$e \quad i$

ν_{ee}

ν_{ei}

ν_{ie}

ν_{ii}

$O(1)$	$O\left(\sqrt{\dfrac{m_e}{m_i}}\right)$	$O\left(\dfrac{m_e}{m_i}\right)$
ν_{ee}	ν_{ii}	ν_{ie}
ν_{ei}		

$e - e$

Fast e e at rest

Before the collision

After " "

$\nu_{Eee} \sim \nu_{ee}$

$\nu_{Eii} \sim \nu_{ii}$

Fast ion against e at rest

After the collision

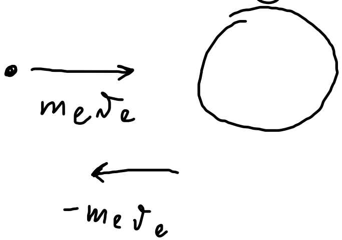
$\underline{v}_e = 2\underline{v}_i$

$\frac{1}{2} m_e v_e^2 = \frac{1}{2} m_e 4 v_i^2$

$= 4 \frac{m_e}{m_i} \left(\frac{1}{2} m_i v_i^2 \right)$

$\nu_{Eei} \sim \frac{m_e}{m_i} \nu_{ei}$

Fast electron against ion at rest



$$m_e v_e^i + \underbrace{m_i v_i^i}_{0} = m_e v_e^f + m_i v_i^f$$

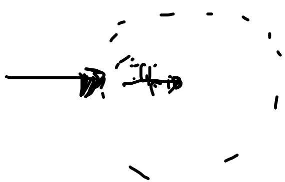
$$v_i^f = \frac{m_e (\overbrace{v_e^i - v_e^f}^{2v_e})}{m_i} \qquad v_i = \frac{2 m_e v_e}{m_i}$$

$$\frac{1}{2} m_i v_i^2 = \frac{1}{2} \cancel{m_i} \frac{4 m_e^2}{m_i^{\cancel{2}}} v_e^2 = \frac{4 m_e}{m_i} \left(\frac{1}{2} m_e v_e^2 \right)$$

$$\nu_{E_{ei}} \sim \frac{m_e}{m_i} \cdot \nu_{E_{ee}}$$

$O(1)$	$O\left(\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}}\right)$	$O\left(\frac{m_e}{m_i}\right)$
ν_{ee}	ν_{ii}	ν_{ie}
ν_{ei}	$\nu_{E_{ii}}$	$\rightarrow \nu_{E_{ie}}$
$\nu_{E_{ee}}$		$\rightarrow \nu_{E_{ei}}$
e.g. ms	100 ms	s

Neutral beam injection $\qquad T_i > T_e$



$$m\frac{d\underline{v}}{dt} = q\underline{E} - m\nu\underline{v}$$ friction due to collisions

$\underline{E}$ required to drive a current

Steady state: $\frac{d\underline{v}}{dt} = 0$

$$\underline{v} = \frac{q}{m\nu}\underline{E}$$

$\eta \propto \nu \propto \frac{1}{T^{3/2}}$ $\underline{E} = \eta\underline{j}$ $\underline{j}$: current density

$\Uparrow$

$v_{th} \propto T^{\frac{1}{2}}$

$\updownarrow$

$\nu \propto \frac{1}{v_{th}^3}$

$$\underline{j} = nq\underline{v} = \frac{nqq\underline{E}}{m\nu} = \frac{nq^2}{m\nu}\cdot\underline{E}$$ $\to \eta^{-1}$

$$\eta = \frac{m\nu}{nq^2}$$

$\nu = \Gamma\cdot\sigma \to \sigma^*$

$= n\cdot v\cdot\sigma^*$ $\nu \propto \frac{1}{v^3}$

$$\sigma^* = \frac{q_T^2 q_F^2}{2\pi\varepsilon_0^2\mu^2 v^4}\ln\Lambda$$

Collisions with neutrals

1) Elastic collision

2) Inelastic collisions $\to$ ionization $\quad e + N \to e + i + e^-$

$\to$ excitation of a neutral

3) Charge exchange

$$\underset{\text{fast}}{i} + N \to \underset{\text{fast}}{N} + \underset{\text{slow}}{i}$$

light

$$\sigma \sim \pi a_0^2 \approx 10^{-20}\ m^2$$

Å

Charged particle motion in $\underline{E}(\underline{x},t)$ and $\underline{B}(\underline{x},t)$

$$m\frac{d\underline{v}}{dt} = q(\underline{v}\times\underline{B})$$

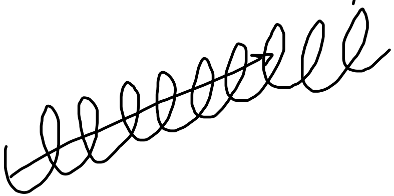
If $\underline{B}$ is $\overrightarrow{\text{const}}$ or it is not

$$\underbrace{\underline{v}\cdot\left[m\frac{d\underline{v}}{dt}\right]}_{} = q(\underline{v}\times\underline{B})\cdot\underline{v}$$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = 0 \implies \frac{1}{2}mv^2 = \text{const} \implies |\underline{v}| = \text{const}$$

$\underline{B} = \underline{\text{const}}$

$\begin{cases} \text{along the field line: uniform straight motion} \quad v_{\parallel} \\ \text{in a plane} \perp \text{to the field line: circular motion} \end{cases}$

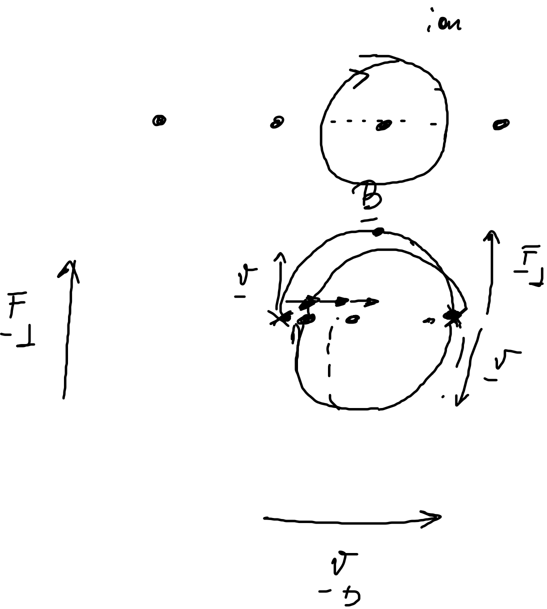


$$r_L = \frac{m v_\perp}{qB} \qquad \omega_L = \frac{qB}{m}$$

$\underline{B}$ + an external $\underline{F} = \vec{\text{const}}$

$$\underline{F} = \underline{F}_{\parallel} + \underline{F}_{\perp}$$

↳ accelerated straight motion along $\underline{B}$



$\underline{F}_\perp = \underline{0}$ $\quad r_L = \frac{m v_\perp}{qB}$

$r_{L\,up} > r_L$

$r_{L\,down} < r_L$

$$m\frac{d\underline{v}}{dt} = q(\underline{v}\times\underline{B}) + \underline{F}$$

$\parallel$ $\quad m\, dv_\parallel/dt = F_\parallel \Rightarrow v_\parallel(t) = \frac{F_\parallel}{m}\cdot t + v_\parallel(0)$

$\perp$ $\quad m\, d\underline{v}_\perp/dt = q(\underline{v}_\perp\times\underline{B}) + \underline{F}_\perp$

Assume that

$$\underline{v}_{\perp} = \underline{v}_L + \underline{v}_D$$

$$\underbrace{\cancel{m \frac{d\underline{v}_{\perp}}{dt}}}_{\text{Larmor motion}} + m \frac{d\underline{v}_D}{dt} = \underbrace{\cancel{q \underline{v}_L \times \underline{B}}}_{\text{Larmor motion}} + q \underline{v}_D \times \underline{B} + \underline{F}_{\perp}$$

$$\underline{v}_D = \underline{\text{const}}$$

$$\Rightarrow \frac{d\underline{v}_D}{dt} = 0$$

$$m \frac{d\underline{v}_D}{dt} = q(\underline{v}_D \times \underline{B}) + \underline{F}_{\perp}$$

$\times \underline{B}$

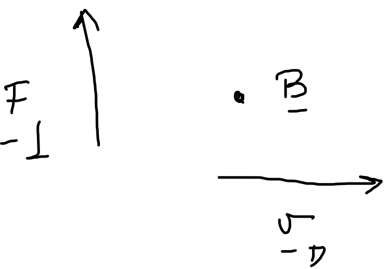
$$0 = q(\underline{v}_D \times \underline{B}) + \underline{F}_{\perp} \quad \times \underline{B}$$

$$0 = \underbrace{q(\underline{v}_D \times \underline{B}) \times \underline{B}} + \underline{F}_\perp \times \underline{B}$$

$$\underline{A} \times (\underline{B} \times \underline{C}) = (\underline{C} \times \underline{B}) \times \underline{A} = (\underline{A}\cdot\underline{C})\underline{B} - (\underline{A}\cdot\underline{B})\underline{C}$$

$$(\underline{v}_D \times \underline{B}) \times \underline{B} = \underbrace{\cancel{(\underline{v}_D \cdot \underline{B})}}_{0}\underline{B} - B^2\underline{v}_D = -B^2\underline{v}_D$$

$$0 = -qB^2\underline{v}_D + \underline{F}_\perp \times \underline{B} \Rightarrow \underline{v}_D = \frac{\underline{F}_\perp \times \underline{B}}{qB^2}$$



If $\underline{F}_\perp = m\vec{g}$

If $\underline{F}_\perp = q\underline{E}_\perp$

$\underline{v}_{De}$ and $\underline{v}_{Di}$ will be opposite

$$\underline{v}_D = \underline{E} \times \underline{B} / B^2$$