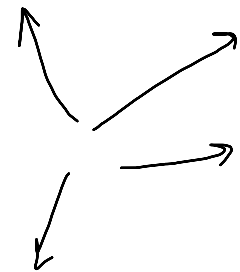
$$\frac{\partial n}{\partial t} = D \frac{d^2 n}{dx^2}$$ Diffusion equation

$$D \sim \frac{(\Delta x)^2}{\Delta t} \qquad [D] = m^2/s$$

Plasma w/o $\underline{B}$

$$\Delta x \sim \lambda_{mfp} \sim v_{th} \cdot \Delta t$$



$$D \sim \frac{v_{th}^2 (\Delta t)^{\cancel{2}}}{\cancel{\Delta t}} \sim \frac{T}{m\nu}$$

Plasma with $\underline{B}$

$$\Delta x \sim r_L$$

$$D \sim r_L^2 \cdot \nu$$

Weakly vs Fully ionized plasmas

Weakly → significant amount of neutrals

→ Collisions $\left\{ \begin{array}{l} e-n \\ i-n \end{array} \right.$ are most dominant

Fully ionized plasmas → (almost) no neutrals

Coulomb collisions

$e-e$ $i-i$ like particles

$e-i$ unlike particles

Weakly ionized plasmas

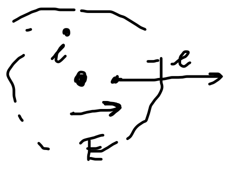
$$\nu_{e,i} = n \cdot v_{rel} \cdot \sigma \underset{\text{weakly ionized plasma}}{\approx} n_n \cdot v_{th_{e,i}} \cdot \sigma_n$$

$$\frac{\nu_{en}}{\nu_{in}} = \frac{v_{the}}{v_{thi}} \underset{T_e \sim T_i}{=} \sqrt{\frac{m_i}{m_e}} \gg 1$$

$B = 0$

$$\frac{D_e}{D_i} = \frac{\cancel{T}}{m_e \nu_e} \cdot \frac{m_i \nu_i}{\cancel{T}} = \frac{m_i}{m_e} \underbrace{\frac{\nu_i}{\nu_e}}_{\sqrt{\frac{m_e}{m_i}}} = \sqrt{\frac{m_i}{m_e}} \gg 1$$

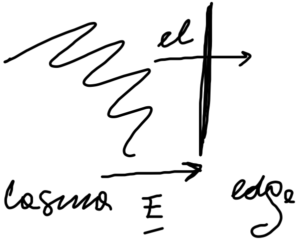
electrons diffuse more than ions
(initially)



an $\underline{E}_{\text{ambipolar}}$ is generated from charge sep.

- slow down el. diffusion
- increase ion diffusion

} ion and el. diffuse at the same rate at equilibrium



$V_p > 0$

$(V_{edge} = 0)$

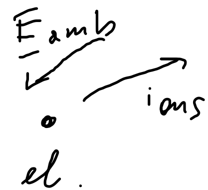
$$r_L^2 = \frac{m^2 v_\perp^2}{e^2 B^2} \approx \frac{m^{\cancel{2}} \frac{T}{\cancel{m}}}{e^2 B^2} \sim \frac{mT}{B^2}$$

$B \neq 0$

$$D \sim r_L^2 \nu$$

$$\frac{D_e}{D_i} \sim \frac{r_{Le}^2 \nu_{en}}{r_{Li}^2 \nu_{in}} \sim \frac{m_e \cancel{T}}{m_i \cancel{T}} \sqrt{\frac{m_i}{m_e}} \sim \sqrt{\frac{m_e}{m_i}} \ll 1$$

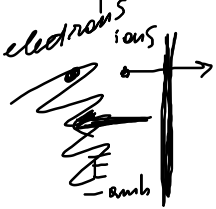
Ions diffuse more (initially)



$\underline{E}_{amb}$ is generated

$$\underline{E}_{amb} \begin{cases} \nearrow \text{slows down ions} \\ \searrow \text{accelerates electrons} \end{cases} \Bigg\} \text{ ions and electrons diffuse at the same rate}$$

$V_{p_s} < 0$



plasma edge

$\underline{E}_{amb}$ is towards the plasma

$$D \sim r_L^2 \nu \sim \frac{mT}{B^2} n v_{th} \sigma \sim \frac{n T^{3/2}}{B^2}$$

Fully ionized plasma
($B \neq 0$)

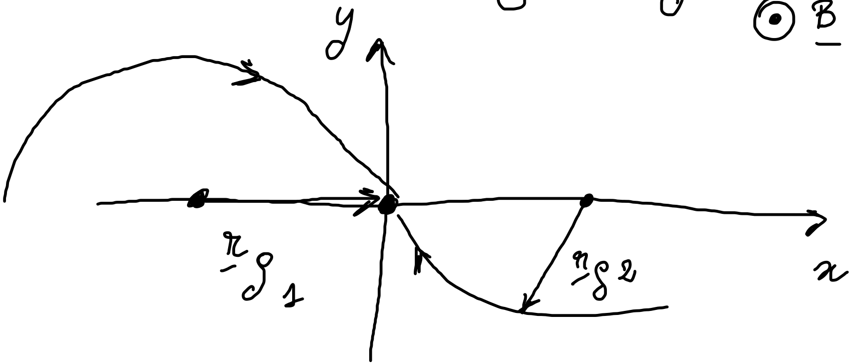
i i
e e } do not lead to transport

$D \sim r_{Li}^2 \nu_i$ ~~strong~~

i-e $\longrightarrow$ lead to transport

C.m. is unchanged by the collisions

$\odot \underline{B}$ Collision occurs at the origin



For simplicity, after the collision, both ions are on the initial plane c.m. before the collision

$$v_x = v_\perp \cos(\omega_L t - \phi)$$

$$v_y = - v_\perp \sin(\omega_L t - \phi)$$

$$x = x_{gc} + v_\perp / \omega_L \sin(\omega_L t - \phi)$$

$$y = y_{gc} + v_\perp / \omega_L \cos(\omega_L t - \phi)$$

$$v_\perp / \omega_L = \frac{m v_\perp}{qB} = r_L$$

Collision at the origin:

$$0 = x_{gc} + v_\perp / \omega_L \sin(\omega_L t - \phi) \; ; \; x_{gc} = - r_L \sin(\omega_L t - \phi)$$

$$0 = y_{gc} + v_\perp / \omega_L \cos(\omega_L t - \phi)$$

$$y_{gc} = - r_L \cos(\omega_L t - \phi)$$

$$\underline{r}_{gc_{1,2}} = -r_L \sin(\omega_L t - \phi)\,\hat{\underline{i}} - r_L \cos(\omega_L t - \phi)\,\hat{\underline{j}} =$$

$$= \frac{v_y}{\omega_L}\hat{\underline{i}} - \frac{v_x}{\omega_L}\hat{\underline{j}} = \frac{\underline{v}_{1,2} \times \hat{\underline{e}}_z}{\omega_L}$$

$v_x = v_1 \cos(\quad)$

$v_y = -v_1 \sin(\quad)$

$$\underline{r}_{cm} = \frac{m_1 \underline{r}_{gc_1} + m_2 \underline{r}_{gc_{1,2}}}{m_1 + m_2} \xrightarrow{m_1 = m_2} \frac{\underline{r}_{gc_1} + \underline{r}_{gc_2}}{2}$$

How does $\underline{v}_{12}$ change after a collision?

$$\underline{v}_{rel} = \underline{v}_1 - \underline{v}_2 \qquad \underline{V}_{cm} = \frac{m_1 \underline{v}_1 + m_2 \underline{v}_2}{m_1 + m_2} \xrightarrow{m_1 = m_2} \frac{\underline{v}_1 + \underline{v}_2}{2}$$

Invert the previous relations

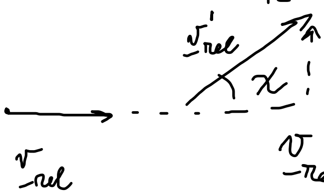
$$\underline{v}_1 = \underline{V}_{cm} + \underline{v}_{rel}/2$$

$$\underline{v}_2 = \underline{V}_{cm} - \underline{v}_{rel}/2$$

' : after the collision

c.m. frame $\quad \underline{v}'_{rel}$ experiences a rotation by an angle χ (arbitrary)

$|\underline{v}'_{rel}| = |\underline{v}_{rel}|$



$$\underline{V}'_{cm} = \underline{V}_{cm}$$

$$\underline{v}'_{rel} = \underline{v}_{rel}\cos\chi + \underline{v}_{rel}\sin\chi \times \hat{\underline{e}}_t$$

$$\underline{v}_1' = \underline{V}_{cm}' + \frac{\underline{v}'_{rel}}{2} = \underline{V}_{cm} + \underline{v}'_{rel}/2$$

$$\underline{v}_2' = \underline{V}_{cm}' - \frac{\underline{v}'_{rel}}{2} = \underline{V}_{cm} - \underline{v}'_{rel}/2$$

$$\underline{r}_{cm}' = \frac{\underline{r}'_{gc1} + \underline{r}'_{gc2}}{2} \qquad \underline{r}'_{gc1} = \frac{\underline{v}_1' \times \hat{\underline{e}}_z}{\omega_L} \qquad \underline{r}'_{gc2} = \frac{\underline{v}_2' \times \hat{\underline{e}}_z}{\omega_L} =$$

$$= \frac{\underline{V}_{cm} \times \hat{\underline{e}}_z}{\omega_L} \qquad = \frac{\underline{V}_{cm} \times \hat{\underline{e}}_z + \frac{\underline{v}'_{rel}}{2} \times \hat{\underline{e}}_z}{\omega_L} \Bigg) = \frac{\underline{V}_{cm} \times \hat{\underline{e}}_z - \frac{\underline{v}'_{rel}}{2} \times \hat{\underline{e}}_z}{\omega_L}$$

$$\underline{r}_{cm} = \frac{\underline{r}_{gc_1} + \underline{r}_{gc_2}}{2} = \frac{\underline{v}_1 \times \hat{\underline{e}}_z}{2\omega_L} + \frac{\underline{v}_2 \times \hat{\underline{e}}_z}{2\omega_L} =$$

$$= \frac{1}{\omega_L} \left(\frac{\underline{v}_1 + \underline{v}_2}{2}\right) \times \hat{\underline{e}}_z = \frac{1}{\omega_L} \underline{V}_{cm} \times \hat{\underline{e}}_z = \underline{r}'_{cm}$$

No diffusion from like particle collisions

e - i

$$\underline{r}_{gc_{1,2}} = \frac{\underline{v}_{12} \times \hat{\underline{e}}_z}{\omega_L}$$

$$\underline{v}_1 = \underline{V}_{cm} + \underline{v}_{rel}/2$$

$$\underline{v}_2 = \underline{V}_{cm} - \underline{v}_{rel}/2$$

m_i m_e

$$\underline{r}_{cm} = \frac{m_1 \underline{v}_1 + m_2 \underline{v}_2}{m_1 + m_2}$$

$$\underline{V}'_{cm} = \underline{V}_{cm}$$

$$\underline{v}'_{rel} = \underline{v}_{rel}\cos\chi + \underline{v}_{rel}\times\hat{\underline{e}}_z \sin\chi$$

$$\underline{v}'_1 \qquad \underline{v}'_2$$

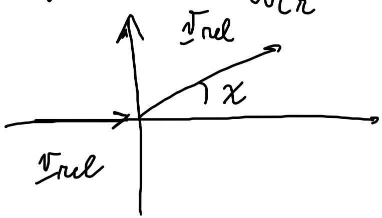
$$\Delta \underline{r}_{cm} = -\frac{1}{\omega_{cr}}\left[(\underline{v}_{rel}\times\hat{\underline{e}}_z)(\cos\chi-1)+\underline{v}_{rel}\sin\chi\right]$$

$$\omega_{cr} = \frac{qB}{\mu} \approx \omega_{ce} \qquad \mu = \frac{m_e m_i}{m_e+m_i} \approx m_e$$

average over χ

average over ξ

$$\langle \Delta \underline{r}_{cm}\rangle = +\frac{1}{\omega_{cr}}(\underline{v}_{rel}\times\hat{\underline{e}}_z) = 0$$



$$\langle \Delta r^2_{cm}\rangle = ?$$