Chapter 2 - Charged particle orbits in electric and magnetic fields

1 Charge particle motion in the Earth magnetic field

Assume that the Earth magnetic field is $B = 3 \cdot 10^{-5}\ T$ at the equator and that it decreases as function of the distance r from the centre of the Earth as $1/r^3$, similarly to a magnetic dipole. Assume that, on the equatorial plane, there is an isotropic population of protons with energy 1 eV and of electrons with energy 30 keV. Each species has a density $n = 10^7\ m^{-3}$, at a distance of 5 Earth radii from the centre of the Earth. If the drift due to the magnetic field curvature is negligible, find

a) The drift velocities of the ions and the electrons due to the gradient of the magnetic field.

b) Is the electron drift eastward or westward?

c) The time taken by an electron to make a full turn around the Earth.

d) The current density around the equator. Is this dominated by the electron or ion contribution?

2 Charge particle orbits in a mirror machine

The magnetic field $\mathbf{B}$ of a mirror machine is described by

$$\mathbf{B} = \frac{1}{2\pi} \nabla\psi(r, z, t) \times \nabla\theta \tag{1}$$

when the cylindrical coordinates (r, z, θ) are used. The flux function $\psi(r, z, t)$ is

$$\psi(r, z, t) = B_{\min}\pi r^2 \left[1 + \frac{2\lambda\zeta^2}{\zeta^4 + 1}\right] \tag{2}$$

and $\zeta = z/L(t)$ where $L(t)$ is the (time dependent) lenght of the mirror.

a) Show that $\mathbf{B}$ has only r and z components and that $\nabla \cdot \mathbf{B} = 0$.

b) Evaluate the flux $\phi(\mathbf{B})$ of the magnetic field described by equation 1 through a circle of radius r and at a height z. Show that $\phi(\mathbf{B}) = \psi(r, z, t)$. This property explains why $\psi(r, z, t)$ is called a *flux function*.

c) Show that the z component of the magnetic field has a minimum $B_{\min}$ at $z = 0$ and maximum $B_{\max} = (1 + \lambda)B_{\min}$ at $z = \pm L(t)$.

d) If $\lambda = 1$, find the aperture of the loss cone.

e) Since the lenght of the mirror is time dependent, there must be an induced electric field, besides the magnetic field. Show that the electric field is in the poloidal direction and that its magnitude $|E_\theta|$ is

$$|E_\theta| = \frac{2r\lambda B_{\min}\zeta^2(1-\zeta^4)}{(1+\zeta^4)^2}\frac{1}{L}\frac{dL}{dt} \tag{3}$$

f) Call θ_0 the pitch angle of a particle where $B = B_{\min}$. Using the numerical integrator provided, show that, when $\lambda = 1$ and the lenght of the mirror is constant, a particle with $\theta_0 = 50°$ is confined, while a particle with $\theta_0 = 40°$ is unconfined. If, instead, the lenght of the mirror slowly decreases as a function of time, show that a particle having $\theta_0 = 50°$ will experience a Fermi acceleration until it becomes unconfined.