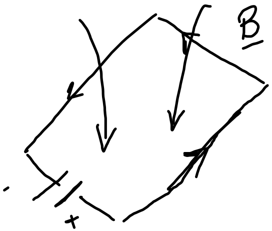
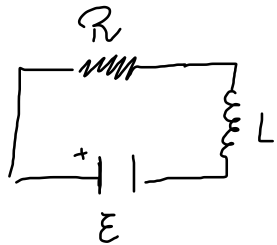


$$fem = L \frac{dI}{dt}$$

Circuito RL



Pr. cons. energia

$$\mathcal{E} = \Delta V_R + \Delta V_L$$

legge di Ohm

legge FNL

$$\mathcal{E} = \overbrace{Ri}^{\text{legge di Ohm}} + \overbrace{L\frac{di}{dt}}^{\text{legge FNL}}$$

Separazione di variabili

$$\mathcal{E} - Ri = L\,di/dt$$

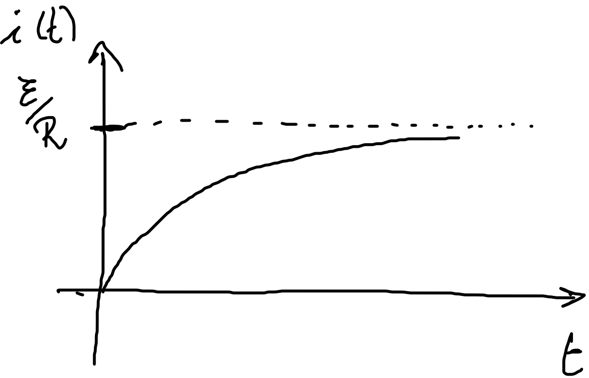
$$\mathcal{E} - Ri = L\frac{di}{dt} \; ; \quad dt = L\,di\,\frac{1}{\mathcal{E}-Ri} \; ; \quad \frac{dt}{L} = \frac{di}{\mathcal{E}-Ri}$$

$$\frac{dt}{L} = \frac{di}{-R\left(i-\frac{\mathcal{E}}{R}\right)} \; ; \quad \frac{-R\,dt}{L} = \frac{di}{i-\frac{\mathcal{E}}{R}} \; ; \quad -\int_0^t \frac{dt}{\tau} = \int_0^{i(t)} \frac{di}{i-\frac{\mathcal{E}}{R}} \qquad \tau \stackrel{\text{def}}{=} \frac{L}{R}$$

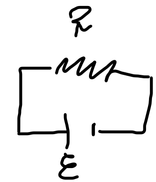
$$-\frac{1}{\tau}\,t\Big|_0^t = \ln\left(i-\frac{\mathcal{E}}{R}\right)\Big|_0^{i(t)}$$

$$-\frac{t}{\tau} = \ln\left[\frac{i(t)-\frac{\mathcal{E}}{R}}{-\frac{\mathcal{E}}{R}}\right]$$

$$e^{-t/\tau} = \frac{i(t) - \mathcal{E}/R}{-\mathcal{E}/R} \; ; \quad \boxed{i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)}$$



Per $t \to +\infty$ $\quad i(t) \to \mathcal{E}/R$



Se $L = 0$

$i = \text{cost} = \mathcal{E}/R$

Per quale t^* si ha

$i(t^*) = 99\% \; i_{max} = 0.99 \cdot \mathcal{E}/R$

$$0.99 \cancel{\frac{\mathcal{E}}{R}} = \cancel{\frac{\mathcal{E}}{R}}\left(1 - e^{-t^*/\tau}\right)$$

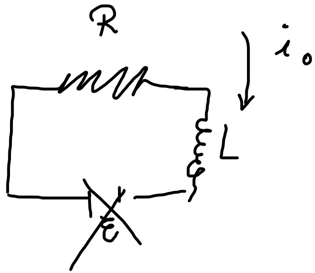
$$e^{-t^*/\tau} = 1 - 0.99 = 0.01$$

$$-t^*/\tau = \ln(0.01)$$

$$t^* \approx 4.6 \cdot \tau$$

$$\tau = L/R$$

Discesa della corrente in un circuito LR



Pr. cons. energia

$$\underbrace{\Delta V_L}_{\text{legge FNL}} + \underbrace{\Delta V_R}_{\text{legge Ohm}} = \cancel{\mathcal{E}} = 0$$

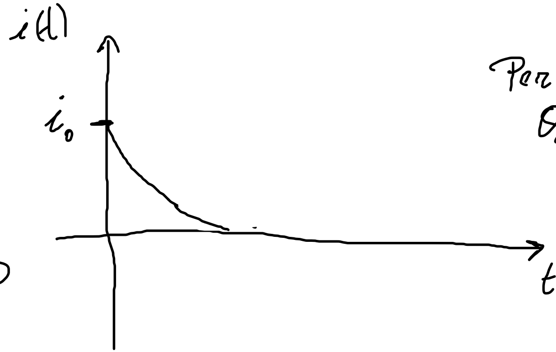
$$L\frac{di}{dt} + Ri = 0 \;; \quad L\frac{di}{dt} = -Ri \;;$$

$$\frac{di}{i} = -dt\frac{R}{L} \qquad \tau \stackrel{def}{=} \frac{L}{R} \qquad -\frac{L}{R}\frac{di}{i} = dt \;;$$

$$\int_{i_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{dt}{\tau} \qquad \tau \stackrel{def}{=} \frac{L}{R}$$

$$\ln\left[\frac{i(t)}{i_0}\right] = -t/\tau \; ; \qquad \frac{i(t)}{i_0} = e^{-t/\tau}$$

$$\boxed{i(t) = i_0 e^{-t/\tau}} \qquad \tau = \frac{L}{R}$$



Per $t \to +\infty$ $\quad i(t) \to 0$

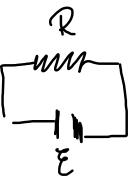
Quando succede che

$$i(t^*) = \frac{1}{100} i_0$$

$$\boxed{\text{Se } L = 0 \text{ allora } \tau = 0}$$

$$\frac{\cancel{i_0}}{100} = \cancel{i_0} e^{-t^*/\tau}$$

$$t^* = -\tau \ln(0.01) \simeq 4.6\,\tau$$



Se $L = 0$

$\varepsilon = R i$

Se $\varepsilon = 0 \Rightarrow i = 0$

Circuito RL

$$\mathcal{E} = Ri + L\frac{di}{dt}$$

$$\text{Potenza} = \mathcal{E}\cdot i = Ri^2 + L\frac{di}{dt}i = Ri^2 + L\frac{d}{dt}\left(\frac{1}{2}i^2\right) = Ri^2 + \frac{d}{dt}\left(\frac{1}{2}Li^2\right)$$

fornita dalla batteria

Potenza dissipata per effetto Joule

$$W_L = \frac{1}{2}Li^2$$

$$i = \frac{\ell B}{\mu_0 N}$$

Esempio

$$L = \mu_0 \frac{N^2}{\ell} S$$

solenoide

$$B = \mu_0 \frac{N}{\ell} i$$

solenoide

$$W_L = \frac{1}{2}\frac{\mu_0 N^2 S}{\ell}\underbrace{\left(\frac{\ell B}{\mu_0 N}\right)^2}_{i^2}$$

$$W_L = \frac{1}{2}\frac{\cancel{\mu_0}\cancel{N^2}S}{\cancel{\ell}}\frac{\ell^{\cancel{2}}B^2}{\mu_0^{\cancel{2}}\cancel{N^2}} = \left(\frac{B^2}{2\mu_0}\right)\cdot \underbrace{S\ell}_{\text{volume}}$$

$$W_L = \left(\frac{B^2}{2\mu_0}\right) \cdot \left(\begin{array}{c}\text{Volume} \\ \text{dove } B \neq 0\end{array}\right)$$

Analogia: $\underline{E} \neq \underline{0}$

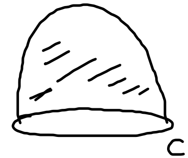
$$W_E = \int_V dV \underbrace{\frac{1}{2}\varepsilon E^2}_{\text{densità di energia}}$$

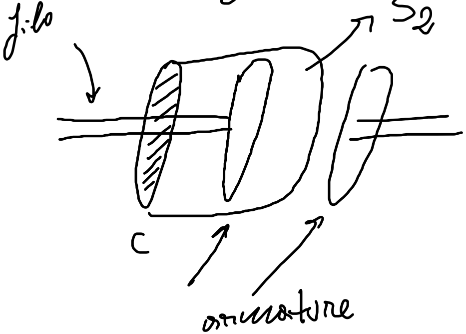
$\underline{B} \neq \underline{0}$

$$W_B = \int_V dV \left(\frac{B^2}{2\mu_0}\right) \rightarrow \text{densità di energia}$$

Si può generare un campo magnetico da un campo el. variabile nel tempo?

Th Ampere $\oint_C \underline{B}\cdot d\underline{l} = \mu_0 I^{conc}$





Condensatore durante la carica

$$\underline{I} = \int_S \underline{j}\cdot d\underline{S}$$

j dens. corrente

S: una qualunque sup. che poggia sul circuito C

S_2: calotta che poggia su C senza la base e che passa tra le armature $I^{conc} = 0$

S_1: area del cerchio racchiuso da C

$I^{conc} = I^{filo}$

$$\oint_C \underline{B} \cdot d\underline{l} = \mu_0 \left(I^{conc} + I^{spost} \right)$$

corrente di spostamento

$$\mu_0 I^{conc} = \mu_0 \frac{dq}{dt} = \mu_0 \frac{d}{dt}(\sigma S) = \mu_0 S \frac{d\sigma}{dt}$$

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0 E$$

$q = \sigma \cdot S \rightarrow$ sup. armatura

$$= \varepsilon_0 \mu_0 S \frac{dE}{dt} = \varepsilon_0 \mu_0 \frac{d}{dt}(SE)$$

Indoviniamo: $I_S = \varepsilon_0 \frac{d}{dt} \int_S \underline{E} \cdot d\underline{S}$

$$\oint_C \underline{B} \cdot d\underline{l} = \mu_0 I^{conc} + \varepsilon_0 \mu_0 \frac{d}{dt} \int_S \underline{E} \cdot d\underline{S}$$

Per fenomeni statici $I_S = 0$

Calotta S_2

$I^{conc} = 0$

"schiaccio" S_2 fino a coincidere con armatura

$$I^S = \varepsilon_0 \frac{d}{dt} \underbrace{\int_S \underline{E} \cdot d\underline{S}}_{\propto \text{ \# linee di campo che attraversano } S_2} = \varepsilon_0 \frac{d}{dt}(ES) = \cancel{\varepsilon_0} \frac{d}{dt}\left(\frac{\sigma}{\cancel{\varepsilon_0}} \cdot S\right) = \frac{dq}{dt} = I$$

cond. piano $E = \frac{\sigma}{\varepsilon_0}$

S_1:

$I^{conc} = I$

$I^S = 0$ perché, fuori dal cond., $\underline{E} = 0$