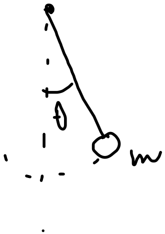


$l = l(t)$

$$\theta(t) = \theta_0(0)\sqrt{\frac{\omega_0}{\omega(t)}}\cos\left(\int_0^t \omega(t')\,dt'\right)$$

$$\omega_0 = \sqrt{g/l_0}$$

$$\omega(t) = \sqrt{g/l(t)}$$

$$\theta(t) = \mathrm{Re}\left[\theta_0(t)\, e^{i\int_0^t \omega(t')\,dt'}\right]$$

$$\ddot{\theta} = -\frac{g}{l(t)}\,\theta$$

$$\cancel{\ddot{\theta}_0} + 2i\dot{\theta}_0\,\omega + i\dot{\omega}\,\theta_0 = 0$$

$$\theta_0(t) = \theta_0(0)\sqrt{\frac{\omega_0}{\omega(t)}}$$

$$\frac{d\theta_0}{dt} = -\theta_0\sqrt{\omega_0}\,\frac{1}{2}\,\frac{1}{\omega^{3/2}}\cdot\frac{d\omega}{dt}$$

$$\frac{d^2\theta_0}{dt^2} = -\frac{\theta_0}{2}\sqrt{\omega_0}\left[-\frac{3}{2}\frac{1}{\omega^{5/2}}\left(\frac{d\omega}{dt}\right)^2 + \cancel{\frac{1}{\omega^{3/2}}\frac{d^2\omega}{dt^2}}\right]$$

What does it mean that

$$\underbrace{\frac{d^2\theta_0}{dt^2}}\ll \omega \underbrace{\frac{d\theta_0}{dt}}$$

$$\frac{1}{\omega^2}\cancel{\theta_0}\left(\frac{d\omega}{dt}\right)^{\cancel{2}} \ll \cancel{\omega}\cdot\frac{1}{\cancel{\omega}}\cancel{\theta_0}\,\cancel{\frac{d\omega}{dt}}$$

$$\frac{d\omega}{dt} \approx \frac{\Delta\omega}{\Delta t}$$

$$\frac{1}{\omega}\left(\frac{d\omega}{dt}\right) \ll \omega$$

If $\Delta t \sim T$

then $\Delta\omega$: change of ω within a T

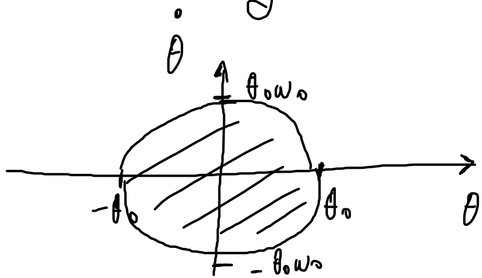
$$\omega = \frac{2\pi}{T}$$

$$\frac{1}{\omega}\frac{\Delta\omega}{\Delta t} \ll \omega$$

$$\frac{\Delta\omega}{\omega} \ll \omega\,\Delta t \sim \frac{\Delta t}{T} \sim 1$$

Energy is <u>not</u> conserved!

Is there something else which is conserved?



<u>Unperturbed case</u>

$$\theta(t) = \theta_0 \cos(\omega_0 t) \Rightarrow \frac{\theta}{\theta_0} = \cos(\omega_0 t)$$

$$\dot\theta(t) = -\theta_0 \omega_0 \sin(\omega_0 t) \Rightarrow -\frac{\dot\theta}{\theta_0\omega_0} = \sin(\omega_0 t)$$

$$\frac{\theta^2}{\theta_0^2} + \frac{\dot\theta^2}{\theta_0^2\omega_0^2} = 1 \quad \text{ellipse}$$

Area $= \pi \cdot \theta_0 \cdot \theta_0\omega_0 = \pi\,\omega_0\,\theta_0^2 =$ const

? Area in the perturbed case?

$$\text{Area} = \int \dot\theta \, d\theta \qquad \text{area} = \int f(x)\,dx$$

$$\oint_{1\text{ period}} \dot{\theta}\, d\theta = \int \dot{\theta}\, \underbrace{\frac{d\theta}{dt}}_{\dot{\theta}}\, dt = \oint \dot{\theta}^2\, dt$$

$$\frac{d}{dt}\left(\theta \frac{d\theta}{dt}\right) = \underbrace{\left(\frac{d\theta}{dt}\right)^2}_{\dot{\theta}^2} + \theta \frac{d^2\theta}{dt^2} \Rightarrow \dot{\theta}^2 = \frac{d}{dt}\left(\theta \frac{d\theta}{dt}\right) - \theta \frac{d^2\theta}{dt^2} \qquad \frac{d^2\theta}{dt^2} = -\omega^2 \theta$$

$$= \int \left[\frac{d}{dt}\left(\theta \frac{d\theta}{dt}\right) - \theta \frac{d^2\theta}{dt^2}\right] dt = \cancel{\theta \frac{d\theta}{dt}\Big|_{t=0}^{t=T}} - \int \theta \frac{d^2\theta}{dt^2}\, dt = +\int \omega^2 \theta^2\, dt$$

$$\int \omega^2 \theta^2 dt = \int_0^T \omega^{\cancel{2}} \theta_0^2 \frac{\omega_0}{\cancel{\omega}} \cos^2\left(\underbrace{\int_0^t \omega(t')\,dt'}\right) dt$$

$$\theta(t) = \theta_0 \sqrt{\frac{\omega_0}{\omega(t)}} \cos\left(\int_0^t \omega(t')\,dt'\right)$$

$$\xi = \int_0^t \omega(t')\,dt' \Rightarrow \frac{d\xi}{dt} = \omega(t)$$

$$\Rightarrow dt = \frac{d\xi}{\omega}$$

$$t = 0 \Rightarrow \xi = \int_0^0 \ldots = 0$$

$$t = T \Rightarrow \xi = \int_0^T \frac{d\theta}{\cancel{dt'}} \cancel{dt'} = 2\pi$$

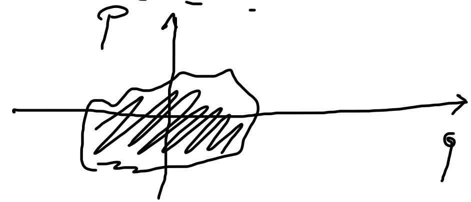
$$= \int_0^{2\pi} \cancel{\omega}\, \theta_0^2\, \omega_0 \cos^2\xi \frac{d\xi}{\cancel{\omega}} = \omega_0 \theta_0^2 \int_0^{2\pi} \cos^2(\xi)\,d\xi = \pi \omega_0 \theta_0^2 = \text{const}$$

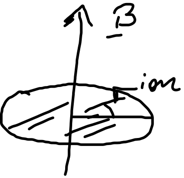
Adiabatic invariant

If q periodic + adiabatic perturbation
(θ) (quantities do not change significantly within T_q)

~~Energy~~

$$\boxed{\oint p\,dq = \text{const}} \qquad p = \frac{\partial \mathcal{L}}{\partial \dot q}$$





T_{Larmor} $\qquad J = \int p_\varphi \, d\varphi$ $\qquad \underline{E} = -\underline{\nabla}\phi - \partial \underline{A}/\partial t$

φ periodic $\qquad p_\varphi = \dfrac{\partial \mathcal{L}}{\partial \dot\varphi}$ $\qquad \underline{B} = \underline{\nabla} \times \underline{A}$

$$\mathcal{L} = \frac{1}{2} m v^2 + q\,\underline{A}\cdot\underline{v} - q\phi = \frac{1}{2} m\left(v_R^2 + v_z^2 + R^2\dot\varphi^2\right) + q\left(A_R v_R + A_z v_z + A_\varphi R\dot\varphi\right) \cancel{-q\phi}$$

$$\frac{\partial \mathcal{L}}{\partial \dot\varphi} = \cancel{\frac{1}{2}} m R^2 \cancel{2} \dot\varphi + qRA_\varphi = p_\varphi \qquad \underbrace{\frac{d\varphi}{dt}\frac{d\varphi}{dt}}\cdot dt = \omega_L^2\, dt$$

$$J = \oint \left(m\underline{R}^2\dot\varphi + qRA_\varphi\right) d\varphi = m r_L^2 \int^{T_L} \dot\varphi \, d\varphi + q r_L \int A_\varphi \, d\varphi$$

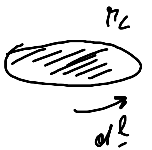
Larmor motion $\qquad R = r_L$

$$= m r_L^2 \omega_L^2 \int_0 dt + q r_L \int A_\varphi \, d\varphi$$

$$= m r_L^2 \omega_L^2 \cdot T_L + //$$

$A_\varphi = ?$

$$\underline{B} = \underline{\nabla} \times \underline{A} \qquad \underline{B} \parallel \hat{\underline{k}}$$



$$\int \underline{B} \cdot d\underline{S} \simeq \pi r_L^2 \cdot B_z$$

Larmor circle

$\parallel$

$$\int (\underline{\nabla} \times \underline{A}) \cdot d\underline{S} = \int \underline{A} \cdot d\underline{l} = \int A_\varphi \, dl = \int A_\varphi \, r_L \, d\varphi = r_L A_\varphi \int_0^{2\pi} d\varphi = 2\pi r_L A_\varphi$$

Stokes theorem

Larmor circle

$$dl = r_L \, d\varphi$$

$$\cancel{\pi} r_L^{\cancel{2}} B_z = \cancel{2\pi} \cancel{r_L} A_\varphi \Rightarrow A_\varphi = \frac{r_L}{2} B_z$$

$$\int A_\varphi \, d\varphi = \int \frac{r_L}{2} B_z \, d\varphi = \frac{r_L B_z}{2} \int d\varphi = \pi r_L B_z$$

$$J = m r_L^2 \omega_L^2 T_L + q r_L \pi r_L B_z =$$

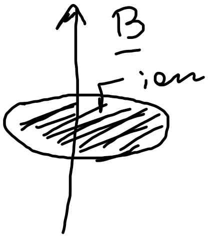
$$\cancel{m} \frac{m^2 v_L^2}{\cancel{q^2} \cancel{B^2}} \frac{\cancel{q^2} \cancel{B^2}}{\cancel{m^2}} \frac{2\pi}{qB} \cancel{m} + q \left(\frac{m v_L}{qB} \right)^2 \pi B$$

$$= \frac{4\pi}{q} m \underbrace{\left(\frac{m v_L^2}{2B} \right)}_{\equiv \mu} + \frac{2\pi m}{q} \underbrace{\left(\frac{m v_L^2}{2B} \right)}_{\equiv \mu} = \left(\frac{6\pi m}{q} \right) \cdot \mu$$

μ is adiabatic invariant

$m = I \cdot A$

$\underline{\mu}$ adiabatic invariant



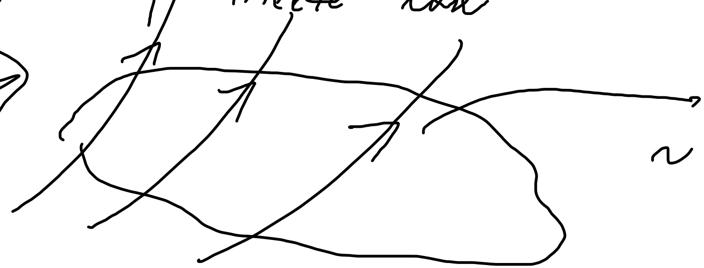
$$\mu = I \cdot A = \frac{q}{T_L} \cdot \pi r_L^2 = \dots = \frac{m v_L^2}{2B}$$

$$r_L = \frac{m v_L}{qB} \qquad r_L \propto \frac{1}{B}$$

$$\Phi = B \pi r_L^2 = \pi \cancel{B} \frac{m^2 v_L^2}{q^2 B^{\cancel{2}}} = \frac{2\pi m}{q^2} (\mu)$$

Flux of B through πr_L^2

"Freeze" law



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