

# Simpson's Paradox in COVID-19 Fatality Rates

## Causal Networks: Learning and Inference

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# Outline

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## Section 1

# Introduction

# A General Overview

- After the outbreak in Wuhan in December 2019, SARS-CoV-2 pandemic started being reported across multiple Countries, all over the world. One of the most cited indicators used to describe and compare the impact of the pandemic in different Countries was **Case Fatality Rate (CFR)**: *the proportion of deaths between SARS-CoV-2 infection confirmed cases*.
- It is important to notice that this measure is strongly influenced by the definition of "*confirmed cases*". Confounding introduced by Country - specific testing policies can strongly impact on the interpretation of the results. This was the case when comparing first-wave CFR between China and Italy!

# A General Overview

- After the outbreak in Wuhan in December 2019, SARS-CoV-2 pandemic started being reported across multiple Countries, all over the world. One of the most cited indicators used to describe and compare the impact of the pandemic in different Countries was **Case Fatality Rate (CFR)**: *the proportion of deaths between SARS-CoV-2 infection confirmed cases*.
- It is important to notice that this measure is strongly influenced by **the definition of "confirmed cases"**. **Confounding** introduced by Country - specific testing policies can strongly impact on the interpretation of the results. This was the case when **comparing first-wave CFR between China and Italy!**

# Observing the Paradox

- For all age groups, CFRs in Italy are lower than those in China;
- But the **total CFR** in Italy is **higher** than that in China.

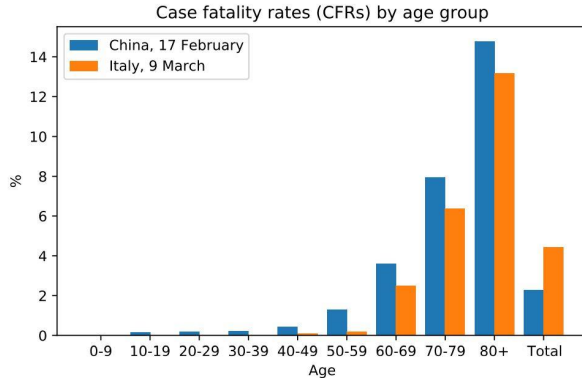


Figure: Simpson's paradox comparing CFR in Italy and China by age group.

# Explaining the Paradox

Table 2: Proportion of confirmed cases from Table 1 by age group, larger proportion highlighted in bold.

Age	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	≥ 80
Italy	0.5%	1.0%	3.5%	5.6%	10.7%	17.4%	17.7%	<b>21.4%</b>	<b>18.4%</b>
China	<b>0.9%</b>	<b>1.2%</b>	<b>8.1%</b>	<b>17.0%</b>	<b>19.2%</b>	<b>22.4%</b>	<b>19.2%</b>	8.8%	3.2%

Table 3: Age demographic (of the general population) for Italy and China, larger proportion highlighted in bold.

Age	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	≥ 80
Italy	8.3%	9.5%	10.1%	11.6%	14.9%	<b>15.8%</b>	<b>12.4%</b>	<b>10%</b>	<b>7.5%</b>
China	<b>11.9%</b>	<b>11.6%</b>	<b>12.9%</b>	<b>15.9%</b>	<b>15%</b>	15.4%	10.5%	5%	1.8%

**Table:** Simpson's paradox is partially explained by the different age demographic, but the biggest difference is in the proportion of confirmed cases by age group.

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- In the context of a pandemic, you may want to evaluate and compare different strategies by asking **interventional** and **counterfactual** questions.
- What we observed by now, however, is only correlation data and "*correlation does not imply causation*", but there is a link between them.

## Common cause principle

Any statistical dependence between two variables  $X$  and  $Y$  must have a causal explanation in that either (i)  $X$  causes  $Y$ , (ii)  $Y$  causes  $X$ , or (iii)  $X$  and  $Y$  have a common cause  $Z$ .

- Multiple **different causal models** could be employed with **different causal interpretations**: choosing a model is the first problem that needs to be address to answer to any causal question.



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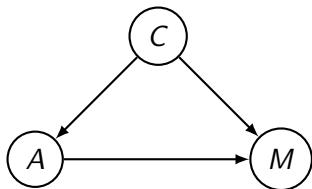
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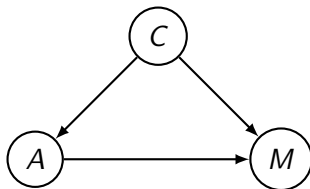
# Causal Model



**Figure:** Assumed causal graph for the relationship between country C, age group A, and mortality M.

- $C \rightarrow A$  : age distribution of the confirmed cases depends on the Country,
- $A \rightarrow M$  : the disease is more dangerous in the elderly,
- $C \rightarrow M$  : country-specific influence on mortality other than age.
- Assumptions
  - Causal sufficiency (no hidden confounding).
  - CFR and age-distribution are based on an observational sample.

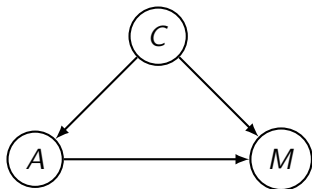
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# Total Causal Effect (TCE)



**Figure:** Assumed causal graph for the relationship between country C, age group A, and mortality M.

① Q: *What would be the effect on mortality of changing country from China to Italy?*

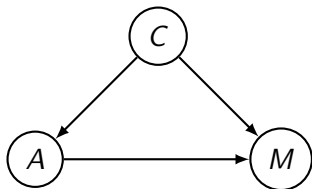
② Average effect, influenced by the different weights.

③ The total causal effect (TCE):

$$\begin{aligned} TCE_{0 \rightarrow 1} &= \\ &= \mathbb{E}[Y | do(T = 0)] \\ &\quad - \mathbb{E}[Y | do(T = 1)] \end{aligned}$$

④ Overall, it looks safer staying in China instead of changing Country.

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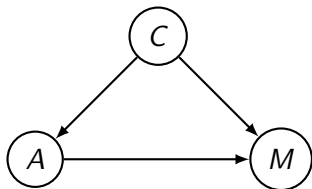


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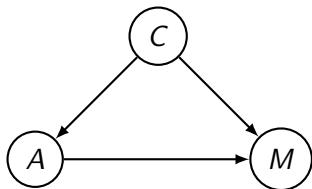


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$$\begin{aligned} TCE_{CN \rightarrow IT} &= \\ &= \mathbb{E}[M | do(C = IT)] \\ &\quad - \mathbb{E}[M | do(C = CN)] \\ &\approx 4.3\% - 2.3\% = 2\% \end{aligned}$$

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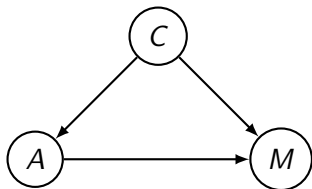


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# Mediation Analysis

- TCE does not help us understand *what drives the difference*, the *mechanism* behind different CFRs.
- We need to better understand the role of age ( $A$ ) as a mediator, to do that, we need to decompose the TCE of the Country ( $C$ ) on mortality ( $M$ ) between:

- **Direct effect** through to the path

$$C \rightarrow M$$

- **Indirect effect**, mediated by  $A$ , through the path

$$C \rightarrow A \rightarrow M$$

- To estimate both *direct* and *indirect* effect we will use tools and concepts of **mediation analysis**.

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# Mediation Analysis

# Controlled Direct Effect (CDE)

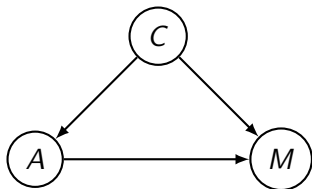


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1 Q: For **50-59 year-olds**, is it safer to get the disease in China or in Italy?

2 Control the value of the mediator.

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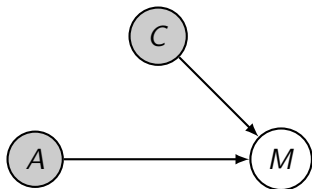


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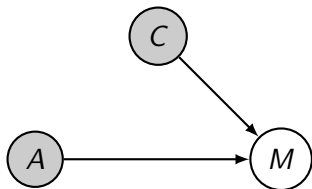


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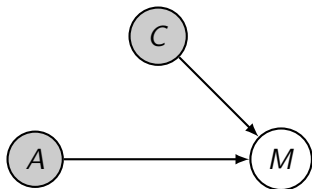


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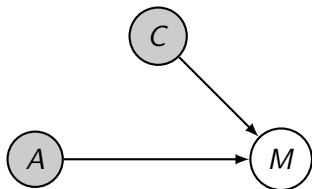


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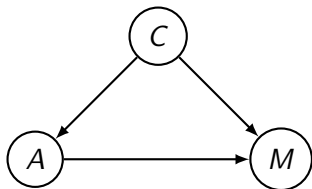


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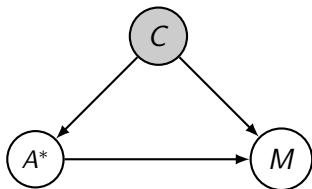


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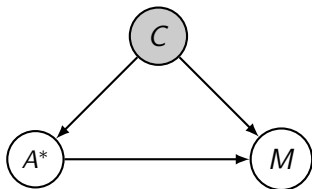


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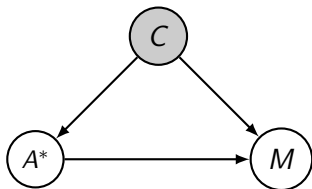


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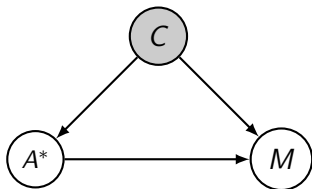


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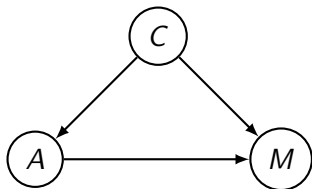


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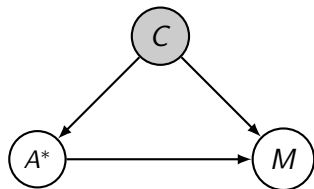


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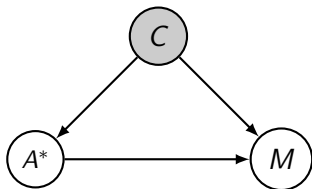


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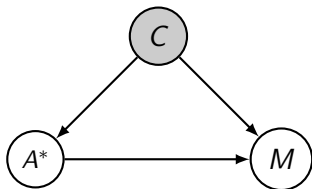


Figure: Assumed causal graph for the relationship between country C, age group A, and mortality M.

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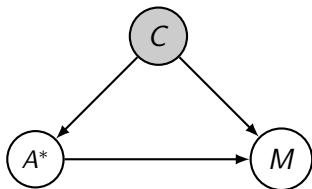


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# Interactions between TCE, NDE and NIE

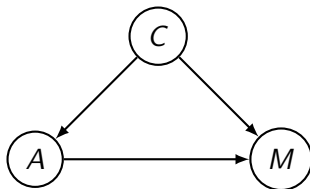


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- 1 Q: *Can the total causal effect be decomposed into a sum of direct and indirect contributions?*

$$TCE_{0 \rightarrow 1} \stackrel{?}{=} NDE_{0 \rightarrow 1} + NIE_{0 \rightarrow 1}$$

- 2 True for simple linear models, where causal effects correspond to path coefficients.
- 3 False in general, due to possible interactions between treatment and mediator in non-linear models.

# Interactions between TCE, NDE and NIE

Simpson's  
Paradox in  
COVID-19  
Fatality Rates

Julius von  
Kügelgen, Luigi  
Gresele, Bernhard  
Schölkopf

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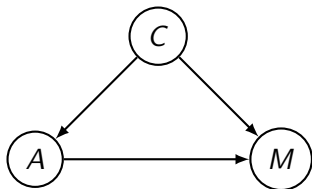


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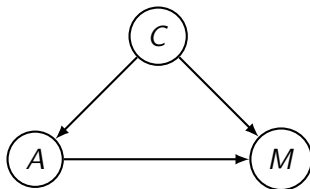


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# Case Study: CFR between China and Italy

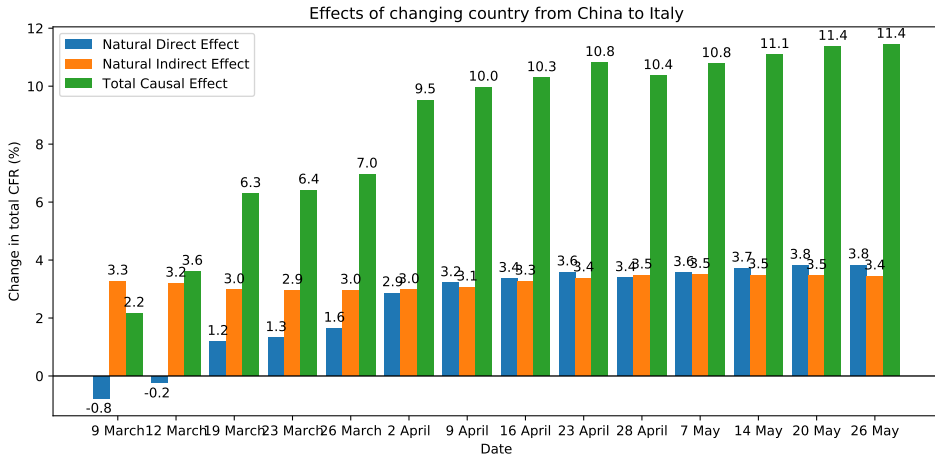


Figure: Evolution of TCE, NDE, and NIE of changing country from China to Italy on total CFR over time.

# Discussions & Conclusions

- 1 **NDE flips sign:** switching increase the total CFR on the Chinese case demographic.
- 2 **NIE consistent** with the of cases amongst elderly in Italy.
- 3 **TCE grows more quickly** than the sum of NDE and NIE, indicating moderation.
- 4 **Overwhelmed** healthcare system in Italy?

## Selection Bias

Elders are more likely to develop symptoms, to get tested, thus introducing bias.

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Counterfactual reasoning can only be performed under causal sufficiency.

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The virus is agnostic to "Countries": the influence on mortality is mediated by other variables.



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- The article illustrated how mediation analysis can be used in a *real-world setting* to answer causal questions.
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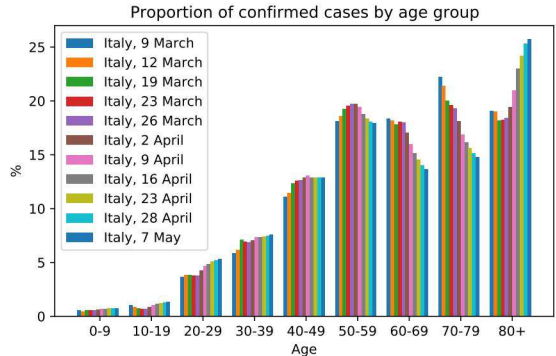


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# Thank You for your Attention!

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