$$\frac{\partial f_T}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2}{4\pi\varepsilon_0^2 m_T^2} \ln\Lambda \left[\underbrace{-\frac{\partial}{\partial \underline{v}} \cdot \left(f_T \frac{\partial h_F}{\partial \underline{v}} \right)}_{\text{friction}} + \underbrace{\frac{1}{2} \frac{\partial}{\partial \underline{v}} \frac{\partial}{\partial \underline{v}} : \left(f_T \frac{\partial^2 g_F}{\partial \underline{v} \partial \underline{v}} \right)}_{\text{"Beam spreading"}} \right]$$

Rosenbluth potentials

$$h_F = \frac{m_T}{\mu} \int \frac{f_F(\underline{v}')}{|\underline{v} - \underline{v}'|} d^3 \underline{v}'$$

$$g_F = \int |\underline{v} - \underline{v}'| f_F(\underline{v}') \, d^3 \underline{v}'$$

$$\underline{u} = \frac{\int d^3 \underline{v} \; \underline{v} \; f_T(\underline{v})}{\int d^3 \underline{v} \; f_T(\underline{v})}$$

$d^3 \underline{v} \; f_T(\underline{v})$: # of particles that have $(\underline{v}, \underline{v} + d\underline{v})$

Multiply by $\underline{v}$ the F-P eq. and integrate

$$\frac{\int \underline{v} \frac{\partial f_T}{\partial t} d^3\underline{v}}{\int f_T d^3\underline{v}} = \frac{\partial}{\partial t} \frac{\int d^3\underline{v}\, \underline{v} f_T}{\int f_T d^3\underline{v}} = \frac{\partial \underline{u}}{\partial t}$$

Friction term

$$-\int \underline{v} \frac{\partial}{\partial \underline{v}} \cdot \left(f_T \frac{\partial h_F}{\partial \underline{v}} \right) d^3\underline{v} = \int f_T \frac{\partial h_F}{\partial \underline{v}} d^3\underline{v}$$

(x) $$-\int dv_x dv_y dv_z \left(v_x \frac{\partial}{\partial v_x} (\quad) + \underbrace{\cancel{v_x \frac{\partial}{\partial v_y}} (\quad)}_{=0} + \underbrace{\cancel{v_x \frac{\partial}{\partial v_z}} (\quad)}_{=0} \right)$$

$$\int dv_x\, dv_y\, dv_z\; v_x \frac{\partial}{\partial v_y}\left(f_T \frac{\partial h_F}{\partial v_y}\right) = \int dv_x\, dv_z\, v_x \underbrace{\int dv_y \frac{\partial}{\partial v_y}\left(f_T \frac{\partial h_F}{\partial v_y}\right)}_{\cancel{f_T \frac{\partial h_F}{\partial v_y}\Big|_{-\infty}^{+\infty}}} = 0$$

"quite fast"

$f_T \to 0$

$v_{x,y,z} \to \pm\infty$

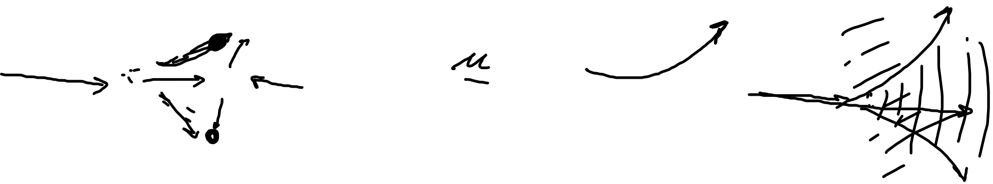
$$-\int dv_x\, dv_y\, dv_z \left(v_x \frac{\partial}{\partial v_x}\left(f_T \frac{\partial h_F}{\partial v_x}\right)\right) = -\int dv_y\, dv_z \int dv_x\; v_x \frac{\partial}{\partial v_x}\left(f_T \frac{\partial h_F}{\partial v_x}\right)$$

$g \quad h'$: integrate by parts

$$= -\int dv_y\, dv_z \left[\cancel{v_x f_T \frac{\partial h_F}{\partial v}\Big|_{-\infty}^{+\infty}} - \int dv_x\; 1 \cdot f_T \frac{\partial h_F}{\partial v_x} \right] =$$

$$= + \left[\int d^3\underline{v}\; f_T \frac{\partial h_F}{\partial \underline{v}} \right]_{x}$$

$$\int \underline{v} \frac{\partial}{\partial \underline{v}} \frac{\partial}{\partial \underline{v}} : \left(f_T \frac{\partial^2 g_F}{\partial \underline{v} \partial \underline{v}} \right) d^3 \underline{v} = 0$$



$$\frac{\partial \underline{u}_T}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln\Lambda}{4\pi\varepsilon_0^2 n_T m_T^2} \int f_T \frac{\partial \underline{h}_F}{\partial \underline{v}} : d^3\underline{v}$$

Field particles at equilibrium: $f_F(\underline{v}) = n_F \left(\frac{m}{2\pi T_F} \right)^{\frac{3}{2}} \exp\left(\frac{-m v^2}{2 T_F} \right)$

$$h_F = [\ldots] = \frac{m_T}{\cancel{m}} \frac{n_F}{v} \operatorname{erf}\left(\frac{v}{v_{th,F}} \right) \qquad v_{th,F} = \left(\frac{2T_F}{m_F} \right)^{\frac{1}{2}}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dy \, \exp(-y^2)$$

Assume: $f_T = n_T \, \delta(\underline{v} - \underline{u}_0)$ → Dirac delta

$\underline{u}_0$

$$\frac{\partial \underline{u}_T}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln\Lambda \cancel{n_T}}{4\pi\varepsilon_0^2 \cancel{n_T} m_F} \frac{m_T}{\mu} n_F \left[\frac{\partial}{\partial \underline{v}} \left(\frac{1}{v} \, \mathrm{erf}\left(\frac{v}{v_{th}} \right) \right) \right] \Bigg|_{\underline{v} = \underline{u}_0}$$

$$\frac{\partial u}{\partial t} = \frac{e^2 q_T^2 \ln\Lambda \, n_e}{4\pi\varepsilon_0^2 m_T^2} \left[Z \left(1 + \frac{m_T}{m_i} \right) \frac{d}{du} \left(\frac{\mathrm{erf}(u/v_{th_i})}{u} \right) + \left(1 + \frac{m_T}{m_e} \right) \frac{d}{du} \left(\frac{\mathrm{erf}(u/v_{th_e})}{u} \right) \right]$$

$$\mu_{i,e} = \frac{m_T m_{i,e}}{m_T + m_{i,e}}$$

$$\frac{m_T}{\mu_{i,e}} = \frac{\cancel{m_T}(m_T + m_{i,e})}{\cancel{m_T} m_{i,e}} = 1 + \frac{m_T}{m_{i,e}}$$

$n_F = n_{i,e}$

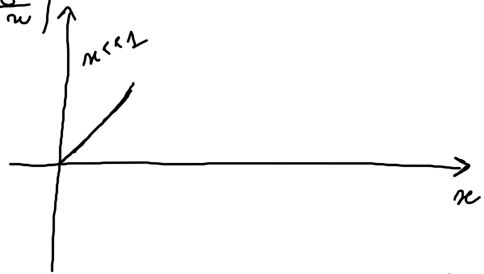
Quasi neutrality: $\underbrace{Ze}\, n_i - e n_e = 0 \Rightarrow n_i = n_e / Z$

$$q_F = \begin{cases} -e & \text{el.} \\ +Ze & \text{ion} \end{cases}$$

"Slowing down" function

$$\frac{d}{dx}\left(\frac{\mathrm{erf}(x)}{x}\right) < 0 \qquad x = u/v_{th_{i,e}}$$

$$-\frac{d}{dx}\left(\frac{\mathrm{erf}(x)}{x}\right)$$



$x \ll 1 \Rightarrow u \ll v_{th_{e,i}}$ $\qquad T_e \simeq T_i$

$x \gg 1 \Rightarrow u \gg v_{th_{e,i}}$ $\qquad v_{th_e} \gg v_{th_i}$

$$v_{th} = \left(\frac{2T}{m}\right)^{\frac{1}{2}}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-y^2)\,dy \approx \frac{2}{\sqrt{\pi}}\int_0^x (1-y^2)\,dy$$

$$= \frac{2}{\sqrt{\pi}}\left(x - x^3/3\right)$$

$0 < y < x$

$x \ll 1 \Rightarrow y \ll 1$

$x \ll 1$

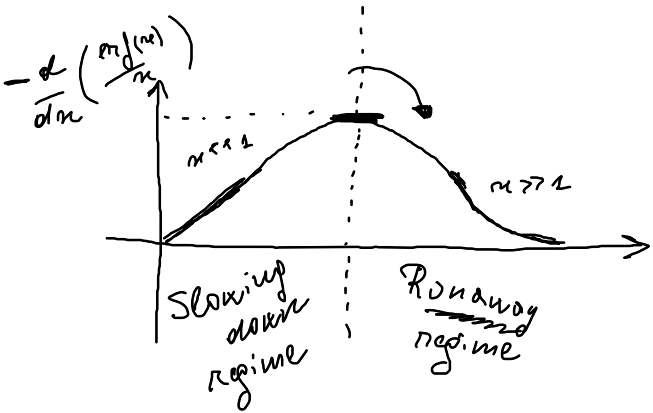
$$\frac{d}{dx}\left(\frac{\mathrm{erf}(x)}{x}\right) = \frac{d}{dx}\left(\frac{2}{\sqrt{\pi}}\left(1 - \frac{x^2}{3}\right)\right)$$

$$= \frac{-4}{3\sqrt{\pi}}\, x$$

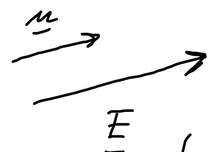
$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dy\, \exp(-y^2) \underset{x \gg 1}{\approx} \frac{2}{\sqrt{\pi}} \int_0^{+\infty} dy\, \exp(-y^2) = 1$$

$$x = u/v_{th}$$

$$\frac{d}{dx}\left(\frac{\mathrm{erf}(x)}{x}\right) \underset{x \gg 1}{\approx} \frac{d}{dx}\left(\frac{1}{x}\right) \approx -1/x^2$$



Slowing down regime:
friction increases with u



There can be a balance between

$\begin{cases} \text{acceleration due to } \underline{E} \\ \text{friction due to plasma} \end{cases}$

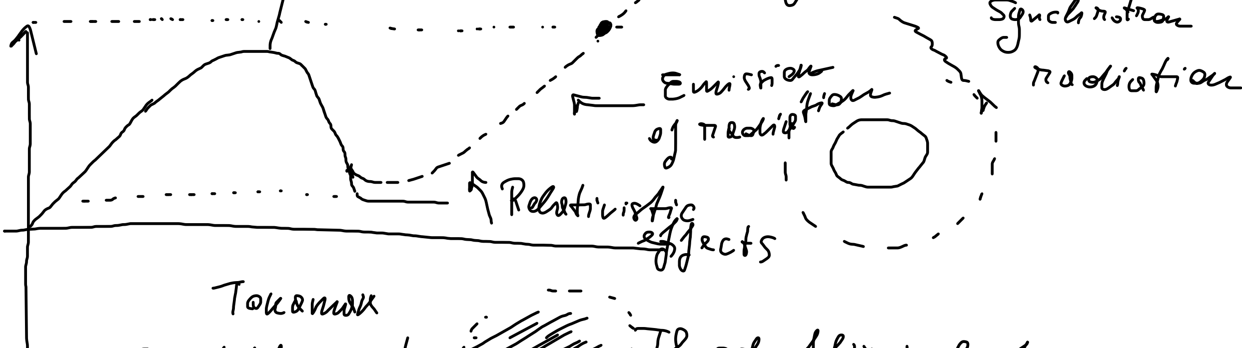
Ions   Electrons can be runaway

$a \propto \frac{qE}{m}$

In the reality:

1) $v_{max} = c$

2) Emission of radiation



Tokamak

Toroidal current

⇒ Poloidal field

If pol. flux is lost

⇒ induce electr. field