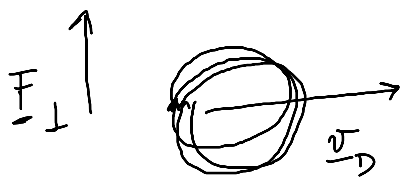
$\underline{F} = \overrightarrow{\text{const}}$

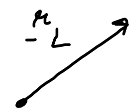
$\parallel \quad a_\parallel = F_\parallel / m \Rightarrow v_\parallel(t) = v_{0\parallel} + a_\parallel t$

$\perp$ Larmor motion + $\underline{v}_D = \dfrac{\underline{F}_\perp \times \underline{B}}{qB^2}$



Non uniform $\underline{E}(\underline{x}, t)$ $\underline{B}(\underline{x}, t)$

Sufficiently small variation



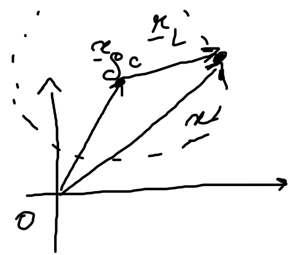
T_L - time scale

scale where → $\Delta x \gg r_L$. $\Delta t \gg T_L$

$|\underline{E}|$ and $|\underline{B}|$ change significantly

$$m \frac{d\underline{v}}{dt} = q\underline{E}(\underline{x}, t) + q\, \underline{v} \times \underline{B}(\underline{x}, t)$$

Small variations: gyro-centre $\underline{x}_{gc}$



$\underline{r}_L$ Larmor radius

$\underline{v} = \underline{v}_{gc} + \underline{v}_L$

$\underline{x} = \underline{x}_{gc} + \underline{r}_L$

$$\underline{E}(\underline{x}, t) = \underline{E}(\underline{x}_{gc} + \underline{r}_L, t) \approx \underline{E}(\underline{x}_{gc}) + (\underline{r}_L \cdot \underline{\nabla})\, \underline{E}(\underline{x}) \Big|_{\underline{x}_{gc}}$$

$r_{Lx}\partial_x + r_{Ly}\partial_y + r_{Lz}\partial_z$

Taylor r_L expansion around $\underline{x}_{gc}$ 1st order

$$\underline{B}(\underline{x}, t) \approx \underline{B}(\underline{x}_{gc}) + (\underline{r}_L \cdot \underline{\nabla})\, \underline{B} \Big|_{\underline{x} = \underline{x}_{gc}}$$

$$(1) \quad m\frac{d\mathbf{v}_{gc}}{dt} + m\frac{d\mathbf{v}_L}{dt} = q\left[\mathbf{E}(\mathbf{x}_{gc}) + (\mathbf{r}_L\cdot\nabla)\mathbf{E}(\mathbf{x})\Big|_{\mathbf{x}_{gc}} + (\mathbf{v}_{gc}+\mathbf{v}_L)\times \left(\mathbf{B}(\mathbf{x}_{gc}) + (\mathbf{r}_L\cdot\nabla)\,\mathbf{B}(\mathbf{x})\Big|_{\mathbf{x}_{gc}}\right)\right]$$

Larmor motion:

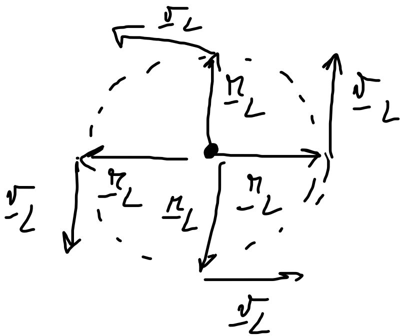
$$(2) \quad m\frac{d\mathbf{v}_L}{dt} = q\left(\mathbf{v}_L \times \mathbf{B}(\mathbf{x}_{gc})\right)$$

$$(1)-(2): \quad \left\langle m\frac{d\mathbf{v}_{gc}}{dt}\right\rangle = \left\langle q\left[\mathbf{E}(\mathbf{x}_{gc}) + \cancel{(\mathbf{r}_L\cdot\nabla)\mathbf{E}(\mathbf{x})\Big|_{\mathbf{x}_{gc}}} + \mathbf{v}_{gc}\times\mathbf{B}(\mathbf{x}_{gc}) + \mathbf{v}_{gc}\times\cancel{(\mathbf{r}_L\cdot\nabla)\mathbf{B}(\mathbf{x})\Big|_{\mathbf{x}_{gc}}} + \mathbf{v}_L\times(\mathbf{r}_L\cdot\nabla)\mathbf{B}\right]\right\rangle$$

Time average over a T_L : $\langle \quad \rangle$

Time average $\langle\ \rangle$

Linear terms $\begin{cases} \underline{r}_L = 0 \\ \underline{v}_L \end{cases}$



after averaging:

$$m \frac{d\underline{v}_{gc}}{dt} = \overset{//,\perp}{q\,\underline{E}(\underline{x}_{gc})} + \overset{\perp}{q\,\underline{v}_{gc} \times \underline{B}(\underline{x}_{gc})} + \overset{//,\perp}{\langle q\,\underline{v}_L \times (\underline{r}_L \cdot \underline{\nabla})\underline{B} \rangle}$$

Separate // and $\perp$ terms

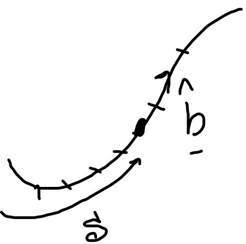
$$\underline{v}_{gc} = \underline{v}_{\perp gc} + v_{/\!/ gc}\,\hat{\underline{b}}$$

$\hat{\underline{b}}$: versor associate to $\vec{\underline{B}}$

$$\frac{d\underline{v}_{gc}}{dt} = \frac{d}{dt}\left(\underline{v}_{gc\perp}\right) + \frac{d}{dt}\left(v_{\parallel gc}\,\hat{\underline{b}}\right)$$

$$= \frac{d\underline{v}_{gc\perp}}{dt} + \frac{dv_{\parallel}}{dt}\,\hat{\underline{b}} + v_{\parallel}\,\frac{d\hat{\underline{b}}}{dt}$$

$$\frac{\partial}{\partial s} = (\hat{\underline{b}}\cdot\underline{\nabla})$$



$$\hat{\underline{b}} = \hat{\underline{b}}(s)$$

$$\frac{d\hat{\underline{b}}}{dt} = \frac{\partial\hat{\underline{b}}}{\partial s}\cdot\underbrace{\frac{\partial s}{\partial t}}_{v_{\parallel}} = v_{\parallel}\frac{\partial\hat{\underline{b}}}{\partial s} = v_{\parallel}\,(\hat{\underline{b}}\cdot\underline{\nabla})\,\hat{\underline{b}}$$

$$\frac{d\underline{v}_{gc}}{dt} = \overset{\perp}{\frac{d\underline{v}_{gc\perp}}{dt}} + \overset{\parallel}{\frac{dv_{\parallel}}{dt}\,\hat{\underline{b}}} + \overset{\perp}{v_{\parallel}^{2}\,(\hat{\underline{b}}\cdot\underline{\nabla})\,\hat{\underline{b}}}$$

$\parallel$ $$m\frac{dv_{\parallel}}{dt} = qE_{\parallel} + \langle q\, \underline{v}_L \times (\underline{r}_L\cdot\nabla)\underline{B}\rangle_{\parallel}$$

$\perp$ $$m\frac{d\underline{v}_{\perp gc}}{dt} = q\,\underline{v}_{gc}\times\underline{B}(\underline{x}_{gc}) + \underbrace{q\underline{E}_{\perp} + \langle q\,\underline{v}_L\times(\underline{r}_L\cdot\nabla)\underline{B}\rangle_{\perp} - m v_{\parallel}^2(\hat{\underline{b}}\cdot\nabla)\hat{\underline{b}}}_{\underline{F}_{\perp}}$$

$$m\frac{d\underline{v}_{\perp gc}}{dt} = q\,\underline{v}_{gc}\times\underline{B}(\underline{x}_{gc}) + \underline{F}_{\perp}$$

motion in a "uniform" magnetic field + a force

If the force is const:

$$\underline{v} = \underline{v}_L + \underline{v}_{gc} \qquad \underline{v}_{gc} = \frac{\underline{F}_{\perp}\times\underline{B}}{qB^2}$$

$$\underline{v}_D = \underbrace{\underline{v}_D^{(0)}}_{\frac{\underline{F}_\perp \times \underline{B}}{qB^2}} + \underline{v}_D^{(1)} \qquad |\underline{v}_D^{(1)}| \ll |\underline{v}_D^{(0)}|$$

$$m \frac{d}{dt}\left(\underline{v}_D^{(0)} + \underline{v}_D^{(1)}\right) = q\left[\left(\underline{v}_D^{(0)} + \underline{v}_D^{(1)}\right) \times \underline{B}\right] + \underline{F}_\perp$$

$$\frac{d\underline{v}_D^{(1)}}{dt} \ll \frac{d\underline{v}_D^{(0)}}{dt}$$

$$\times \underline{B}$$

$$m \frac{d\underline{v}_D^{(0)}}{dt} \approx \cancel{q\,\underbrace{\underline{v}_D^{(0)} \times \underline{B}}} + q\,\underline{v}_D^{(1)} \times \underline{B} + \cancel{\underline{F}_\perp} \quad \times \underline{B}$$

$$(\underline{C} \times \underline{B}) \times \underline{A} = (\underline{A} \cdot \underline{C})\underline{B} - (\underline{A} \cdot \underline{B})\underline{C}$$

$$(\underline{F}_\perp \times \underline{B}) \times \underline{B} = \cancel{(\underline{B} \cdot \underline{F}_\perp)\underline{B}} - B^2 \underline{F}_\perp$$

$$\frac{(\underline{F}_\perp \times \underline{B}) \times \underline{B}}{qB^2} = -\frac{B^2 \underline{F}_\perp}{qB^2} = -\frac{\underline{F}_\perp}{q}$$

$$m\,\frac{d\underline{v}_D^{(0)}}{dt}\times\underline{B} = q\left(\underline{v}_D^{(1)}\times\underline{B}\right)\times\underline{B}$$

$$= q\left(\underline{v}_D^{(1)}\cdot\underline{B}\right)\underline{B} - B^2\,\underline{v}_d^{(1)}\,q$$

$$\boxed{\underline{v}_d^{(1)} = -\frac{m}{qB^2}\,\frac{d\underline{v}_D^{(0)}}{dt}\times\underline{B}}$$

$$\underline{v}_D^{(0)} = \frac{\underline{F}_\perp\times\underline{B}}{qB^2}$$

$$\underline{v}_d^{(1)} = -\frac{m}{qB^2}\,\frac{d}{dt}\left[\frac{\underline{F}_\perp\times\underline{B}}{qB^2}\right]\times\underline{B} \longrightarrow -B^2\,\frac{d\underline{F}_\perp}{dt}$$

$$\underline{v}_D^{(1)} = \frac{m}{q^2B^2}\,\frac{d\underline{F}_\perp}{dt}$$

If $\underline{F}_\perp = q\,\underline{E}_\perp$

$$\underline{v}_D^{(1)} = \frac{m}{qB^2}\frac{d\underline{E}_\perp}{dt} \qquad \underline{v}_D^{(1)} \parallel \underline{E}_\perp$$

$\underline{v}_D^{(1)}$ is charge dependent

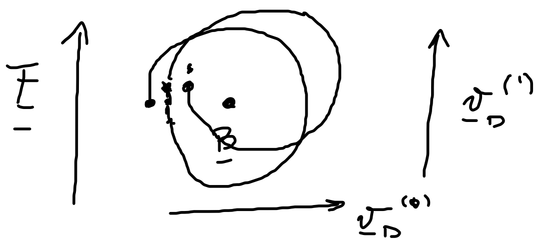
$\Rightarrow$ it drives a current

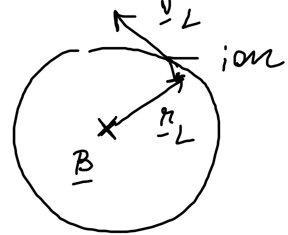
$\pm$

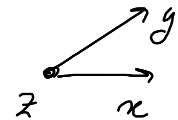
polarization drift

$\underline{E} = \overrightarrow{\text{const}}$

$\underline{E}$ increases as a function of time







$$q\langle \underline{v}_L \times (\underline{r}_L\cdot\underline{\nabla})\underline{B}\rangle = ?$$

$$\underline{B} = B_x\hat{\underline{i}} + B_y\hat{\underline{j}} + B_z\hat{\underline{k}}$$

$$\underline{v}_L = v_L\left[-\sin(\omega_L t)\,\hat{\underline{i}} + \cos(\omega_L t)\,\hat{\underline{j}}\right]$$

$$\underline{r}_L = \frac{v_L}{\omega_L}\left[\cos(\omega_L t)\,\hat{\underline{i}} + \sin(\omega_L t)\,\hat{\underline{j}}\right]$$

$$\frac{v_L}{\omega_L} = \frac{v_L m}{qB} = r_L$$

$$r_L q v_L\left\langle \left[-\sin(\omega_L t)\,\hat{\underline{i}} + \cos(\omega_L t)\,\hat{\underline{j}}\right] \times \left(\left[\cos(\omega_L t)\,\hat{\underline{e}} + \sin(\omega_L t)\,\hat{\underline{j}}\right]\cdot \frac{\partial}{\partial x}\hat{\underline{i}} + \frac{\partial}{\partial y}\hat{\underline{j}} + \frac{\partial}{\partial z}\hat{\underline{k}}\right)\underline{B}\right\rangle$$

$$= q r_L v_L \left\langle \left(-\sin(\omega_L t)\, \hat{\underline{i}} + \cos(\omega_L t)\, \hat{\underline{j}} \right) \times \right.$$

$$\left. \left(\cos(\omega_L t) \left(\frac{\partial B_x}{\partial x} \hat{\underline{i}} + \frac{\partial B_y}{\partial x} \hat{\underline{j}} + \frac{\partial B_z}{\partial x} \hat{\underline{k}} \right) + \sin(\omega_L t) \left(\frac{\partial B_x}{\partial y} \hat{\underline{i}} + \frac{\partial B_y}{\partial y} \hat{\underline{j}} + \frac{\partial B_z}{\partial y} \hat{\underline{k}} \right) \right\rangle \right.$$