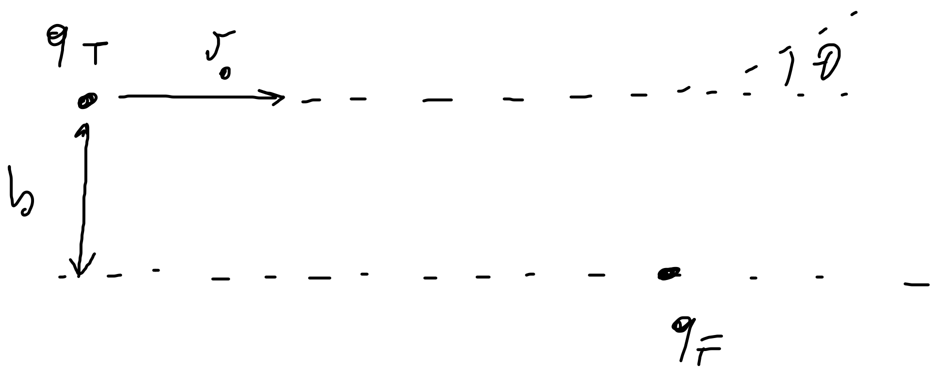
Coulomb collisions



$$\tg\left(\frac{\theta}{2}\right) = \frac{q_T q_F}{4\pi\varepsilon_0 b \mu v_0^2}$$

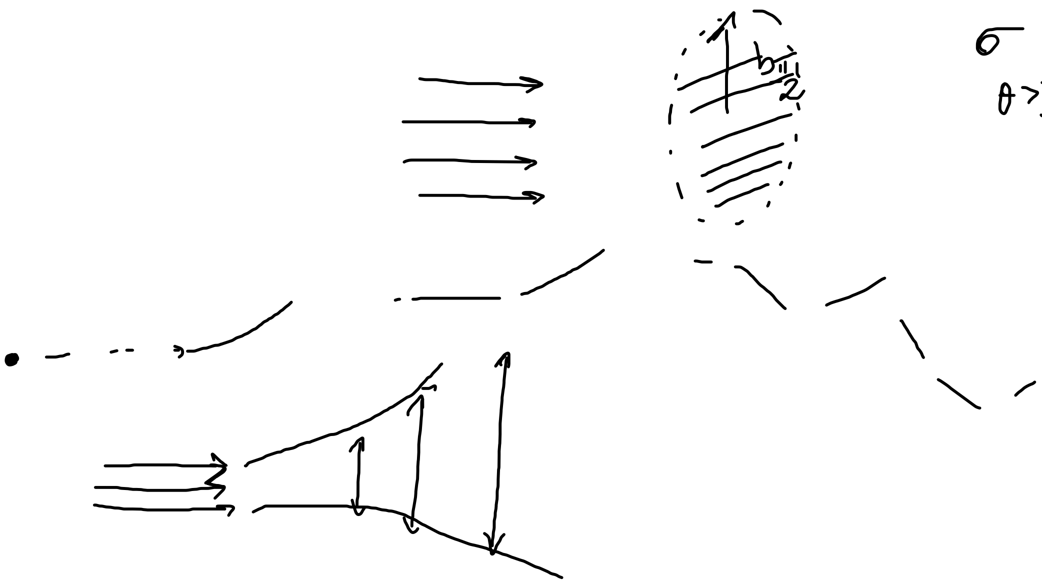
reduced mass $\mu = \frac{m_T m_F}{m_T + m_F}$

Small angle collision $\theta < \pi/2$

Large " " $\theta > \pi/2$

$b_{\frac{\pi}{2}}$: impact parameter corresponding to $\theta = \frac{\pi}{2}$

$$1 = \frac{q_T q_F}{4\pi\varepsilon_0 \mu b_{\frac{\pi}{2}} v_0^2} \rightarrow b_{\frac{\pi}{2}} = \frac{q_T q_F}{4\pi\varepsilon_0 \mu v_0^2}$$



$$\sigma_{\theta > \frac{\pi}{2}} = \pi b_{\frac{\pi}{2}}^2$$

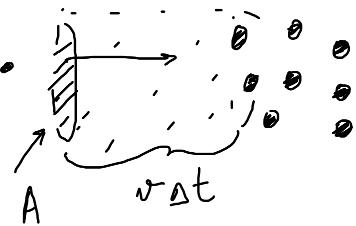
$$\langle \theta \rangle = 0$$

$$\langle \theta^2 \rangle \neq 0$$

$$\sum_{i=1}^{N} \theta_i^2 \approx 1$$

$$\sum \theta^2 \times (\text{prob. to obtain } \theta^2 \text{ in a collision}) \approx 1$$

n density of part.



Δt

$$\#\text{particles} = n \cdot A\, v\, \Delta t$$

$$\text{Probability} = \frac{\sigma \cdot n v \Delta t \cancel{A}}{\cancel{A}} = \sigma n v \Delta t$$

$$\Gamma = \text{Flux} = \frac{\#\text{particles}}{\text{area} \cdot \Delta t} = \frac{n \cancel{A} v \cancel{\Delta t}}{\cancel{A}\, \cancel{\Delta t}} = n \cdot v$$

Assume that an interaction has occurred

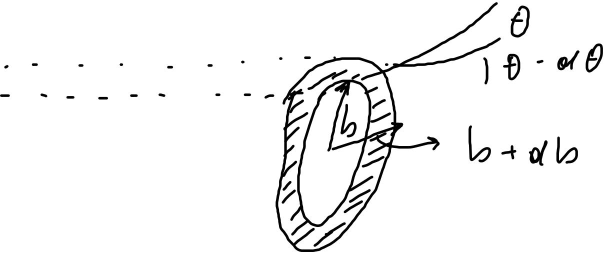
1) Prob $= 1$

2) $\Delta t = \nu^{-1}$

ν : interaction frequency

$$1 = \sigma \overbrace{n v}^{\Gamma} \nu^{-1}$$

$$\Rightarrow \boxed{\nu = \sigma \Gamma}$$



$$\sigma(\theta) = \pi (b + db)^2 - \pi b^2 =$$

$$= \cancel{\pi b^2} + 2\pi b\, db + O\left((db)^2\right) - \cancel{\pi b^2} =$$

$$= 2\pi b\, db$$

$$\nu^{-1}$$

$$\sum \vartheta^2(b) \times (\text{prob}) = \sum \vartheta^2(b)\, \Gamma \cdot \Delta t \cdot \sigma(\theta) = \int \vartheta^2(b)\, \Gamma \cdot \Delta t\, 2\pi b\, db \approx 1$$

$$\sigma^* = \int db\, 2\pi b\, \vartheta^2(b) \qquad \mathrm{tg}\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$$

$$\theta \ll 1$$

$$(\Gamma \cdot \sigma^*)^{-1}$$

$$\sigma^{*} \approx \int db\, 2\pi \cancel{b} \left(\frac{q_T^2 q_F^2}{\frac{4\pi^2 \varepsilon_0^2 \cancel{b^2} \mu^2 v_0^4}{\theta^2}} \right) = (\mathrm{coeff}) \int_{b_{\pi/2}}^{\lambda_D} \frac{db}{b}$$

$$= (\mathrm{coeff}) \ln b \Big|_{b_{\pi/2}}^{\lambda_D} = \frac{q_T^2 q_F^2}{2\pi \varepsilon_0^2 \mu^2 v_0^4} \underbrace{\ln\left(\frac{\lambda_D}{b_{\pi/2}}\right)}_{\substack{\text{Coulomb} \\ \text{logarithm} \\ 15 \div 30}}$$

$$\frac{\sigma^{*}}{\pi b^2_{\pi/2}} = \frac{\frac{\cancel{q_T^2 q_F^2}}{2\cancel{\pi}\varepsilon_0^2 \cancel{\mu^2 v_0^4}} \ln\Lambda}{\cancel{\pi} \frac{\cancel{q_T^2 q_F^2}}{16\pi^2 \cancel{\varepsilon_0^2 \mu^2 v_0^4}}} \approx 8 \ln\Lambda$$

$$\approx 200$$

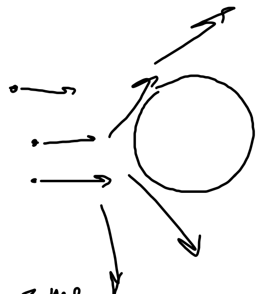
Collision frequencies

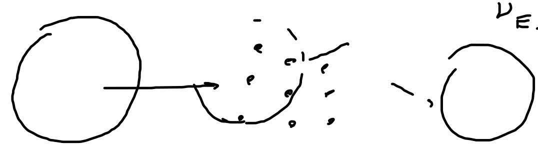
$i \quad e$

$\begin{cases} e-e \\ i-i \\ i-e \\ e-i \end{cases}$

Momentum exchange $\begin{cases} \nu_{ee} \\ \nu_{ii} \\ \nu_{ei} \\ \nu_{ie} \end{cases}$

Energy exchange $\begin{cases} \nu_{Eee} \\ \nu_{Eii} \\ \nu_{Eei} \\ \nu_{Eie} \end{cases}$





$v_{th} = \left(\frac{2T}{m}\right)^{\frac{1}{2}}$

$e-e \quad \mu = \frac{m_e^2}{2m_e} = \frac{m_e}{2} \qquad \nu_{ei} \approx \nu_{ee}$

$\nu = n \cdot \sigma^* \cdot v_{rel} \qquad \nu_{ee} \qquad v_{rel} \approx v_{the}$

$\sigma^* = \frac{1}{2\pi}\left(\frac{q_T q_F}{\varepsilon_0 \mu v_0^2}\right)^2 \ln\Lambda \qquad \nu_{ei} \qquad T_e \approx T_i$

$e-i \quad \mu = \frac{m_e m_i}{m_e + m_i} \approx \frac{m_e \cancel{m_i}}{\cancel{m_i}} \approx m_e$

$v_{rel} \approx v_{the} \qquad v_{the} \gg v_{thi}$

	ν_{ii}	ν_{ee}
μ	$\simeq m_i$	$\simeq m_e$
v_{rel}	$\simeq v_{th\,i}$	$\simeq v_{th\,e}$

$$\frac{\nu_{ee}}{\nu_{ii}} = \frac{v_{th\,e}}{v_{th\,i}} \cdot \frac{\sigma^{*}_{ee}}{\sigma^{*}_{ii}}$$

$$= \frac{\cancel{v_{th\,e}}}{\cancel{v_{th\,i}}} \; \frac{1}{\mu_{ee}^{2} {v^{*}_{th\,e}}^{3}} \cdot \mu_{ii}^{2} \cdot {v^{*}_{th\,i}}^{3}$$

$$v_{th} \sim \frac{1}{m^{\frac{1}{2}}}$$

$$\sim \frac{m_i^2}{m_e^2} \frac{v_{th\,i}^3}{v_{th\,e}^3} \sim \frac{m_i^2}{m_e^2} \frac{m_e^{3/2}}{m_i^{3/2}} \sim \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \gg 1$$

$$\nu_{ee} \sim \nu_{ei} \gg \nu_{ii}$$

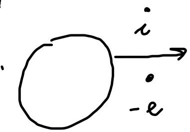
$$\nu_{ie}$$

$$\frac{\Delta v_e}{v_e} \approx 1$$

$$m_i \Delta \underline{v}_i + m_e \Delta \underline{v}_e = 0$$

$$\frac{\Delta v_i}{v_i} \simeq \frac{m_e}{m_i}$$

$$\Delta v_i = -\frac{m_e}{m_i} \Delta v_i \approx -\frac{m_e}{m_i} 2 v_i$$



$$\Delta v_e = 2 v_i$$

For i against e it takes $\frac{m_i}{m_e}$ more collissions to

get $\frac{\Delta v}{v} \approx 1$

$$\nu_{ie} \sim \frac{m_e}{m_i} \nu_{ee}$$

$1 \qquad \nu_{ee} \sim \nu_{ei}$

$\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \qquad \nu_{ii}$

$\left(\frac{m_e}{m_i}\right) \qquad \nu_{ie}$