpha} \underline{u}_{1\alpha}}_{\underline{j}_1} = 0 \Rightarrow \underline{k}\cdot \underline{j}_1 = 0$$

$$\rho_1 = 0 \quad \text{se onde trasv.} \qquad (\underline{k}\cdot\underline{E}_1 = 0 = \rho_1)$$

Per onde trasversali:

$$-i\omega m_\alpha \cancel{n_{0\alpha}} \underline{u}_{1\alpha} = q_\alpha \underline{E}_1 \cancel{n_{0\alpha}} \; ; \quad \underline{u}_{1\alpha} = \frac{i q_\alpha}{m_\alpha \omega} \underline{E}_1$$

$$\underline{j} = \sum_\alpha n_{0\alpha} q_\alpha \underline{u}_{1\alpha} = \sum_\alpha \frac{i n_{0\alpha} q_\alpha^2}{m_\alpha \omega} \underline{E}_1 \Rightarrow \sigma = \frac{i}{\omega} \sum_\alpha \frac{n_{0\alpha} q_\alpha^2}{m_\alpha}$$

$$k^2 - \frac{\omega^2}{c^2} - \cancel{i\mu_0\omega}\,\frac{i}{\cancel{\omega}}\sum_\alpha \frac{n_\alpha q_\alpha^2}{m_\alpha} = 0$$

$$\frac{n_\alpha q_\alpha^2}{m_\alpha} = \omega_{p_\alpha}^2 \cdot \varepsilon$$

$$k^2 - \frac{\omega^2}{c^2} + \sum_\alpha \frac{\omega_{p_\alpha}^2}{c^2} \; ;$$

$$\omega^2 = \underbrace{k^2c^2}_{\text{vuoto}} + \sum_\alpha \omega_{p_\alpha}^2$$

i, e

$$\omega^2 = k^2c^2 + \omega_{p_i}^2 + \omega_{p_e}^2 \simeq c^2k^2 + \omega_{p_e}^2$$

$$\omega \gg \omega_{pe} : \quad \frac{\omega}{k} \simeq c \; ; \quad \text{(prop. come in vuoto)}$$

$$\omega \sim \omega_{pe}$$

$$\underline{\underline{\omega^2 - \omega_{pe}^2 = c^2k^2}}$$

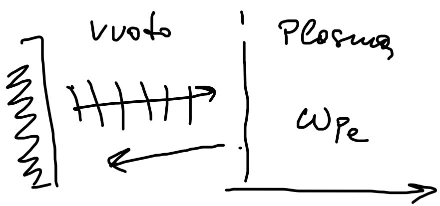
Nel vuoto: $k = \frac{\omega}{c}$ $\quad$ k è reale

Nel plasma: $k^2 = \frac{\omega^2 - \omega_{pe}^2}{c^2}$

$k^2 > 0$ solo se $\omega > \omega_{pe}$ $\therefore$ $k \in \mathbb{R}$ $\quad k = \frac{\sqrt{\omega^2 - \omega_{pe}^2}}{c}$

$k^2 < 0$ se $\omega < \omega_{pe}$: k è immaginario

$$e^{-kx} = e^{-x/L} \qquad L = \frac{1}{k} = \frac{c}{\sqrt{\omega_{pe}^2 - \omega^2}}$$



Se $\omega > \omega_{pe}$ onda passa nel plasma

Se $\omega < \omega_{pe}$ $\qquad \omega \to \omega_{pe}$

$$v_f = \omega/k = \frac{\omega c}{\sqrt{\omega^2 - \omega_{pe}^2}} \longrightarrow +\infty$$

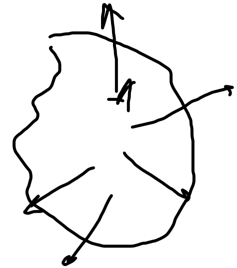
$$\omega \to \omega_{pe}$$

$$v_g = \frac{d\omega}{dk} = \frac{1}{\frac{dk}{d\omega}} = \frac{c}{\omega}\sqrt{\omega^2-\omega_{pe}^2} \longrightarrow 0$$

$$k = \frac{\sqrt{\omega^2-\omega_{pe}^2}}{c} \qquad \frac{dk}{d\omega} = \frac{1}{\cancel{2}c}\,\frac{\cancel{2}\omega}{\sqrt{\omega^2-\omega_{pe}^2}}$$

$$\underline{S} = \underline{E} \times \underline{H} = \frac{1}{\mu_0}\,\underline{E} \times \underline{B}$$

$$\text{Potenza} = \int_A \underline{S} \cdot d\underline{A}$$



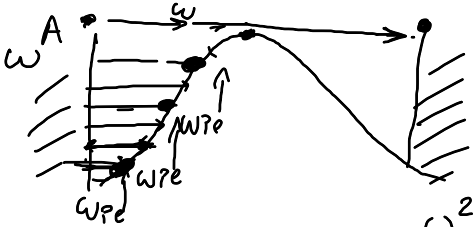
$$\frac{E}{B} = c$$

$$[S] = W/m^2$$

$$\underline{S} = \frac{E_0^2}{\omega\mu_0}\,\underline{k}$$

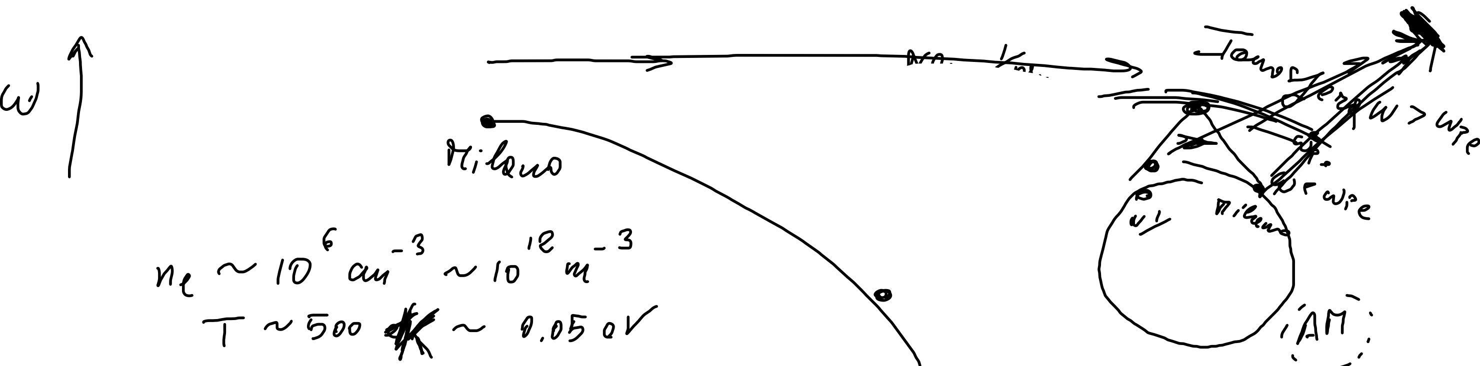
Se $\underline{S}$ è reale: $\mathrm{Re}[\underline{k}] \neq 0$

Se $\omega < \omega_{pe}$: onda riflessa



$$\omega_{pe}^2 = \frac{n_e e^2}{m \varepsilon_0}$$

$\omega < \omega_{pe}$



$n_e \sim 10^6\ cm^{-3} \sim 10^{12}\ m^{-3}$

$T \sim 500\ K \sim 0.05\ eV$

$$\nu_{pe} = \frac{1}{2\pi}\,\omega_{pe} = \frac{1}{2\pi}\sqrt{\frac{n_e e^2}{m \varepsilon_0}} \approx 9\ MHz$$

Se ci sono modi normali

eq. Poisson è ridondante

Eq. continuità: $\sum_\alpha q_\alpha \times \frac{\partial n_\alpha}{\partial t} + \underline{\nabla} \cdot (n_\alpha \underline{u}_\alpha) = 0 \quad \times q_\alpha$

$$\underbrace{\sum_\alpha q_\alpha \frac{\partial n_\alpha}{\partial t}}_{\frac{\partial \rho}{\partial t}} + \underbrace{\sum_\alpha q_\alpha \underline{\nabla} \cdot (n_\alpha \underline{u}_\alpha)}_{\underline{\nabla} \cdot \underline{j}} = 0 \; ; \qquad \frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{j} = 0$$

$$\sum_\alpha q_\alpha n_\alpha = \rho \qquad \sum_\alpha q_\alpha \underline{u}_\alpha n_\alpha = \underline{j} \qquad \underline{\nabla} \cdot \underline{j} = -\frac{\partial \rho}{\partial t}$$

$$\underbrace{\underline{\nabla} \cdot \left[\underline{\nabla} \times \underline{B} \right]}_{=0} = \underline{\nabla} \cdot \left[\mu_0 \underline{j} + \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t} \right]$$

$$\cancel{\mu_0} \underline{\nabla} \cdot \underline{j} + \varepsilon_0 \cancel{\mu_0} \frac{\partial \underline{\nabla} \cdot \underline{E}}{\partial t} = 0 \; ;$$

$$-\frac{\partial \rho}{\partial t} + \varepsilon_0 \frac{\partial}{\partial t} \underline{\nabla} \cdot \underline{E} = 0 \; ; \qquad \frac{\partial}{\partial t} \left[\underline{\nabla} \cdot \underline{E} - \frac{\rho}{\varepsilon_0} \right] = 0$$

$$\underline{\nabla}\cdot\underline{E} - \rho/\varepsilon_0 = \text{const}$$

In generale eq. Poisson non è ridondante
($\text{const} = 0$)

Se però ci sono modi normali: $\frac{\partial}{\partial t} \to -i\omega$

$$\frac{\partial}{\partial t}\left(\underline{\nabla}\cdot\underline{E} - \rho/\varepsilon_0\right) = 0$$

$$-i\omega\left(\underline{\nabla}\cdot\underline{E} - \rho/\varepsilon_0\right) = 0 \Rightarrow \underline{\nabla}\cdot\underline{E} = \rho/\varepsilon_0$$