$\parallel$

$$m\frac{d v_\parallel}{dt} = q\left[E_\parallel + \langle \underline{v}_\perp \times (\underline{r}_\perp\cdot\underline{\nabla})\,\underline{B}\rangle_\parallel\right]$$

$\perp$

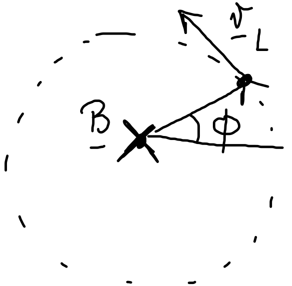
$$m\frac{d\underline{v}_\perp}{dt} = q\,\underline{v}_{gc}\times\underline{B} + \underline{F}_\perp$$

$$\underline{F}_\perp = q\,\underline{E}_\perp + \overbrace{q\,\langle \underline{v}_\perp \times (\underline{r}_\perp\cdot\underline{\nabla})\,\underline{B}\rangle_\perp}^{} \overbrace{- m\,\underline{v}_{gc}^{2}\,(\hat{\underline{b}}\cdot\underline{\nabla})\underline{b}}^{A}$$

$$\underline{v}_D^{(0)} = \frac{\underline{F}_\perp\times\underline{B}}{qB^2} \qquad \underline{v}_D^{(1)} = \frac{m}{q^2B^2}\,\frac{d\underline{F}_\perp}{dt}$$

$q \langle \underline{v}_L \times (\underline{r}_L \cdot \underline{\nabla}) \underline{B} \rangle = ?$

In $\underline{x} = \underline{x}_{gc}$ $\quad \underline{B} \not\parallel -\hat{\underline{k}}$



$\phi = \omega_L t$ at the particle position

$\underline{B} = B_x \hat{\underline{i}} + B_y \hat{\underline{j}} + B_z \hat{\underline{k}}$

$$\underline{v}_L = \left[-\sin(\omega_L t)\, \hat{\underline{i}} + \cos(\omega_L t)\, \hat{\underline{j}} \right] v_L$$

$$\underline{r}_L = \frac{v_L}{\omega_L} \left[\cos(\omega_L t)\, \hat{\underline{i}} + \sin(\omega_L t)\, \hat{\underline{j}} \right]$$

$$\frac{v_L^2 q}{\omega_L} \left\langle \left(-\sin(\omega_L t)\, \hat{\underline{i}} + \cos(\omega_L t)\, \hat{\underline{j}} \right) \times \left(\cos(\omega_L t)\, \hat{\underline{i}} + \sin(\omega_L t)\, \hat{\underline{j}} \right) \cdot \left[\frac{\partial}{\partial x} \hat{\underline{i}} + \frac{\partial}{\partial y} \hat{\underline{j}} + \frac{\partial}{\partial z} \hat{\underline{k}} \right] \left(B_x \hat{\underline{i}} + B_y \hat{\underline{j}} + B_z \hat{\underline{k}} \right) \right\rangle$$

$$= \frac{q v_L^2}{\omega_L} \left\langle \left(-\sin(\omega_L t)\,\hat{\underline{i}} + \underline{\cos(\omega_L t)}\,\hat{\underline{j}}\right) \times \left(\underline{\cos(\omega_L t)} \left(\frac{\partial B_x}{\partial x}\hat{\underline{i}} + \frac{\partial B_y}{\partial x}\hat{\underline{j}} + \frac{\partial B_z}{\partial x}\hat{\underline{k}}\right) + \sin(\omega_L \underline{t}) \left(\frac{\partial B_x}{\partial y}\hat{\underline{i}} + \frac{\partial B_y}{\partial y}\hat{\underline{j}} + \frac{\partial B_z}{\partial y}\hat{\underline{k}}\right)\right)\right\rangle$$

$$\langle \sin(\omega_L t)\rangle = 0$$

$$\langle \cos(\omega_L t)\rangle = 0$$

$$\langle \sin(\omega_L t)\cos(\omega_L t)\rangle = 0$$

$$\langle \sin^2(\omega_L t)\rangle = \langle \cos^2(\omega_L t)\rangle = \frac{1}{2}$$

$$= \frac{q v_L^2}{2\omega_L} \left(-\frac{\partial B_x}{\partial x}\hat{\underline{k}} + \frac{\partial B_z}{\partial x}\hat{\underline{i}} - \frac{\partial B_y}{\partial y}\hat{\underline{k}} + \frac{\partial B_z}{\partial y}\hat{\underline{j}} \right)$$

$$\nabla \cdot \underline{B} = 0 \Rightarrow \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\Rightarrow -\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y} = \frac{\partial B_z}{\partial z}$$

$$= \frac{q v_L^2}{2\omega_L} \cdot \left(\frac{\partial B_z}{\partial x}\hat{\underline{i}} + \frac{\partial B_z}{\partial y}\hat{\underline{j}} + \frac{\partial B_z}{\partial z}\hat{\underline{k}} \right)$$

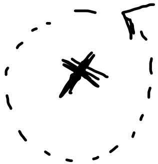
$$= \frac{q v_L^2}{2\omega_L} - \nabla |B| (x_{gc})$$

$$F = -\frac{q v_L^2}{2\omega_L} \nabla |B| (x_{gc})$$

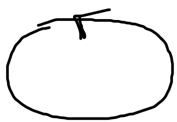
$$\frac{\cancel{q} v_L^2}{2 \cancel{q} B} m = \frac{m v_L^2}{2B} = \frac{K_\perp}{B} = \text{magnetic momentum}$$

μ

ion



$$A = \pi \cdot r_L^2 \qquad m = \pi r_L^2 \frac{q}{T_L}$$



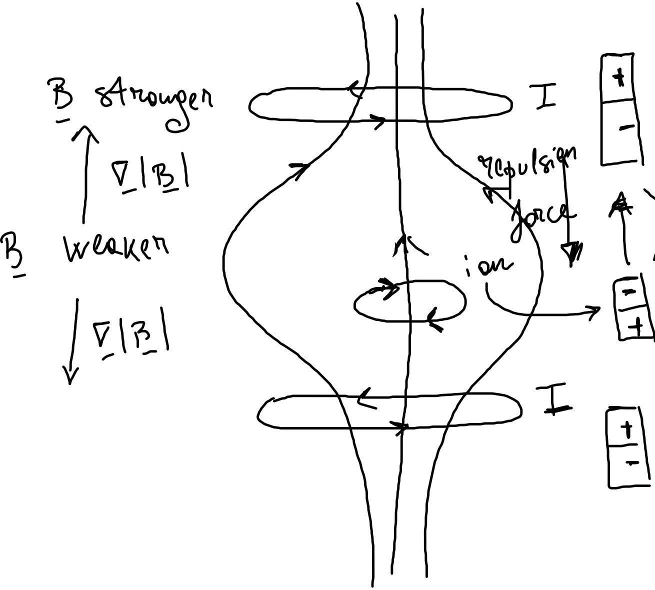
I

$$m = I \cdot A$$

$$= \pi \frac{m^2 v_L^2}{q^2 B^2} \cdot \frac{q}{2\pi} \cdot \omega_L$$

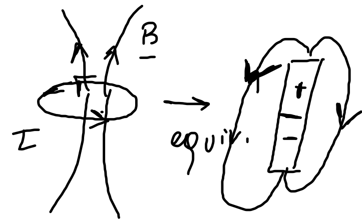
$$= \cancel{\pi} \frac{m^{\cancel{2}} v_L^2}{\cancel{q^2} B^{\cancel{2}}} \frac{\cancel{q}}{2\cancel{\pi}} \frac{\cancel{q} \cancel{B}}{\cancel{m}}$$

$$\mu = \frac{m v_L^2}{2B}$$

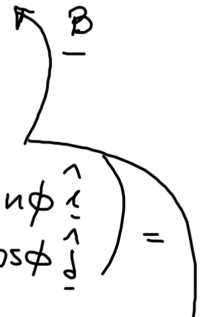


Magnetic bottle

Oersted experiment



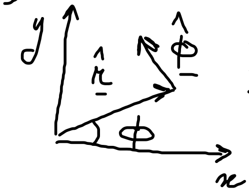
$$\underline{F} = -m v_{\parallel}^2 \left(\hat{\underline{b}}\cdot\underline{\nabla}\right)\hat{\underline{b}}$$



$$\hat{\underline{b}} = \hat{\underline{\phi}}$$

$$\hat{\underline{\phi}} = -\sin\phi\,\hat{\underline{i}} + \cos\phi\,\hat{\underline{j}}$$

$$\underline{F} = -m v_{\parallel}^2 \underbrace{\left(\frac{1}{r}\frac{\partial}{\partial\phi}\right)}_{\hat{\underline{\phi}}\cdot\underline{\nabla}} \begin{pmatrix} -\sin\phi\,\hat{\underline{i}} \\ +\cos\phi\,\hat{\underline{j}} \end{pmatrix} =$$



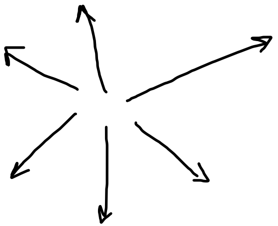
$$\hat{\underline{r}} = \cos\phi\,\hat{\underline{i}} + \sin\phi\,\hat{\underline{j}}$$

$$= -\frac{m v_{\parallel}^2}{r}\left[-\cos\phi\,\hat{\underline{i}} - \sin\phi\,\hat{\underline{j}}\right] = \underbrace{\frac{m v_{\parallel}^2}{r}\hat{\underline{r}}}_{\text{centrifugal force}}$$

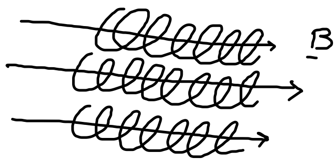
$$\underline{F} = \underline{F}_E + \underline{F}_{\nabla|\underline{B}|} + \underline{F}_{centr.}$$

Consequence of drifts on magnetic confinement

$\underline{B} = \underline{0}$

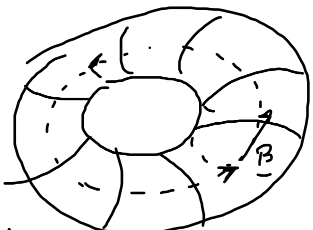


$\underline{B} \neq \underline{0}$



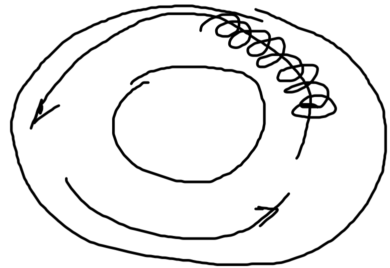
Close the field line

Linear system → toroidal system

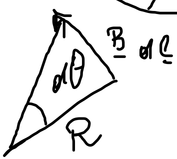


Ampere's law

$$\oint_C \underline{B}\cdot d\underline{l} = \mu_0 I_{chained}$$



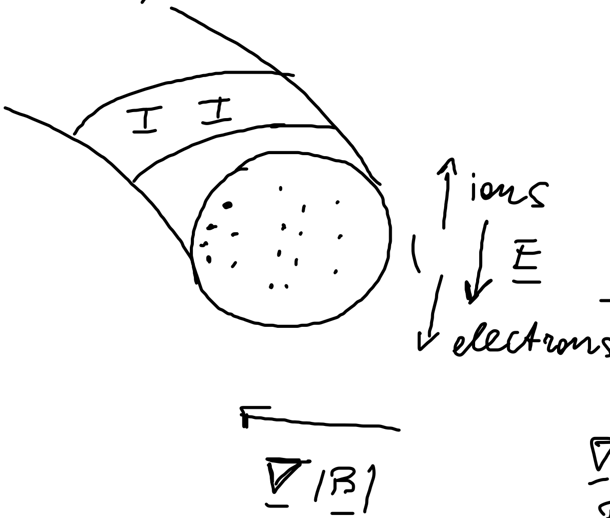
$$I_{chained} = I \cdot N$$



C ≡ circle of radius R

$$\underline{B}\cdot d\underline{l} = BR\,d\theta \qquad \oint_C \underline{B}\cdot d\underline{l} = BR\int_0^{2\pi} d\theta = 2\pi BR$$

$2\pi B R = \mu_0 N I \Rightarrow B(R) = \frac{\mu_0 N I}{2\pi R} \qquad B \propto \frac{1}{R}$



$\underline{F} = -\mu \underline{\nabla}|\underline{B}| \qquad \mu = \frac{m v_\perp^2}{2B}$

$\underline{v}_D = \frac{\underline{F}\times\underline{B}}{qB^2} = -\mu \frac{\underline{\nabla}|\underline{B}|\times\underline{B}}{qB^2}$

$\underline{\nabla}|\underline{B}| \simeq$ radial

$\underline{B} \simeq$ toroidal

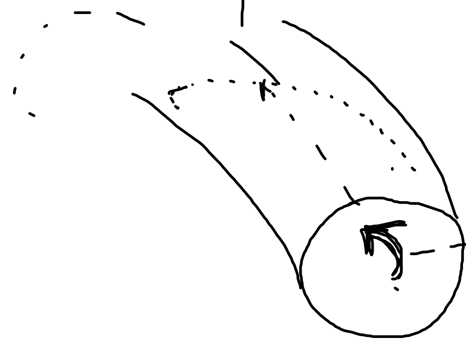
$\Rightarrow \underline{\nabla}|\underline{B}|\times\underline{B} \simeq$ vertical

$\underline{v}_D = \frac{\underline{E}\times\underline{B}}{B^2}$ same direction for ions and electrons

$\underline{E}$ produced by charge separation

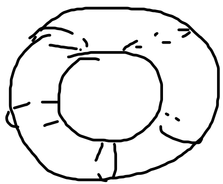
Add a poloidal component to $\vec{B}$

$\underline{\nabla}|\underline{B}| \times \underline{B}$



poloidal component

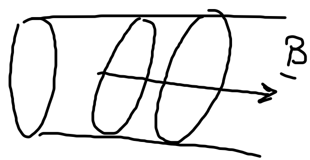
$\underline{\nabla}|\underline{B}|$



Tokamak

"Bend the coil"

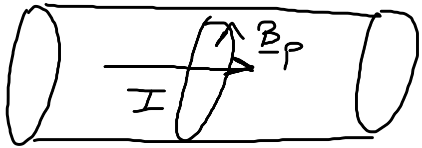
<u>Stellarator</u>

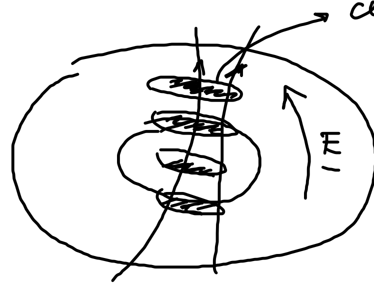


Drawback:
lose symmetry

helps confinement

Tokamak approach: drive a toroidal current





$\underline{E}$ induced by I_{sol}

advantage: symmetry / simplicity

disadvantage: transient system