How does a slowing down function look like?

$$d + t \longrightarrow \underbrace{\alpha} + n \qquad E_\alpha \simeq 3.5\ \mathrm{MeV}$$

$$\frac{\partial f_T}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln\Lambda}{4\pi\varepsilon_0^2 m_T^2}\left[-\frac{\partial}{\partial \underline{v}}\cdot\left(f_T \frac{\partial h_F}{\partial \underline{v}}\right) + \frac{1}{2}\frac{\partial}{\partial \underline{v}}\frac{\partial}{\partial \underline{v}} : \left(f_T \frac{\partial^2 g_F}{\partial \underline{v}\,\partial \underline{v}}\right)\right]$$

$$g_F(\underline{v}) = \int |\underline{v} - \underline{v}'|\, f_F(\underline{v}')\, d^3\underline{v}'$$

$$h_F(\underline{v}) = \frac{m_T}{\mu}\int \frac{f_F(\underline{v}')}{|\underline{v} - \underline{v}'|}\, d^3\underline{v}'$$

Assumptions: a) Uniform plasma

b) Isotropic $f_T(v) \Rightarrow \frac{\partial}{\partial \underline{v}}\frac{\partial}{\partial \underline{v}} : (\quad) = 0$

$$\frac{\partial f_T}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln\Lambda}{4\pi\varepsilon_0^2 m_T} \left[-\frac{\partial}{\partial \underline{v}} \cdot \left(f_T \frac{\partial h_F}{\partial \underline{v}} \right) \right]$$

$$h_F = \frac{m_T}{\mu} \frac{n_F}{v} \operatorname{erf}\left(\underline{\underline{v / v_{th}}} \right)$$

$$\frac{\partial}{\partial \underline{v}} h_F = \left(\frac{\partial}{\partial v} h_F \right) \hat{\underline{e}}_v$$

spherical coordinates

$(\underline{v}, \theta, \varphi)$

$$\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \varphi} = 0$$

$$\underbrace{\frac{\partial}{\partial v} \left(\frac{1}{v} \operatorname{erf}(v / v_{th}) \right)}$$

$$\frac{d}{dx} \left(\frac{\operatorname{erf}(x)}{x} \right)$$

$$v_{thi} \ll v_T \ll v_{the}$$

$E_\alpha \simeq 3.5$ MeV $\Rightarrow$ $v_\alpha = \left(\frac{2E_\alpha}{m_\alpha}\right)^{\frac{1}{2}} \simeq 7 \cdot 10^6$ m/s

$T \approx 10$ keV $\quad v_{th,i} \simeq 10^6$ m/s

$v_{th,e} \simeq 6 \cdot 10^7$ m/s

$$v_{th_i} \overset{\times 10}{<<} v_\alpha \overset{\times 10}{<<} v_{th_e}$$

$x_i = \frac{v_{th_i}}{v_\alpha} << 1$ $\qquad \frac{d}{dx}\left(\frac{\operatorname{erf}(x)}{x}\right) \simeq \begin{cases} -\frac{4}{3\sqrt{\pi}} x_i & \text{ions} \\ -\frac{1}{x_e^2} & \text{electrons} \end{cases}$

$x = \frac{v}{v_{th}}$ $\qquad x_e = \frac{v_{th_e}}{v_\alpha} >> 1$

$\frac{\partial}{\partial x} = \frac{\partial}{\partial v}\frac{\partial v}{\partial x} = v_{th}\frac{\partial}{\partial v}$

$v = x \cdot v_{th}$

$$\frac{\partial f_T}{\partial t} \approx \frac{Z_T^2 e^2 \ln\Lambda\, n_e}{4\pi\varepsilon_0^2 m_T}\frac{\partial}{\partial \underline{v}}\cdot\left[\left(\underbrace{\frac{Z_i}{\cancel{\mu_i}}\frac{1}{v^2 \cancel{m_i}}\frac{v^2 / \cancel{m_i}}{v^{-2}}}_{\text{ion term}} + \underbrace{\frac{4}{3\sqrt{\pi}}\frac{v}{v_{the}^3}\frac{1}{\cancel{\mu_e}}}_{\text{electron term}}\right) f_T \hat{e}_v\right]$$

$$\approx \frac{Z_i Z_T^2 e^4 \ln\Lambda\, n_e}{4\pi\varepsilon_0^2 m_T \cancel{\mu_i}}\frac{\partial}{\partial \underline{v}}\cdot\left[\frac{f_T}{v^2}\left(1+\frac{v^3}{v_{cr}^3}\right)\hat{e}_v\right]$$

$$\frac{\partial}{\partial \underline{v}}\cdot \underline{F}(\underline{v}) = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2 F_v(\underline{v})\right]$$

sph. coordinates

$$v_{cr}^3 = \frac{3\sqrt{\pi}\, Z_i v_{the}^3 \cancel{\mu_e}}{4\cancel{\mu_i}}$$

$$\frac{\partial f_T}{\partial t} \approx \frac{n_e Z_i Z_T^2 e^4 \ln\Lambda}{4\pi \varepsilon_0^2 m_T \mu_i} \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{\cancel{v^2}}{\cancel{v^2}}\left(1+\frac{v^3}{v_{cr}^3}\right) f_T\right]$$

Stationary case $\quad \frac{\partial f_T}{\partial t} = 0 \qquad\qquad d + t \to \alpha + n$

$$S(v) = \frac{S_0}{4\pi v^2}\,\delta(v - v_0) \qquad v_0 = \sqrt{\frac{2E_0}{m_\alpha}}$$

$$\frac{\partial}{\partial t} n_T = \int \frac{\partial f_T}{\partial t}\, d^3\underline{v} = \int (F.P.)\, d^3\underline{v} = 0 \qquad E_0 \simeq 3.5 \text{ MeV}$$

$n_T = \int f_T\, d^3\underline{v}$

With Source term: $\quad \frac{\partial f_T}{\partial t} = (F.P.) + S(v)$

$$\frac{\partial n_T}{\partial t} = \int S(v)\, d^3\underline{v} =$$

$$\frac{\partial n_T}{\partial t} = \int S(v)\, d^3\underline{v} = \int \frac{S_0}{\cancel{4\pi v^2}}\, \delta(v-v_0)\, \overbrace{\cancel{4\pi v^2}\, dv}^{d^3\underline{v}} =$$

sph. coordinates

$$= S_0 \longrightarrow \frac{\#\text{ alphas}}{m^3 \cdot s}.$$

$$S(v) = \frac{S_0}{4\pi v^2}\, \delta(v - v_0)$$

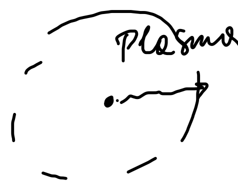
$$S_0 \longrightarrow \frac{\#\text{alphas}}{m^3 \cdot s}$$

$4\pi v^2$: required due to $d^3\underline{v}$ integration

$$\frac{\partial n_T}{\partial t} = S_0$$

all alphas are born at 3.5 MeV

$$\Rightarrow n_T = S_0 \cdot t$$



loss term

$$\mathcal{L} = -\frac{S_0}{4\pi v^2}\, \delta(v - v_m)$$

$$\frac{n_e Z_i Z_T^2 e^4 \ln\Lambda}{4\pi\varepsilon_0^2 m_T \cancel{\mu_i} v}\int_0^{v} \frac{\partial}{\partial v}\left(\left(1+\frac{v^3}{v_{cr}^3}\right) f_T\right) dv$$

$$= \int_0^{v} \frac{S_0}{4\pi}\left[\delta(v-v_m) - \delta(v-v_0)\right] dv$$

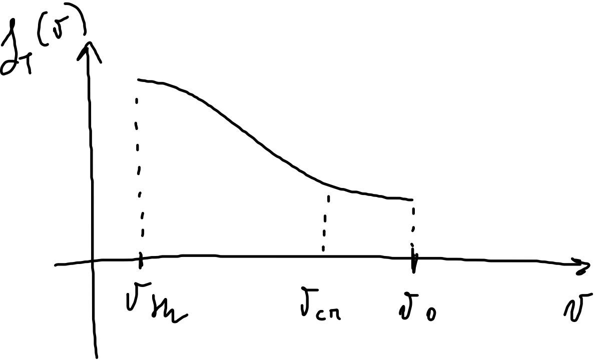
RHS

if $v > v_0$: $(1-1) = 0$

$$\left(1+\frac{v^3}{v_{cr}^3}\right) f_T = 0 \Rightarrow f_T = 0$$

if $v < v_m$: RHS $= 0 \Rightarrow f_T = 0$

$v_m < v < v_0$ $\quad (\text{coeff})\left(1+\frac{v^3}{v_{cr}^3}\right) f_T = \frac{S_0}{4\pi} \Rightarrow f_T = \frac{(\text{coeff})}{v^3 + v_{cr}^3}$



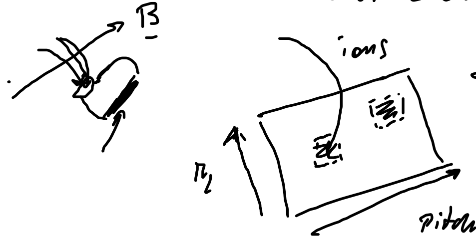
How can we measure the α particle pdf?

α particles should be confined mostly in the core

~ 150 million K

→ Measure the lost α particles "directly"

Fast Ion Loss Detector



Scintillator Plate : Image of velocity space of lost ions

Core αCplas

Indirect measurement

1) γ-ray emission
(Passive diagnostics)

2) Collective Thomson Scattering
(Active diagnostics)

scattered wave

GHz waves α_S

3) Neutral Particle Analyzer

Impurity

$\alpha + {}^{9}Be \to n + {}^{12}C^{*}$ excited state

light particle heavy ion

$^{12}C^{*} \xrightarrow{ns} {}^{12}C + \gamma$, $E_\gamma = 4.44$ MeV

e^-

α + Neutral beam

Neutral α