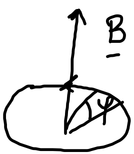
q_j periodic $\Rightarrow \int_{\text{cycle}} p_j \, dq_j$ $\qquad p_j = \dfrac{\partial L}{\partial \dot{q}_j}$

Larmor period $\Rightarrow$ $\mu = \dfrac{mv^2}{2B}$ is invariant



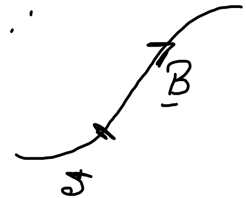
φ

$$= \underbrace{I}_{\frac{q}{T_L}} \cdot \underbrace{A}_{\pi r_L^2}$$

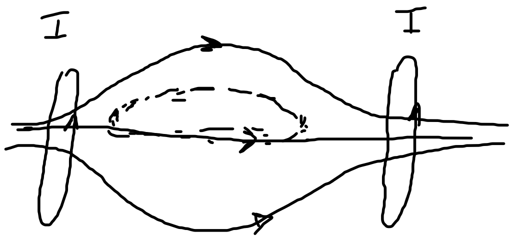
$\Phi(B) = \text{const}$

Parallel motion

$$m\frac{dv_\parallel}{dt} = \underbrace{qE_\parallel}_{=0} - \underbrace{\left(\mu \nabla |B|\right)_\parallel}_{\left[q\langle v_\perp \times (r_\perp\cdot\nabla)B\rangle\right]_\parallel} = -\mu\frac{\partial |B|}{\partial s}$$



Mirror machine



$$v_\parallel \cdot m\frac{dv_\parallel}{dt} = -\mu\frac{\partial B}{\partial s}\cdot \underbrace{v_\parallel}_{\partial s/\partial t}$$

$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{dB}{dt} = -\frac{d}{dt}(\mu B)$$

$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \mu B\right) = 0$$

$$\frac{1}{2} m v_{\parallel}^2 + \mu B = \text{const} = W_0$$

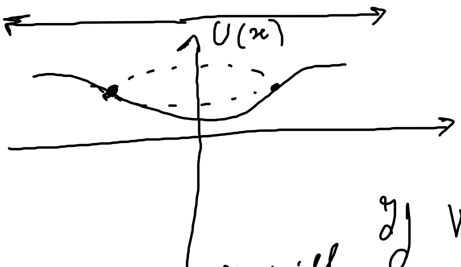
O: a region where $B = B_{min}$

$$\frac{1}{2} m v_{\parallel 0}^2 + \frac{m v_{\perp 0}^2}{2 \cancel{B_{min}}} \cdot \cancel{B_{min}} = \frac{1}{2} m \left(v_{\parallel 0}^2 + v_{\perp 0}^2 \right) = W_0$$

1D motion
conservative force

$$W = \frac{1}{2} m v^2 + U(x)$$

μB : effective potential



$$v^2 = \frac{2}{m} \left(W - U(x) \right)$$

confined
If $W = U(\tilde{x})$ at some point $\tilde{x}$, then at that point $v = 0$
Or turning point exists

If $W > U(x)$
v will never be $= 0$: particle is unconfined

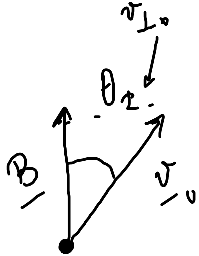
Recipe:

if $W > U_{max}$: particle is unconfined

if $W < U_{max}$: " confined

(if U is symmetric)

$$W_0 = \frac{1}{2} m v_{\parallel}^2 + \underbrace{\mu B(s)}_{U(s)}$$



θ : pitch angle

$v_{\perp 0} = v_0 \sin\theta_r$

Boundary: $W_0 = \mu B_{max}$

$$\cancel{\frac{1}{2}} \cancel{m}(\underbrace{v_{\parallel 0}^2 + v_{\perp 0}^2}_{v_0^2}) \lessgtr \underbrace{\frac{\cancel{m} v_{\perp 0}^2}{\cancel{2} B_{min}}}_{\mu} \cdot B_{max}$$

< confined

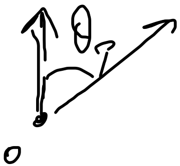
> unconfined

; $\dfrac{v_{\perp 0}^2}{v_0^2} > \dfrac{B_{min}}{B_{max}}$

$$\sin^2\theta_p = \frac{B_{min}}{B_{max}}$$

Confinement: $\sin^2\theta_p > \frac{B_{min}}{B_{max}}$

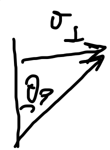
No confinement: $\sin^2\theta_p < \frac{B_{min}}{B_{max}}$



$$F_{friction} = -\mu \frac{\partial B}{\partial s}$$

$\mu \propto m v_{\perp 0}^2$

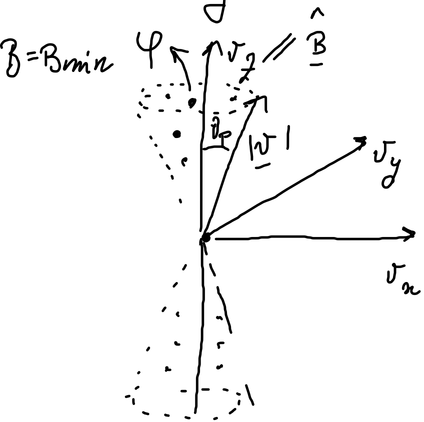
More $v_\perp$ $\Rightarrow$ more friction



$$\sin\theta_p = \frac{v_\perp}{v}$$

more $v_\perp$ $\rightarrow$ θ_p is larger

No confinement



$$\sin^2\theta_p < \frac{B_{min}}{B_{max}}$$

Loss cone

$$f_{confined} = \frac{(\text{particles out of the loss cone})}{(\text{all particles})}$$

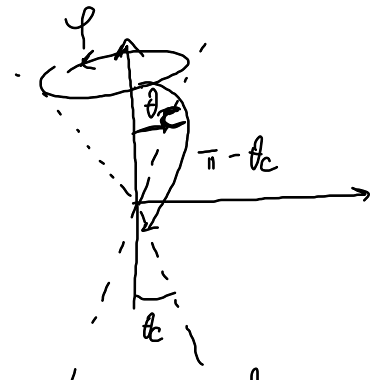
$$f(v) = f_M \propto \exp\left(\frac{-mv^2}{2T}\right) \qquad d^3\underline{v}$$

$$f_{conf} = \int_{\text{out of loss cone}} \overbrace{dv\, v^2\, d(\cos\theta)\, d\varphi}^{d^3 v} \cdot f_M(v)$$

Spherical coordinates

$$\int_0^{+\infty} dv\, v^2 f_M(v) \overbrace{\int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\varphi}^{4\pi}$$

$$\eta_{conf} = \frac{\cancel{\int_0^{+\infty} dv\, v^2 f_M(v)} \int_{\theta_c}^{\pi-\theta_c} d(\cos\theta) \int_0^{2\pi} d\varphi}{4\pi \cancel{\int_0^{+\infty} dv\, v^2 f_M(v)}}$$

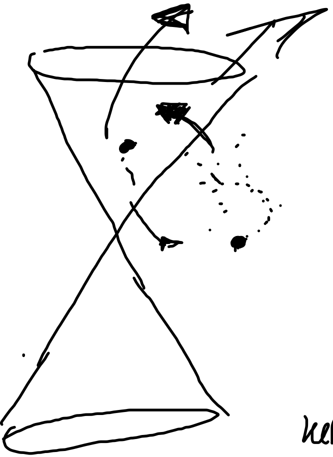


$$\sin^2\theta_c = \frac{B_{min}}{B_{max}} \qquad = \frac{\cancel{2\pi} \int_{\theta_c}^{\pi-\theta_c} d\theta_p \sin\theta_p}{2\, \cancel{4\pi}} = \frac{1}{2} \left. -\cos\theta_p \right|_{\theta_c}^{\pi-\theta_c} =$$

$$= -\frac{1}{2}\left(-\cos\theta_c - \cos\theta_c\right) = \cos\theta_c = \sqrt{1-\sin^2\theta_c}$$

If $\frac{B_{min}}{B_{max}} = \frac{1}{2}$

$$\eta_{conf} \approx \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \approx 70\% \qquad = \sqrt{1 - \frac{B_{min}}{B_{max}}}$$



μ

Time scale

$T \sim keV$ Diffusion time scale $\overline{ms}$ enter/exit loss cone

Loss time scale $\approx \mu s$

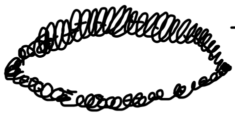
$keV = 1.6 \cdot 10^{-16}\, J$ $T \sim 1\, keV$ $v_{th} = \sqrt{\frac{2T}{m}} \sim 4 \cdot 10^{5}\, m/s$

$m \sim m_p = 1.67 \cdot 10^{-27}\, kg$

$L \sim 1\, m$

$\Delta t_{loss} \sim L / v_{th} \sim \mu s$

Confined particles in a mirror machine

Turning point  Turning point

Periodic motion

$\Rightarrow \int p\,dq$

$q = s$

$= \int p_\parallel\, ds$

$p_\parallel = m v_\parallel + \cancel{q A_\parallel}$

Coulomb gauge

If there is no $j_\parallel$, then $p_\parallel = m v_\parallel$ $\quad (A_\parallel = 0)$

If $\rho = 0 \Rightarrow \underline{\nabla} \cdot \underline{A} = 0$

$(\underline{\nabla} \times \underline{B})_\parallel = (\mu_0 \underline{j})_\parallel \qquad \underline{B} = \underline{\nabla} \times \underline{A}$

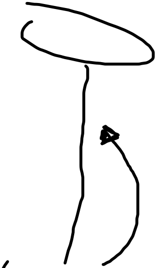
$\left[\underline{\nabla} \times (\underline{\nabla} \times \underline{A})\right]_\parallel = \mu_0 j_\parallel \Rightarrow \cancel{\left[\underline{\nabla}(\underline{\nabla} \cdot \underline{A})\right]_\parallel} - \nabla^2 \underline{A} = \mu_0 j_\parallel = 0 \Rightarrow \nabla^2 \underline{A}_\parallel = 0$

$\Rightarrow A_\parallel = 0$

$J = \int m v_{\parallel} ds$

$I_{coil} \uparrow$

$B_{max} \uparrow$



Trajectory gets shorter

$\Rightarrow$ particle bounces sooner

$J \sim v_{\parallel} \cdot L_{trajectory} \rightarrow v_{\parallel} \uparrow \rightarrow$ particle moves towards the loss cone

Fermi acceleration