

# **Causal Inference in Genetic Trio Studies**

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Causal Networks PhD Course 2021

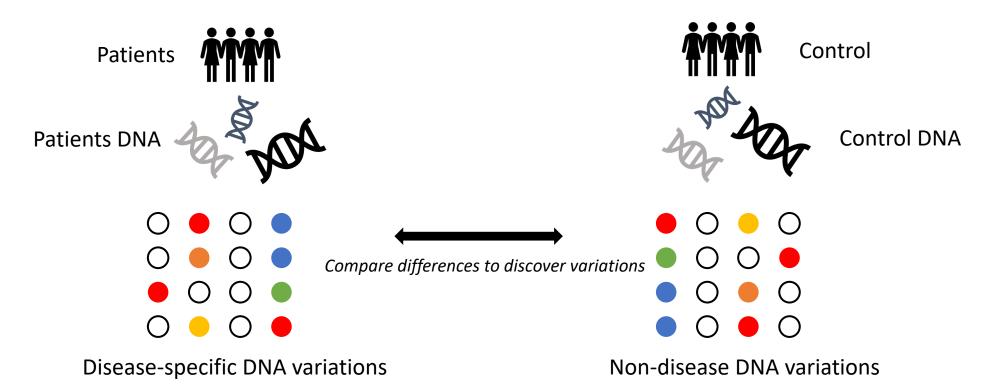
### Outline

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- Background knowledge
- Main contribution
- Causality in trio design
- The Digital Twin Test (DTT): full-chromosome and local
- Case study: Autism Spectrum Disorder (ASD)
- Final remarks

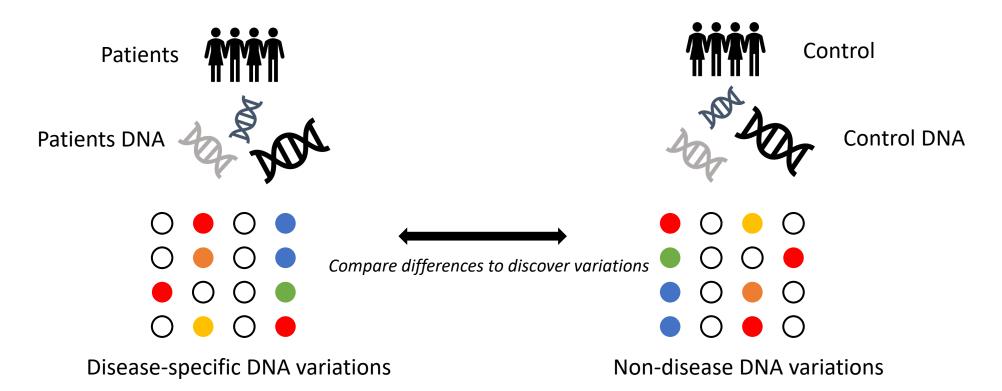
### Genome-wide Association Studies (GWAS)

**GWAS** discover regions of the genome containing *variants* that causally affect a *phenotype*, that is, identifying meaningful relationships between *genotypes* and *outcomes* of interest



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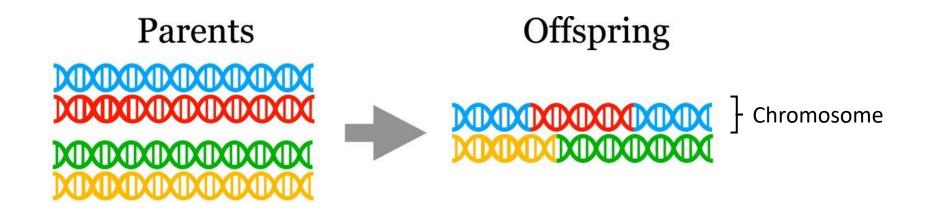
**GWAS** discover regions of the genome containing *variants* that causally affect a *phenotype*, that is, identifying meaningful relationships between *genotypes* and *outcomes* of interest



N.B.: all true statistical associations represent relevant biological activity, irrelevant but true associations can arise from the confounding effect of environmental conditions or other factors

### Background Knowledge: Meiosis

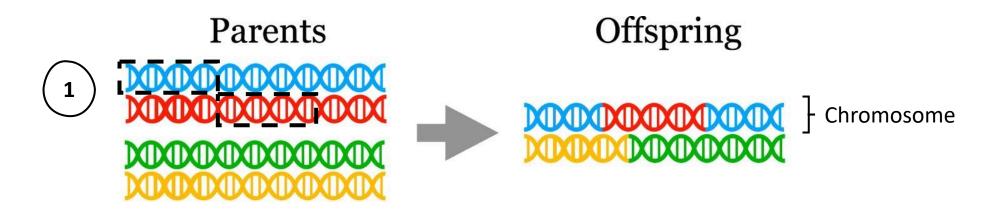
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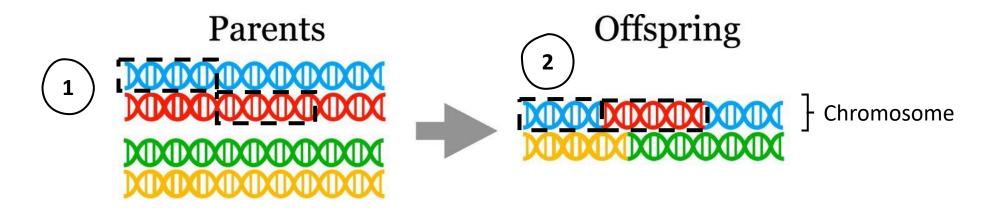
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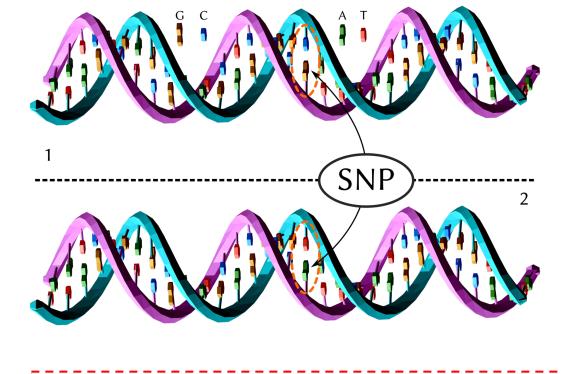
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Creates new combinations of code on each chromosome

# Background Knowledge: Single-nucleotide Polymorphisms (SNPs)

SNPs are sites on the genome where 2 possible alleles occur in the population



Allele: one of the variants of a gene

N.B.: SNPs on the same chromosome are dependent

Haplotype: set of observed alleles for an entire strand



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formalization of family studies immunity w.r.t. population structure



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localization of causal variants within windows in the full genome



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possibility of exploiting several multivariate models and domain information to increase power



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**Digital Twin Test**: a method for finding causal regions that are immune to confounding variables

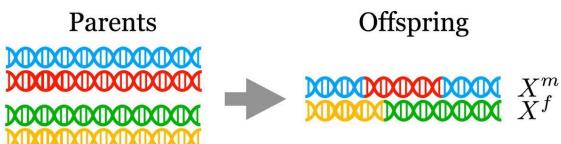
### Notation

 $M^{a}$ 

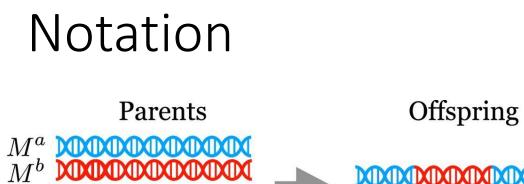
 $M^{b}$ 

 $F^{a}$ 

 $F^{b}$ 



Subjects:  $(X_1^m, ..., X_p^m) \in \{0,1\}^{n \times p}, (X_1^f, ..., X_p^f) \in \{0,1\}^{n \times p};$ Mothers:  $(M_1^a, ..., M_p^a) \in \{0,1\}^{n \times p}, (M_1^b, ..., M_p^b) \in \{0,1\}^{n \times p};$ Fathers:  $(F_1^a, ..., F_p^a) \in \{0,1\}^{n \times p}, (F_1^b, ..., F_p^b) \in \{0,1\}^{n \times p}.$ 



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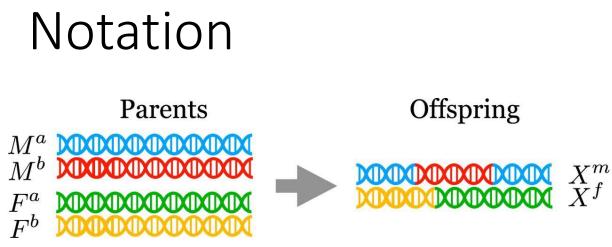
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**Offspring** *genotypes* matrix:  $X = X^m + X^f$ 

 $X^{f}$ 

 $X_j$ : *j*-th column of X representing the *j*-th genome site  $X^{(i)}$ : *i*-th row of X representing the subject *i* 

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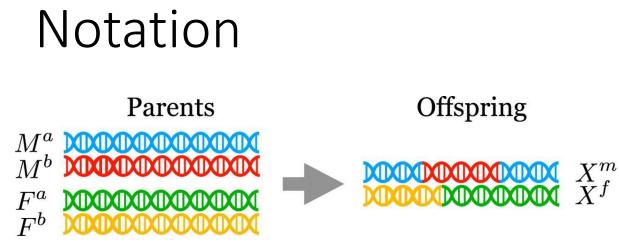


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**SNP**  $X_i^m$ : single-nucleotide polymorphism inherited either from  $M_i^a$  or  $M_i^b$  with equal probability

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**External confounder**  $X \mid (A, Z = z) \stackrel{\text{def}}{=} X \mid (A, Z = z') \text{ for any } z \text{ and } z'$   $\downarrow \downarrow$   $Z \amalg X_j \mid A$  The confounder Z is independent of the SNP j given A  $\downarrow \downarrow$   $Y \not \perp X_j \mid A \implies Y \not \perp X_j \mid (A, Z)$ If X and Y are associated after conditioning on A, the association is not due to confounder Z

Let Z be an external confounder, then any valid test of the null hyphothetis  $H_0$  is also a valid test of the stronger null hypothesis that accounts for the confounder Z:

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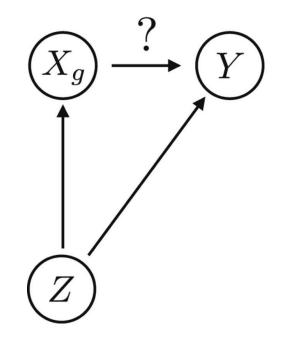
**Note 1**: if we reject  $H_0$ , the dependence between  $X_i$  and Y cannot be due to an external confounder Z

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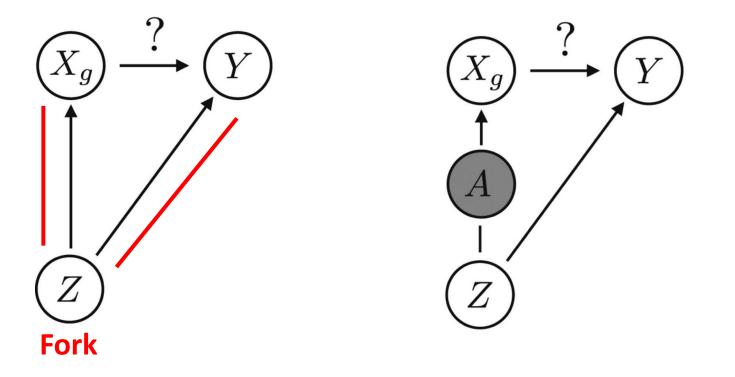
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**Note 1**: if we reject  $H_0$ , the dependence between  $X_i$  and Y cannot be due to an external confounder Z

Note 2: if we reject  $H_0$ , it does not yet imply that  $X_j$  is the causal *SNP*, but it implies that there is an association on the chromosome that is not the result of external confounding

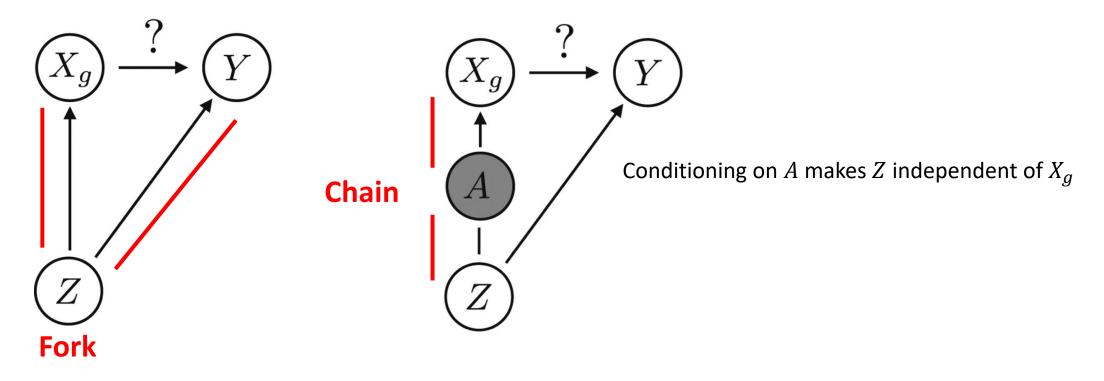


Z is an external confounder that can create an association between  $X_g$  and Y even if there is no causal effect (due to the fork structure)



Structural Equation Model M:  $(A, Z) = f_{AZ}(N_{AZ}),$   $X = f_X(A, N_X),$   $Y = f_Y(X, Z, N_Y)$  $f_{AZ}, f_X, f_Y$  are fixed functions  $N_{AZ}, N_X, N_Y$  are the exogenous variables

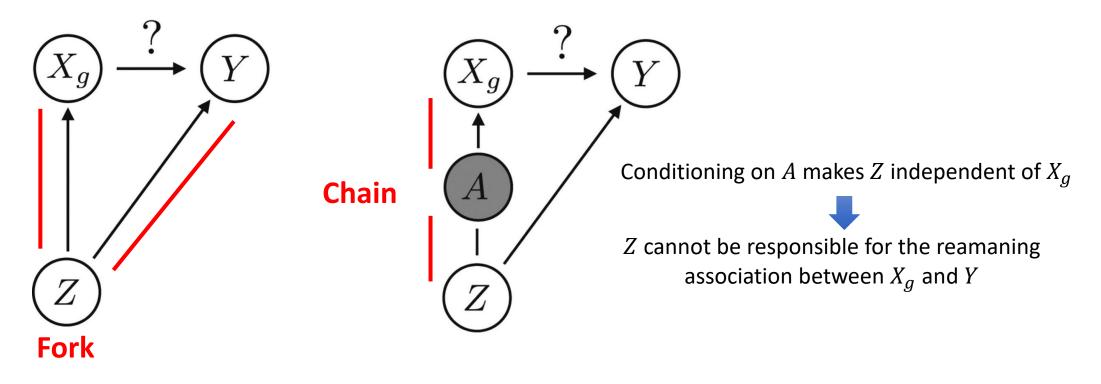
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### Discussion of Possible Confounders

(virtually) all *confounders* in genetic studies do not affect the transmission of the genetic information

thus *external confounders* which are <u>correctly accounted</u> for in the trio design

### Discussion of Possible Confounders

#### **Examples of external confounders**



Environmental conditions after conception



Population structure, ethnicity, location



Cryptic relatedness



Family effects, altruistic genes



Assortative mating

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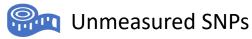


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#### **Examples of not external confounders**



Germline mutations

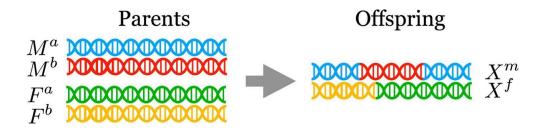


### The Randomness in Inheritance

The process by which subject's two haplotypes arise from the parental haplotypes is modelled as a **hidden Markov model (HMM**)

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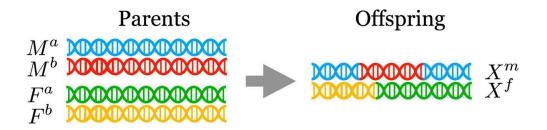


Model for a single observation on one chromosome (e.g.:  $X^m$ )

 $X_j^m = \begin{cases} a \text{ if site } j \text{ is copied from } M^a \\ b \text{ if site } j \text{ is copied from } M^b \end{cases}$ 

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$$X_j^m = \begin{cases} a \text{ if site } j \text{ is copied from } M^a \\ b \text{ if site } j \text{ is copied from } M^b \end{cases} \qquad P(X_1^m = a) = \frac{1}{2} \\ P(X_j^m = x_{j-1}^m | X_{1:(j-1)}^m = x_{1:(j-1)}^m) = \frac{1}{2} (1 + e^{-2d_j}) \end{cases}$$

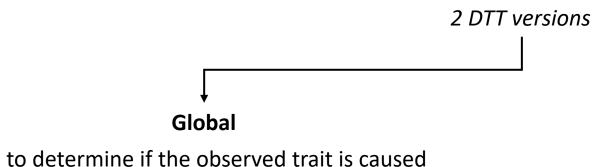
 $d_j$  is the genetic distance between SNPs j - 1 and j

HMM describes the distribution of  $X^m$  given  $M^a$  and  $M^b$  ( $F^a$ ,  $F^b$ )

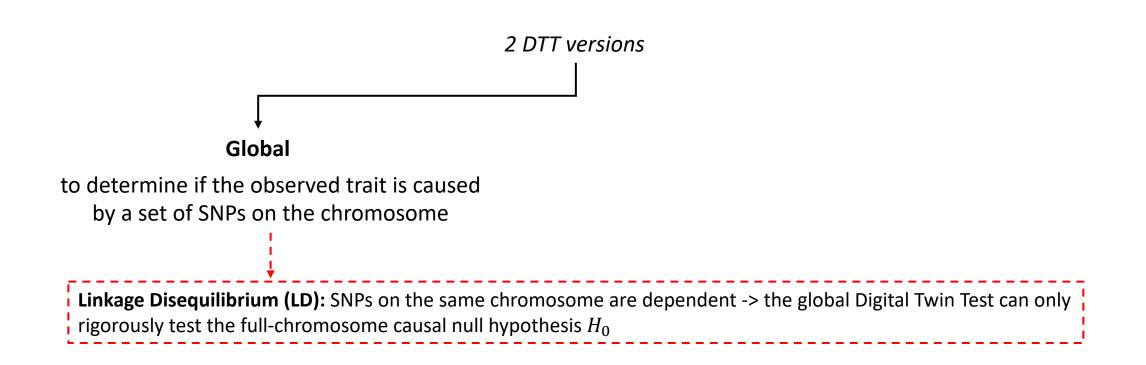
#### Goals

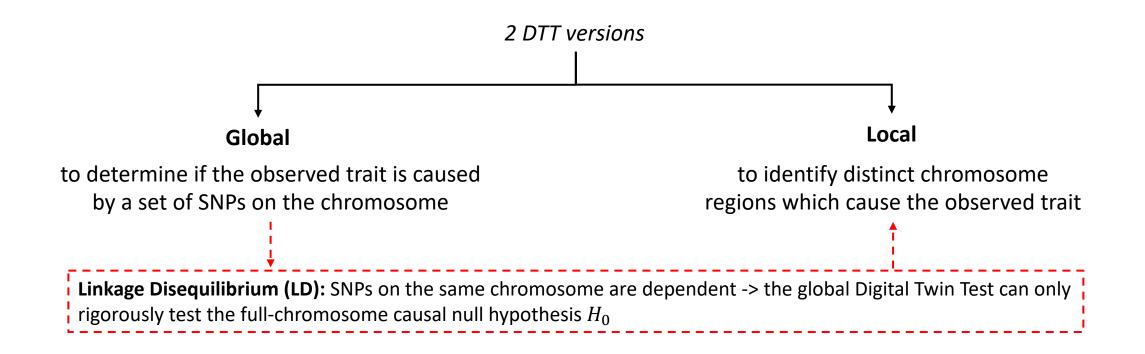
5 to determine whether a observed trait has any genetic basis, that is, to test the  $H_0: X_j \perp Y \mid A$ 

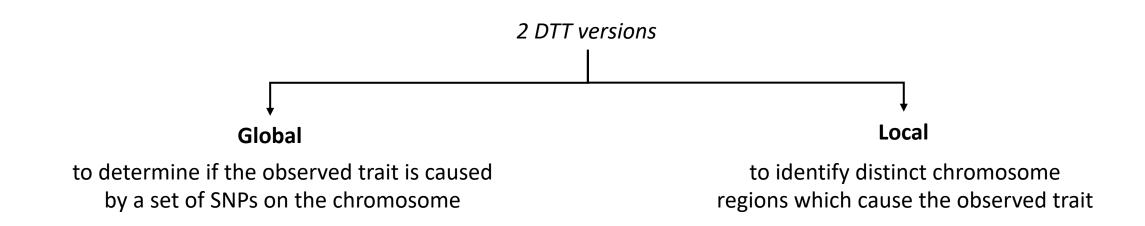
to find regions of the genome that contain causal variants



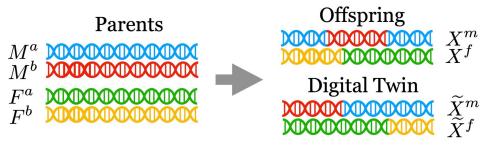
to determine if the observed trait is caused by a set of SNPs on the chromosome



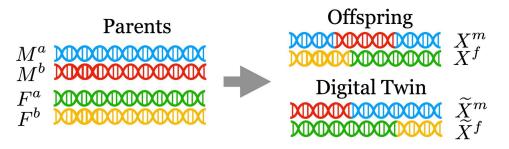




**N.B.**: the DTT is a natural randomization test in a trio design because it replicates the random mechanism generating the data!



 $H_0: X_C \amalg Y \mid A$ 



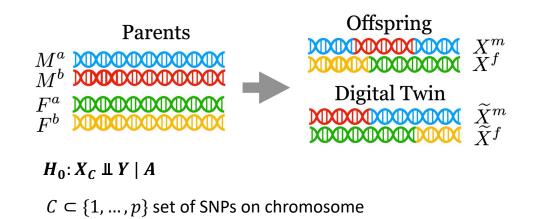
#### Compute $t^* = T((X_{-C}, X_C), Y)$

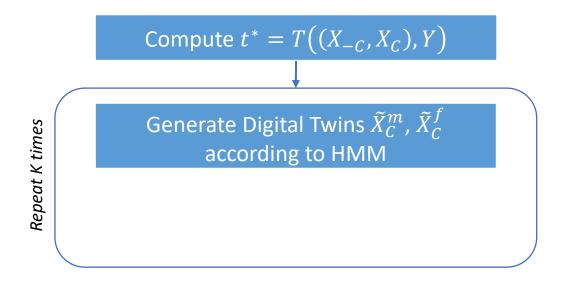
 $H_0 {:} X_C \mathbin{\amalg} Y \mid A$ 

 $C \subset \{1, ..., p\}$  set of SNPs on chromosome

 $X_C$  denotes  $(X_j)_{j \in C}$ 

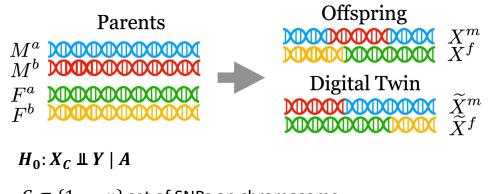
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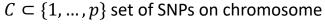




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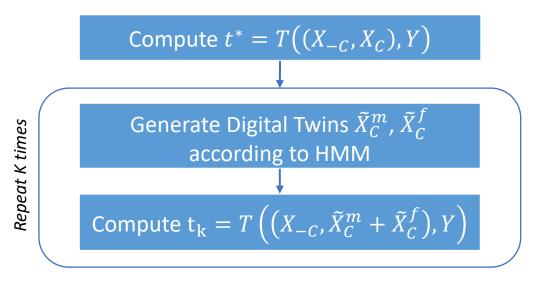
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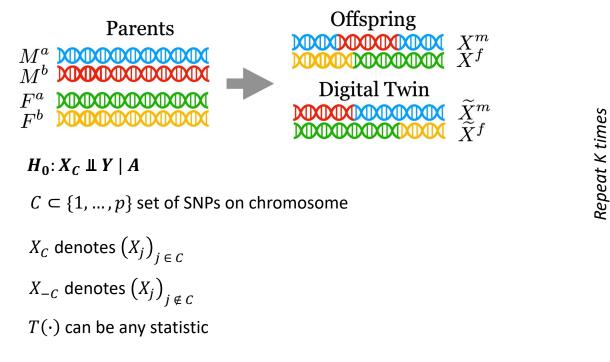


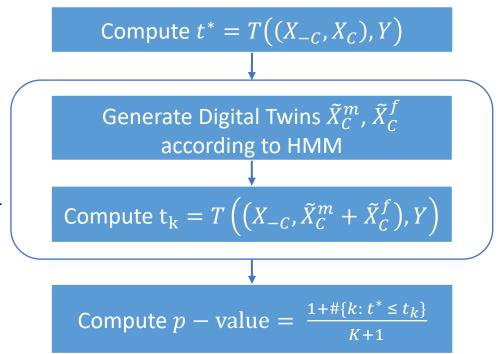


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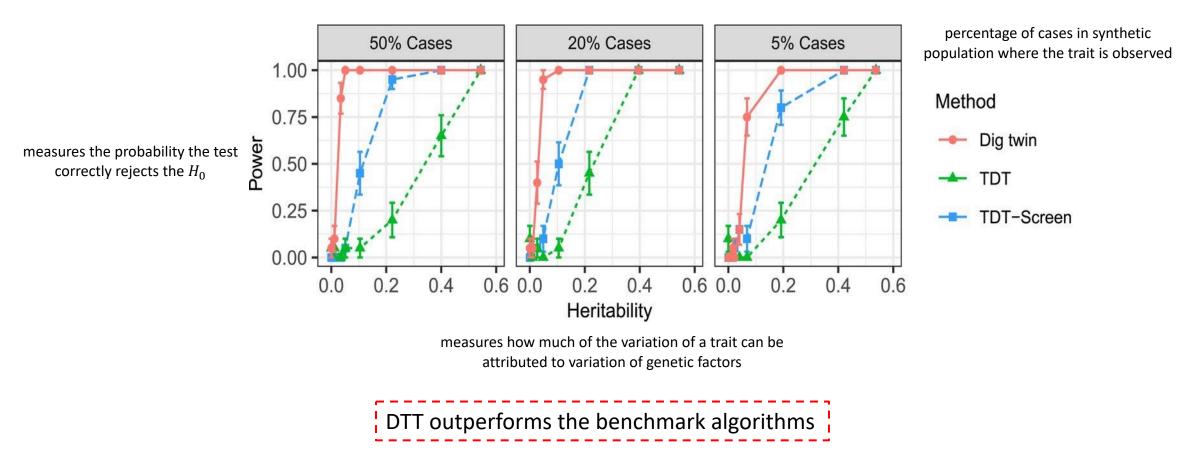
The algorithm returns a *p*-value, and the corresponding level  $\alpha$  hypothesis test rejects when this *p*-value is less than  $\alpha$ .

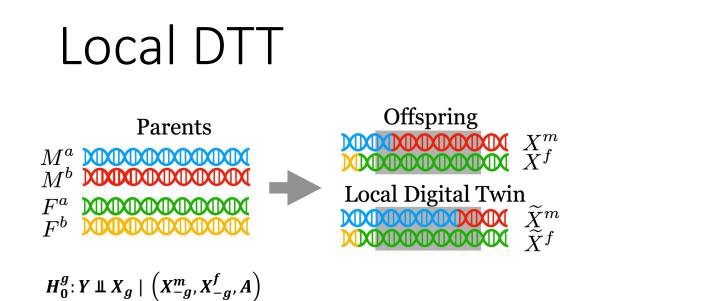
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- 2. Synthetic population of 2500 parent-offspring trios

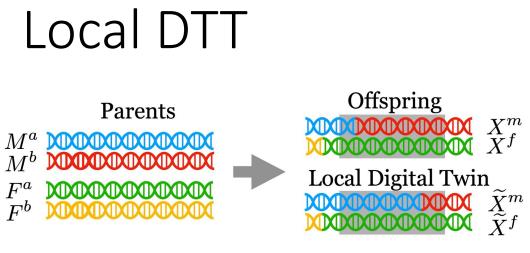
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- 5. Comparison with TDT / TDT-Screen algorithms







Compute 
$$t^* = T((X_{-g}, X_g), Y)$$

 $H_0^g: Y \perp X_g \mid \left(X_{-g}^m, X_{-g}^f, A\right)$ 

G is a partition of  $\{1, \dots, p\}, g \in G$  is group of SNPs

 $X_g$  denotes  $(X_j)_{j \in g}$  $X_{-g}$  denotes  $(X_j)_{j \notin g}$ 

#### Local DTT Offspring Parents $M^b$ Local Digital Twin $F^{a}$ $F^b$ $H_0^g: Y \perp X_g \mid \left(X_{-g}^m, X_{-g}^f, A\right)$

 $\widetilde{\widetilde{X}}^m_{\widetilde{X}^f}$ 

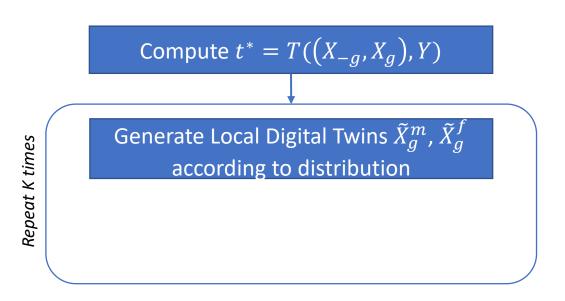
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Sampling distribution of Local Digital Twins  $(X_g^m, X_g^f) | (X_{-g}^m, X_{-g}^f, A)$ 



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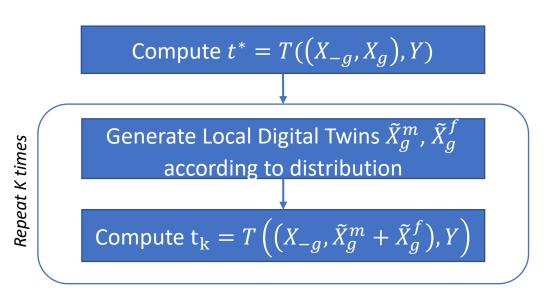
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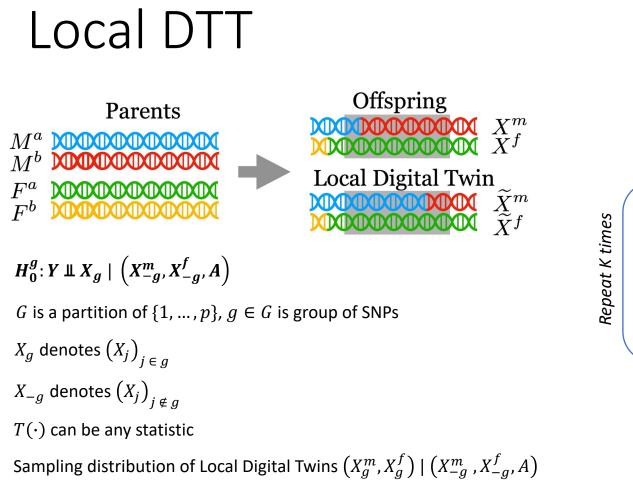
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Sampling distribution of Local Digital Twins  $(X_g^m, X_g^f) \mid (X_{-g}^m, X_{-g}^f, A)$ 





Compute  $t^* = T((X_{-g}, X_g), Y)$ Generate Local Digital Twins  $\tilde{X}_g^m, \tilde{X}_g^f$ according to distribution Compute  $t_k = T((X_{-g}, \tilde{X}_g^m + \tilde{X}_g^f), Y)$ Compute  $p - value = \frac{1 + \#\{k: t^* \le t_k\}}{K+1}$ 

The algorithm returns *p*-values that require statistical corrections (e.g., Bonferroni or BH) to ensure independency

(a slightly modification of the algorithm leads to independent p-values)

#### **Experimental setting**

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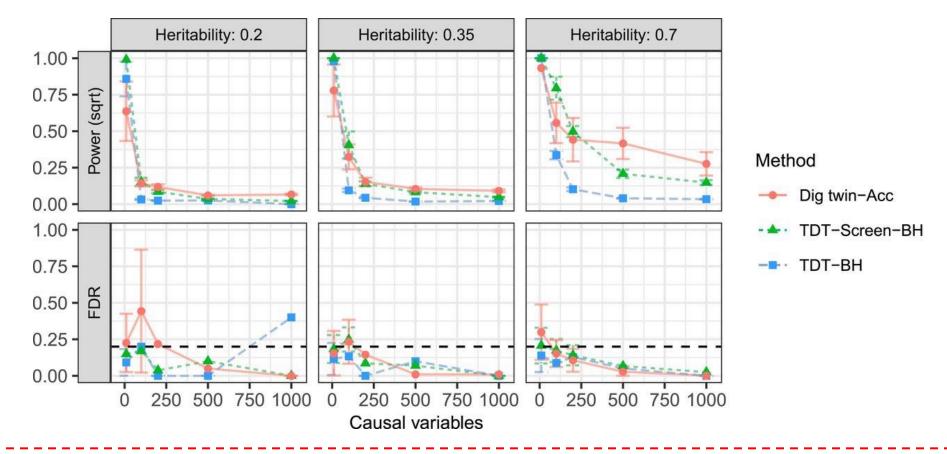
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- 5. DTT on each group and accumulation test to produce the set of discoveries

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- 3. FDR nominal level  $\alpha = 0.2$
- 4. 591, 513 SNPs on chromosomes 1 22, split into 532 pre-determined groups of size  $\sim$ 5 Mb
- 5. DTT on each group and accumulation test to produce the set of discoveries
- 6. Comparison with TDT / TDT-Screen algorithms applying statistical corrections to *p*-values

- 1. Synthetic population of 10,000 parent-offspring trios
- 2. Logistic regression model to generate parent-offspring trait
- 3. FDR nominal level  $\alpha = 0.2$
- 4. 591, 513 SNPs on chromosomes 1 22, split into 532 pre-determined groups of size  $\sim$ 5 Mb
- 5. DTT on each group and accumulation test to produce the set of discoveries
- 6. Comparison with TDT / TDT-Screen algorithms applying statistical corrections to *p*-values
- 7. Benchmark algorithms do not have formal guarantees for localization due to full-chromosome null test



TDT / TDT-Screen lead to spurious discoveries because they cannot give reliable information about SNPs locality due to intra-chromosome SNPs depencency (linkage disequilibrium)

# Case Study: Autism Spectrum Disorder (ASD)

- 1. Local Digital Twin Test applied to a dataset of 2,565 parent-child trios to study whether the intergenic variant *rs910805* on chromosome 20 can be the cause of ASD
- 2. They applied Local DTT to groups centered around SNP *rs910805* of size ranging from 1Mb to full-chromosome
- 3. Significance level  $\alpha = 0.05$

### Case Study: Autism Spectrum Disorder (ASD)

Resolution	1 Mb	2 Mb	3 Mb	4 Mb	5 Mb	full-chromosome
<i>p</i> -value	0.237	0.146	0.100	0.0168	0.0244	0.011

Results show that H<sub>0</sub> cannot be rejected at finer resolutions but is rejected for larger groups

## Case Study: Autism Spectrum Disorder (ASD)

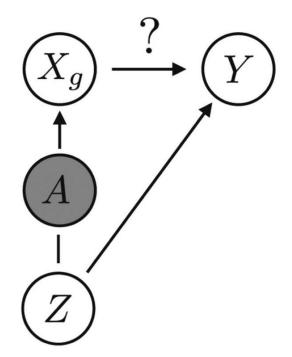
Resolution	1 Mb	2 Mb	3 Mb	4 Mb	5 Mb	full-chromosome
<i>p</i> -value	0.237	0.146	0.100	0.0168	0.0244	0.011

Results show that H<sub>0</sub> cannot be rejected at finer resolutions but is rejected for larger groups

Observed association between the vicinity of SNP *rs910805* and ASD is not due to external confounders

### Final Remarks

DTT aims to establish if a genomic region contains causal SNPs



#### Findings

- 1. the haplotypes *A* block the external confounders
- 2. given *A* we extactly know the distribution of *X*
- 3. given such causal model if A satisfy the backdoor criterion w.r.t. X and Y,  $H_0: X_g \perp Y \mid A \text{ only detects causality}$

#### Limits

- 1. computational phased haplotypes used by DTT can be subject to phased errors
- 2. in some case it might be harder to collect parent-offspring data than unrelated individuals

# Thank you!

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