Chapter 1 - Introduction to plasma physics

1 Rutherford scattering

Consider a Rutherford scattering process between two particles having velocities $\mathbf{v_1}$, $\mathbf{v_2}$, charges q_1, q_2 and masses m_1,m_2, respectively.

a) Show that, in the centre of mass frame, the equation of motion is given by

$$\mu\ddot{\mathbf{r}} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}} \tag{1}$$

where $\mathbf{r}$ is the relative position vector.

b) Define the angular momentum $\mathbf{L} = \mu\mathbf{r} \times \dot{\mathbf{r}}$. By taking its derivative with respect to the time, show that this is a constant vector. In particular, note that this implies that the scattering process must occur in the plane where $\mathbf{r}$ and $\dot{\mathbf{r}}$ lie.

c) Using the geometry shown in 1, show that $|\mathbf{L}| = \mu b v_\infty = \mu r^2\dot{\phi}$, from which $\dot{\phi} = b v_\infty / r^2$.

d) Call $\mathbf{v_i}$ and $\mathbf{v_f}$ the initial and final velocity of the charged particle shown in the figure and note that $|\mathbf{v_i}| = |\mathbf{v_f}|$. From symmetry, the x component of the velociy before and after scattering must be unchanged, whereas the y component must flip. Starting from these considerations, evaluate the change of the y velocity component before and after the collision (Δv_y) as a function of v_{yi}.

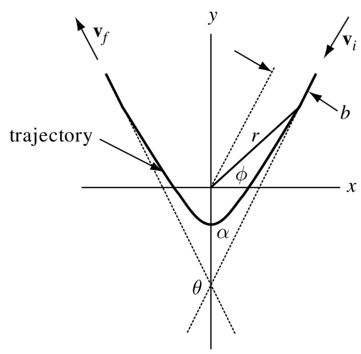

Figure 1: Scattering geometry in the centre of mass frame. The scattering centre is in the origin and θ denotes the scattering angle.

e) Starting from the y component of equation 1, find a relation between dv_y and $d\cos\phi$. (Hint: consider the conservation of the angular momentum to write dt as a function of $d\phi$). Call ϕ_i and ϕ_f the initial and final values of ϕ. By integrating dv_y, find Δv_y for the whole collision and as a function of ϕ_i and ϕ_f.

f) Using the figure, find the relation between the angles θ and α and between the angles ϕ_i,ϕ_f and α. By combining the different formulas for Δv_y found above, show that

$$\tan\frac{\theta}{2} = \frac{q_1 q_2}{4\pi\epsilon_0 \mu b v_\infty^2} \tag{2}$$

and find an approximation for small angle collisions.