$\underline{\underline{M}} \cdot \underline{\xi} = 0$ $\qquad$ $\underline{\xi}$: spostamento del plasma risp. all'equilibrio

$$\begin{bmatrix} \omega^2 - v_A^2 k_{\parallel}^2 & 0 & 0 \\ 0 & \omega^2 - k^2 v_A^2 - c_S^2 k_{\perp}^2 & -k_{\parallel} k_{\perp} c_S^2 \\ 0 & -c_S^2 k_{\parallel} k_{\perp} & \omega^2 - c_S^2 k_{\parallel}^2 \end{bmatrix} \begin{pmatrix} \xi_x \\ \xi_y \\ \xi_z \end{pmatrix} = 0$$

$\underline{k} = k_{\parallel} \hat{\underline{e}}_z + k_{\perp} \hat{\underline{y}}$ $\qquad$ $\underline{B} = B_0 \hat{\underline{e}}_z$

$v_A^2 = \dfrac{B_0}{\mu_0 \rho_0}$ $\qquad$ $c_S^2 = \dfrac{\gamma p_0}{\rho_0}$

$\det \underline{\underline{M}} = 0$

$$\left(\omega^2 - k_{\parallel}^2 v_A^2\right) \det \begin{bmatrix} \omega^2 - k^2 v_A^2 - c_s^2 k_\perp^2 & -k_{\parallel} k_\perp c_s^2 \\ -k_{\parallel} k_\perp c_s^2 & \omega^2 - c_s^2 k_{\parallel}^2 \end{bmatrix} = 0$$

Onda Alfven torsionale
(shear Alfven wave)

$\omega = \pm k_{\parallel} v_A$

$$\left(\omega^2 - k_{\parallel}^2 v_A^2\right) \xi_x = 0$$

$$\left\{ \begin{bmatrix} 2 \times 2 \end{bmatrix} \begin{pmatrix} \xi_y \\ \xi_z \end{pmatrix} = 0 \right.$$

$$\det \begin{bmatrix} -k_\perp^2 v_A^2 - C_S^2 k_\perp^2 & -k_\parallel k_\perp C_S^2 \\ -C_S^2 k_\parallel k_\perp & (v_A^2 - C_S^2) k_\parallel^2 \end{bmatrix} =$$

$$= -k_\perp^2 k_\parallel^2 v_A^2 (v_A^2 - C_S^2) - k_\parallel^2 k_\perp^2 C_S^2 (v_A^2 - C_S^2) - k_\parallel^2 k_\perp^2 C_S^4$$

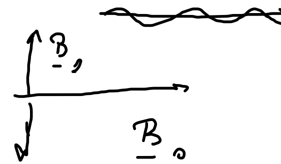
$$= -k_\parallel^2 k_\perp^2 (v_A^2 + C_S^2)(v_A^2 - C_S^2) - k_\parallel^2 k_\perp^2 C_S^4 =$$

$$= -k_\parallel^2 k_\perp^2 \left[(v_A^4 - C_S^4) + C_S^4 \right] \neq 0 \Rightarrow \xi_y = 0 \quad \xi_z = 0$$

$$\underline{B}_1 = i \underline{k} \times (\underline{\xi}_1 \times \underline{B}_0) \qquad \Rightarrow \xi_x \neq 0$$

$$\rho_1 = -i \rho_0 (\underline{k} \cdot \underline{\xi})$$

$$\underline{B}_1 = i\left(k_\parallel \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y\right) \times \left(\xi_{1x} \hat{\underline{e}}_x \times B_0 \hat{\underline{e}}_z\right) =$$



$$= i\left(k_\parallel \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y\right) \times \left(-B_0 \xi_{1x}\right) \hat{\underline{e}}_y$$

$$= i k_\parallel B_0 \xi_{1x} \hat{\underline{e}}_x \qquad B_1 \neq 0 \qquad \underline{B}_1 \propto \hat{\underline{e}}_x$$

$$\rho_1 = -i\rho_0 \left(k_\parallel \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y\right) \cdot \xi_{1x} \hat{\underline{e}}_x = 0$$

$$v_A^2 = \frac{B_0^2}{\mu_0 \rho_0} = \frac{2 \cdot p_M}{\rho_0} \qquad c_s^2 = \frac{\gamma p_0}{\rho_0}$$

$$[p] = Pa = \frac{N}{m^2} = \frac{J}{m^3}$$

$$\equiv \qquad p_M = \frac{B_0^2}{2\mu_0} \qquad \gamma = 2 \qquad \equiv$$

$$\gamma = \frac{2+N}{N} \Rightarrow \gamma = 2 \text{ allora } N = 2$$

Corda vibrante

$$v^2 = \frac{\text{Tensione}}{m/L} \quad ; \quad v_A^2 = \frac{B_0^2}{\mu_0 \rho_0}$$

$$= \frac{\text{Tensione}}{\underbrace{\frac{m}{V}}_{\rho_0} \cdot S}$$

$$\frac{\text{Tensione}}{S} = \frac{B_0^2}{\mu_0}$$

$$\text{Tensione} = \frac{B_0^2}{\mu_0} \cdot S$$

$$\omega = k_\parallel v_A$$

$$\underline{v}_g = \frac{\partial}{\partial k_\parallel} \omega(k)\, \hat{\underline{e}}_z = v_A \hat{\underline{e}}_z$$

altre 2 onde:

$$\omega^2 \neq k_\parallel^2 v_A^2 \to \xi_r = 0$$

$$- c_s^2 \omega^2 k^2$$

$$\left(\omega^2 - k^2 v_A^2 - c_s^2 k_\perp^2\right)\left(\omega^2 - c_s^2 k_\parallel^2\right) - k_\parallel^2 k_\perp^2 c_s^4 = 0$$

$$\omega^4 - c_s^2 k_\parallel^2 \omega^2 - k^2 v_A^2 \omega^2 + k^2 k_\parallel^2 v_A^2 c_s^2 - \omega^2 c_s^2 k_\perp^2 + \cancel{c_s^4 k_\parallel^2 k_\perp^2} - \cancel{k_\parallel^2 k_\perp^2 c_s^4} = 0$$

$$\omega^4 - \omega^2 k^2 \left(v_A^2 + c_s^2\right) + k^2 k_\parallel^2 v_A^2 c_s^2 = 0$$

$$\omega^2_{1,2} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$a = 1$$

$$b = -k^2\left(v_A^2 + c_s^2\right)$$

$$c = k^2 k_\parallel^2 v_A^2 c_s^2$$

$$= -\frac{b}{2a} \pm \frac{b}{2a}\sqrt{1 - \frac{4ac}{b^2}} = -\frac{b}{2a}\left(1 \pm \sqrt{1 - \frac{4ac}{b^2}}\right)$$

$$\alpha^2 = \frac{4ac}{b^2} = \frac{4\cancel{K^2} K_{\parallel}^2 v_A^2 c_s^2}{K^{\cancel{4}\,2} (c_s^2 + v_A^2)^2} = 4 \frac{K_{\parallel}^2}{K^2} \left(\frac{v_A^2 c_s^2}{(c_s^2 + v_A^2)^2} \right) \leq 1$$

$$\leq 1$$

$$\frac{K_{\parallel}}{K} \leq 1$$

$$(c_s^2 + v_A^2)^2 = c_s^4 + v_A^4 + 2 v_A^2 c_s^2 \geq v_A^2 c_s^2$$

+ : onda magnetosonica (onda Alfven compressionale)

− : onda acustica

$$\frac{c_s^2}{v_A^2} = \frac{\gamma p_0}{\cancel{\rho_0}} \frac{\mu_0 \cancel{\rho_0}}{B_0^2} \simeq \frac{\gamma p_0}{B^2/\mu_0} \sim \frac{p_0}{B^2/2\mu_0} \sim \beta < 1$$

$$\alpha^2 = 4\frac{k_{\parallel}^2}{K^2} \frac{c_s^2/v_A^2}{(1 + c_s^2/v_A^2)^2} << 1 \qquad \left(\frac{c_s^2}{v_A^2} << 1 \right)$$

Segue +:

$$\omega^2 \approx \left(k_{\parallel}^2 + k_{\perp}^2\right) v_A^2$$

$\xi_x = 0$

$$\begin{bmatrix} -c_s^2 k_{\perp}^2 & -k_{\parallel} k_{\perp} c_s^2 \\ -c_s^2 k_{\parallel} k_{\perp} & k^2 v_A^2 - c_s^2 k_{\parallel}^2 \end{bmatrix} \begin{pmatrix} \xi_{1y} \\ \xi_{1z} \end{pmatrix} = 0$$

$$-c_s^2 k_{\perp}^2 \xi_{1y} - k_{\parallel} k_{\perp} c_s^2 \xi_{1z} = 0 \quad ; \quad \frac{\xi_{1z}}{\xi_{1y}} = \frac{-\cancel{c_s^2} k_{\perp}^2}{k_{\parallel} k_{\perp} \cancel{c_s^2}}$$

$$\leq k^2$$

$$-c_s^2 k_{\parallel} k_{\perp} \xi_{1y} + \left(k^2 v_A^2 - c_s^2 k_{\parallel}^2\right) \xi_{1z} = 0 \qquad \frac{\xi_{1z}}{\xi_{1y}} = \frac{c_s^2 k_{\parallel} k_{\perp}}{k^2 v_A^2 - c_s^2 k_{\parallel}^2} \approx \frac{c_s^2 k_{\parallel} k_{\perp}}{k^2 v_A^2} \ll 1$$

$$\underline{B}_1 = i\,\underline{k}\times\left(\underline{\xi}_1\times\underline{B}_0\right)$$

$$= i\left(k_\parallel\,\underline{\hat{e}}_z + k_\perp\,\underline{\hat{e}}_y\right)\times\left(\xi_{1y}\,\underline{\hat{e}}_y\times B_0\,\underline{\hat{e}}_z\right)$$

$$= i\left(k_\parallel\,\underline{\hat{e}}_z + k_\perp\,\underline{\hat{e}}_y\right)\times \xi_{1y}B_0\,\underline{\hat{e}}_x$$

$$= iB_0\xi_{1y}\left(k_\parallel\,\underline{\hat{e}}_y - k_\perp\,\underline{\hat{e}}_z\right)$$

$$\rho_1 = -i\rho_0\left(\underline{k}\cdot\underline{\xi}\right) = -i\rho_0\left(k_\parallel\,\underline{\hat{e}}_z + k_\perp\,\underline{\hat{e}}_y\right)\cdot\xi_{1y}\,\underline{\hat{e}}_y = -i\rho_0\,k_\perp\,\xi_{1y} \neq 0$$

$B_1 \parallel \underline{\hat{e}}_x$

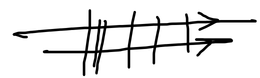
Onda acustica: ($\ominus$ nella rel. disp.)

$$\xi_{1x} = 0$$

$$\omega^2 \simeq \frac{k^2 v_A^2}{2}\left(1 - \sqrt{1-\alpha^2}\right)$$

$$\simeq \frac{k^2 v_A^2}{2}\left(\cancel{1} - \cancel{1} + \frac{\alpha^2}{2}\right) \simeq \frac{k^{2}}{\cancel{2}}\frac{v_A^{2}}{\cancel{2}}\,\frac{\cancel{v_A^2}\, c_s^2\, \cancel{4} k_\parallel^2}{\cancel{v_A^4}\, \cancel{k^2}} \simeq c_s^2 k_\parallel^2$$

$$\begin{bmatrix} k_\parallel^2 c_s^2 - k^2 v_A^2 - c_s^2 k_\perp^2 & -k_\parallel k_\perp c_s^2 \\ -c_s^2 k_\parallel k_\perp & \cancel{k^2 c_s^2} \cancel{- c_s^2 k_\parallel^2} \end{bmatrix} \begin{pmatrix} \xi_{1y} \\ \xi_{1z} \end{pmatrix} = 0$$



$$\left(k_\parallel^2 c_s^2 - k^2 v_A^2 - c_s^2 k_\perp^2\right)\xi_{1y} - k_\parallel k_\perp c_s^2 \xi_{1z} = 0 \; ; \quad \frac{\xi_{1y}}{\xi_{1z}} \simeq \frac{k_\parallel k_\perp c_s^2}{k^2 v_A^2} \ll 1$$

$$\xi_{1y} \ll \xi_{1z}$$

Plasma $B=0$
2 fluidi

onde elettrostatiche $\underline{k}\cdot\underline{E}_1 \neq 0$ $\exists$ almeno 1 comp. $\underline{k} \parallel \underline{E}$

Richiedono $\rho_1 \neq 0$

onde e.m.

$\underline{k}\cdot\underline{E}_1 = 0$

$\omega^2 \cong k^2c^2 + \omega_{pe}^2$

$\frac{\omega}{k} \ll v_{the}, v_{thi}$: Schermatura di Debye

$v_{thi} \ll \frac{\omega}{k} \ll v_{the}$: onde ione-acustiche ($T_e \gg T_i$)

$\frac{\omega}{k} \gg v_{the}, v_{thi}$: onde Langmuir

Plasma $B\neq 0$ in MHD

onda Alfven torsionale

" magnetoacustica

onda sonora