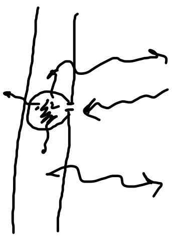
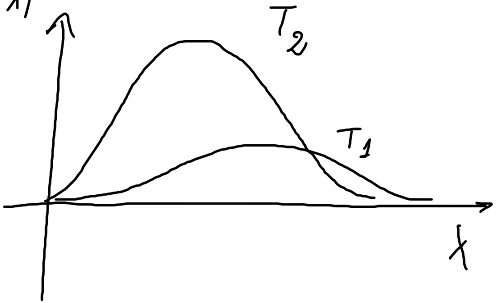
Natura corpuscolare della radiazione

Corpo nero

eq. termodinamica $T = cost$





$T_2 > T_1$

$I = \sigma T^4$ legge di Stefan-Boltzmann

$[T] = K \quad [I] = W/m^2$

Legge di Wien

$$\lambda_{max} T = 2.898 \cdot 10^{-7}\ m \cdot K$$

$$[\sigma] = 5.67 \cdot 10^{-8}\ W/m^2K^4$$

$T = 300\ K \Rightarrow$ $\lambda \approx 10^{-5}\ m$

legge di Wien

infrarosso

se

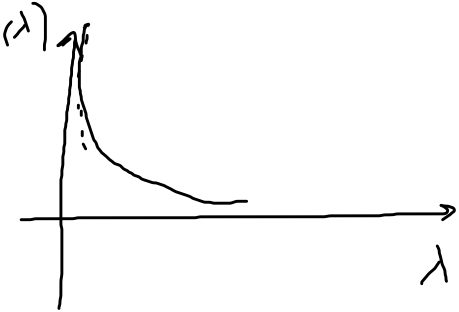
$\lambda_{max} = 500\ nm \Rightarrow T = 6000\ K$

$1.38 \cdot 10^{-23}\ J/K$

Teoria:

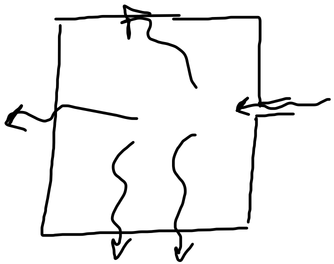
$u(\lambda) = \frac{2\pi c}{\lambda^4} k_B T$

densità di potenza



$I \propto \int_0^{+\infty} u(\lambda)\,d\lambda \propto \int_0^{+\infty} \frac{d\lambda}{\lambda^4}$

Ipotesi di Planck



$$E = \underbrace{n}_{1,2,3\dots} \, \underbrace{h}_{\text{cost. di Planck } 6.63\cdot 10^{-34}\, J\cdot s}\nu$$

ν → frequenza radiazione

$$u(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

Legge di Wien

$$\frac{du}{d\lambda} = 0$$

$$\frac{d}{d\lambda}\left[\frac{1}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right] = 0$$

$$5\lambda K_B T \left(\underbrace{e^{\frac{hc}{\lambda K_B T}} - 1}_{}\right) = hc\, e^{\frac{hc}{\lambda K_B T}}$$

$$\frac{hc}{\lambda K_B T} \gg 1$$

$$12 \quad e^{\frac{hc}{\lambda K_B T}}$$

$$5\lambda K_B T\, \cancel{e^{\frac{hc}{\lambda K_B T}}} \approx hc\, \cancel{e^{\frac{hc}{\lambda K_B T}}} \;;\quad \lambda T \approx \frac{hc}{5 K_B} \approx 2.9\cdot 10^{-3}\ mK$$

Legge di Stefan - Boltzmann

$$I = \int_0^{+\infty} u(\lambda)\, d\lambda = 2\pi c^2 h \int_0^{+\infty} d\lambda \frac{1}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda K_B T}} - 1}$$

$$x \overset{def}{=} \frac{hc}{\lambda K_B T}$$

$$dx = \frac{hc}{K_B T}\, d\left(\frac{1}{\lambda}\right) = -\frac{hc}{K_B T} \frac{d\lambda}{\lambda^2}$$

Se $\lambda \to 0$: $x \to +\infty$

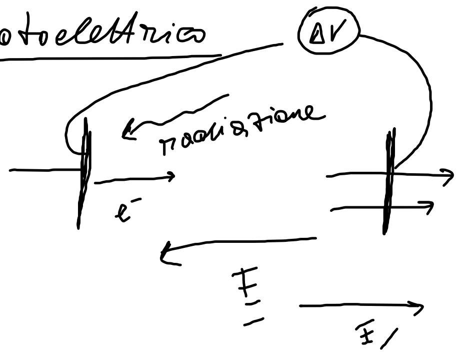
Se $\lambda \to +\infty$: $x \to 0$

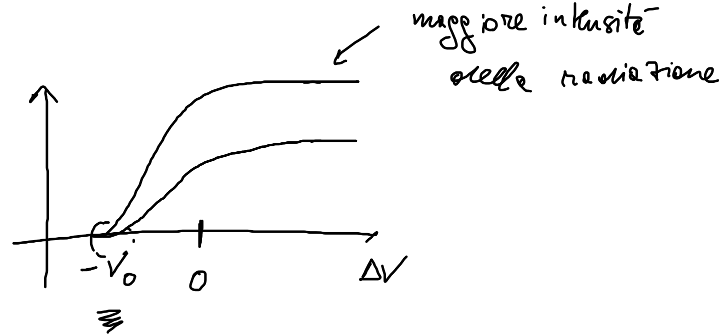
$$= 2\pi c^2 h \, \frac{k_B T}{hc} \int_0^{+\infty} dx \left(\frac{k_B T}{hc}\right)^3 \frac{x^3}{e^x - 1} = \sigma \cdot T^4$$

$$\hookrightarrow \sigma = \frac{2\pi^4 k_B^4}{15 c^3 h^3} \approx 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$$

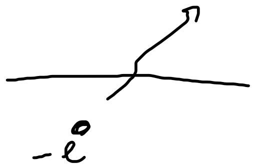
$$\int_0^{+\infty} dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15}$$

Effetto fotoelettrico





Esiste ν_{min} : $\nu_{rad} < \nu_{min} \Rightarrow I = 0$ indipend. dall'intensità della radiazione



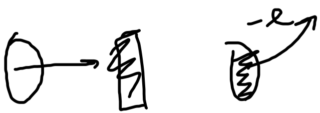
ϕ: funzione lavoro

$h\nu$

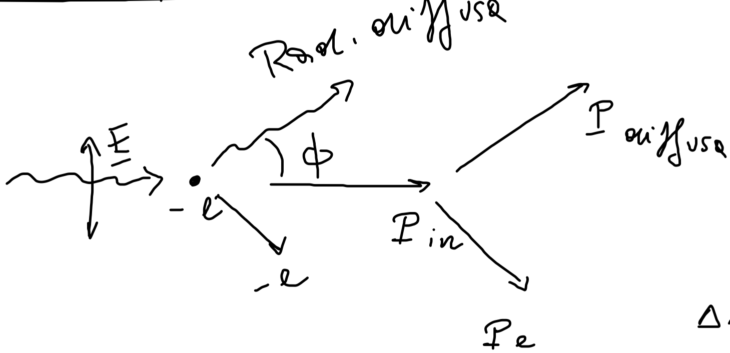
$$\underbrace{h\nu}_{\text{en. radiazione}} = \phi + \underbrace{K}_{\text{en. cinetica}}$$

Situazione limite: $K = 0$ –

$$h\nu_{min} = \phi \; ; \quad \nu_{min} = \phi / h$$



Effetto Compton



$$\underline{E} \propto \cos(\omega t - \underline{kx})$$

$$\underline{a} = \frac{e\underline{E}}{m}$$

$$\Delta\lambda = \lambda_{diffusa} - \lambda_{incidente} = \underbrace{\frac{h}{mc}}_{\simeq 2.4 \cdot 10^{-12}\,m}(1-\cos\phi)$$

Se __non__ cambia ν:

$$\cancel{h\nu} = \cancel{h\nu} + K_e \Rightarrow K_e = 0$$

$K_e > 0$ $\qquad K_e = h(\nu_{diff} - \nu_{inc}) > 0 \Rightarrow \nu_{diff} > \nu_{inc}$

$$\lambda_{diff} < \lambda_{inc}$$

$\Delta\lambda \approx 10^{-12}\ m$

$\lambda_{visibile} \approx 400 \div 700\ nm \doteq 4 \div 7 \cdot 10^{-7}\ m$

$400\ nm + 10^{-12}\ m = 4 \cdot 10^{-7} + 10^{-12}$

$= (4.00001) \cdot 10^{-7}\ m$

Raggi X: $\lambda \approx 10^{-10}\ m$

(Å)