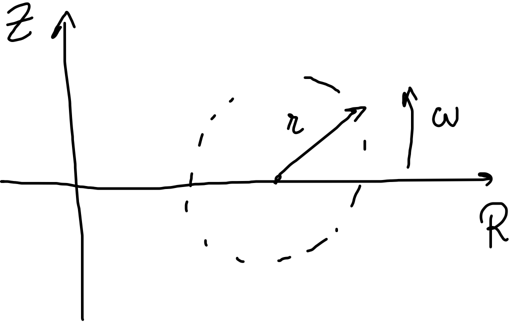
w/o drift



$$\begin{cases} R - R_0 = r\cos(\omega t) \\ z = r\sin(\omega t) \end{cases}$$

$$\begin{cases} \dot R = -r\omega\sin(\omega t) \\ \dot z = +r\omega\cos(\omega t) \end{cases}$$

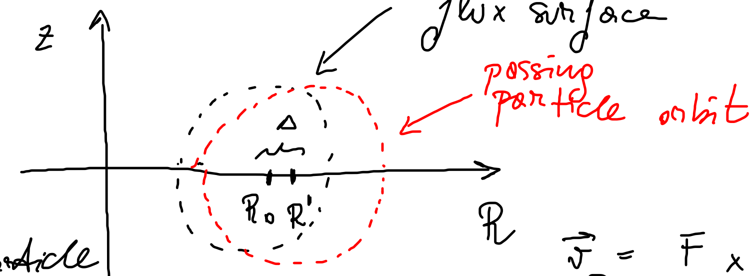
with drift: v_D along z

$$\begin{cases} \dot R = -r\omega\sin(\omega t) \\ \dot z = \underbrace{r\omega\cos(\omega t)}_{(R-R_0)} + v_D = \omega(R-R_0) + v_D = \omega(R-R') \end{cases}$$

$-\omega R_0 + v_D = -\omega R'$; $R' = R_0 - v_D/\omega$

With drift: circular motion around $(R', 0)$ $\qquad \omega = \frac{v_\parallel}{r} B_P / B_T$

$$R' - R_0 = \underbrace{-v_D/\omega}_{\Delta}$$



If $\Delta \ll a$: good particle confinement

$$\vec{v}_D = \frac{\underline{F} \times \underline{B}}{qB^2}$$

$$\vec{v}_{\nabla|B|} = -\mu \frac{\underline{\nabla}|\underline{B}| \times \underline{B}}{qB^2}$$

$$\vec{v}_D = \vec{v}_{D\nabla|B|} + \vec{v}_{centr}$$

$$B \simeq \frac{B_0 R_0}{R} \qquad \underline{\nabla}|\underline{B}| \approx \frac{B\hat{R}}{R}$$

$$|\vec{v}_{\underline{\nabla}|\underline{B}|}| \approx \frac{mv_\perp^2}{2B}\,\frac{\cancel{B}}{R}\cdot\frac{\cancel{B}}{q\cancel{B^2}} \approx \frac{mv_\perp^2}{2qBR}$$

$$\vec{v}_{centr} = +\frac{mv_\parallel^2}{R_0}\,\hat{\underline{R}} \times \frac{\underline{B}}{qB^2}$$

$$\mu = \frac{mv_\perp^2}{2B}$$

$$|\vec{v}_{centr}| \approx \frac{m v_{=}^2}{qRB} \qquad |\vec{v}_{\nabla|B|}| \approx \frac{m v_\perp^2}{2qRB} \qquad \omega = \frac{v_\parallel}{r} \frac{B_p}{B_T}$$

$$v_D \approx |\vec{v}_{centr}| + |\vec{v}_{\nabla|\underline{B}|}| \approx m\left(v_{=}^2 + \frac{v_\perp^2}{2}\right)\frac{1}{qRB} \approx \frac{mv^2}{qRB}$$

$v_\parallel \approx v_\perp$

saf$\,$ety factor

$q \approx$ some units

$$\Delta = \frac{v_D}{\omega} \approx \frac{m v^{\cancel{2}}}{qR\cancel{B}} \frac{r \cancel{B}_T}{\cancel{v_\parallel} B_p} \approx \frac{mv}{qB_p} \cdot \frac{r}{R} \approx \varepsilon \cdot r_{L_p}$$

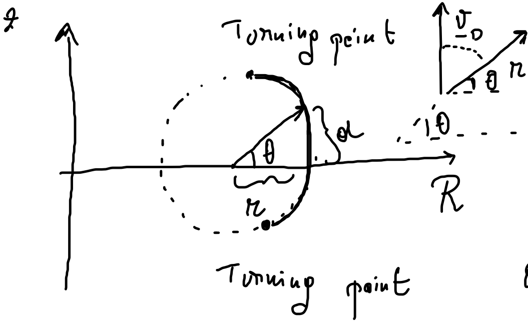
$\frac{mv}{qB_p} \to r_{L,p}$

$\frac{r}{R} \to \varepsilon$

$$\frac{mv}{qB_T} \frac{B_T}{B_p} \frac{r}{R} \approx \text{some } r_L \ll a$$

$\frac{mv}{qB_T} \to r_L$

Trapped particles



With drift $v_D \parallel z$

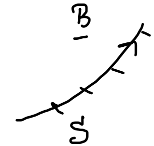
$$\frac{v_{\parallel 0}}{v_{\perp 0}} < \sqrt{2\varepsilon} \Rightarrow v_\parallel \ll v_\perp$$

$$\dot r = v_D \sin(\theta)$$

$$v_D \approx \frac{m v_\perp^2}{2qRB}$$

$$\dot r \approx \frac{m v_\perp^2}{2qRB} \sin\theta$$

Along the field line



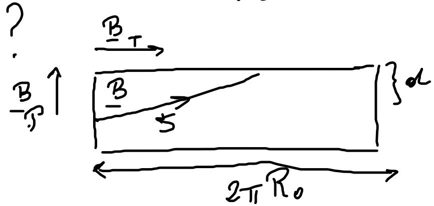
$$m \dot v_\parallel = -\mu \frac{\partial B}{\partial s} = -\mu \frac{\partial B}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} = -\mu \varepsilon B_0 \sin\theta \cdot \frac{B_p}{B} \frac{1}{r}$$

$$B(\theta) \approx B_0 (1 - \varepsilon \cos\theta)$$

$$\frac{\partial B}{\partial \theta} = \varepsilon B_0 \sin\theta$$

$$\frac{\partial \theta}{\partial s} = \frac{B_p}{B} \frac{1}{r}$$

$$\frac{\partial \theta}{\partial s} = ?$$



$$\frac{d}{B_p} = \frac{s}{B}$$

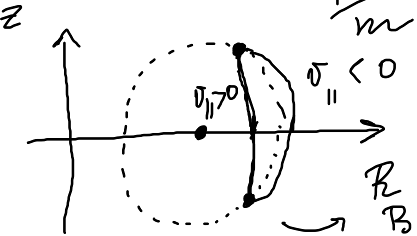
$$\theta = d/r = s \frac{B_p}{B} \frac{1}{r} = \theta(s)$$

$$\dot r \approx \frac{v_\perp^2}{2\omega_L R}\sin\theta$$

$$\dot v_\parallel \approx -\frac{1}{\cancel{m}}\underbrace{\frac{\cancel{m} v_\perp^2}{2B}}_{\mu}\frac{\cancel{R}B_0}{R_0}\frac{B_p}{\cancel{B}}\frac{1}{\cancel{R}}\sin\theta \approx -\frac{v_\perp^2}{2R_0 B}B_p\sin\theta$$

$$\frac{\dot r}{\dot v_\parallel} \approx -\frac{\cancel{v_\perp^2}}{\cancel{2\omega_L R}}\cdot\frac{\cancel{2R_0 B}}{\cancel{v_\perp^2}\, B_p} \approx -\frac{m}{qB_p}$$

$$\omega_L = \frac{q\cancel{B}}{m}$$



Banana orbit

$$\frac{dr}{dv_\parallel} \approx -\frac{m}{qB_p}$$

$$\int_r^{r_{TP}} dr \approx -\frac{m}{qB_p}\int_{v_\parallel}^{0} dv_\parallel$$

$$r_{TP} - r \approx +\frac{m}{qB_p}v_\parallel \; ; \quad \overbrace{\boxed{r \approx r_{TP} - \frac{m v_\parallel}{qB_p}}}^{\Delta}$$

Banana width

$$= \Delta = \frac{m v_{\parallel}}{q B_p} = \frac{m v_\perp}{q B_T} \frac{B_T}{v_\perp} \frac{v_{\parallel}}{B_p} = r_L \underbrace{\frac{B_T}{B_p}}_{r_{L_p} = \frac{m v_\perp}{q B_p}} \frac{v_{\parallel}}{v_\perp} < r_{L_p} \cdot \sqrt{2\varepsilon}$$

$$\frac{m v_\perp}{q B_T} \rightarrow r_L$$

$r \ll a$

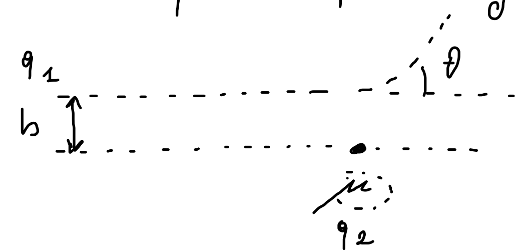
Passing particle

$$\Delta \sim \varepsilon \cdot r_{L_p} \ll a$$

$$\varepsilon \ll 1 \Rightarrow \sqrt{\varepsilon} > \varepsilon$$

Collisions in plasmas

Coulomb collisions (fully ionized plasma) c.m. frame



$$\tan\left(\frac{\theta}{2}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0 v_{rel}^2 \mu b}$$

Small angle collisions are dominant $\theta \ll 1$: if $\theta > \pi/2$ "large angle"

$$\theta \simeq \frac{q_1 q_2}{2\pi\varepsilon_0 v_{rel}^2 \mu b}$$

Debye shielding

$$\phi(r) = \overbrace{\frac{1}{4\pi\varepsilon_0}\frac{q}{r}}^{\phi_{vacuum}} e^{-r/\lambda_D}$$

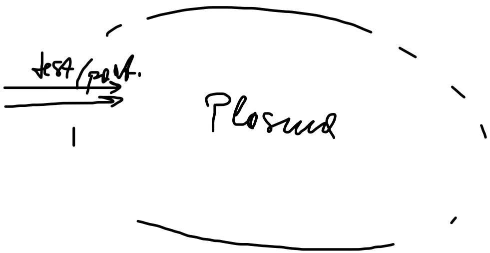
$b_{max} \simeq \lambda_D$

λ_D: Debye length $b_{min} = b_{\pi/2}$

1) Assume a population of test particles with $f_T(\underline{v})$ enters the plasma

2) Number of test particles $\ll$ number of plasma particles

→ f_{plasma} unchanged by the test particles



? How does $f_T(\underline{v})$ change due to collisions ?