Fully ionized plasmas
($B \neq 0$)

Coulomb collisions

e - i

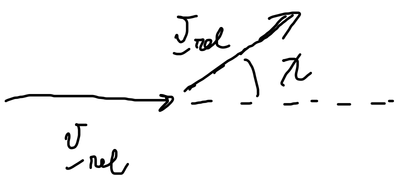
e - e } like particle collisions
i - i

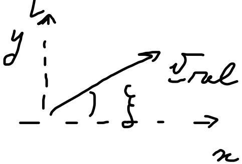
e - i → unlike part. coll.

Like part. coll. : $\Delta \underline{r}_{gc\,cm} = 0$ Do not contribute to transport

Unlike " " :

$$\Delta \underline{r}_{gc\,cm} = -\frac{1}{\omega_{cr}}\left[(\underline{v}_{rel} \times \hat{\underline{e}}_z)(\cos\chi - 1) + \underline{v}_{rel} \sin\chi \right]$$





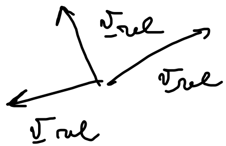
(ξ, χ)

Average over χ:

$$\langle \Delta \underline{r}_{gcm} \rangle \Big|_{\text{over } \chi} = \frac{1}{w_{cr}} \left(\underline{v}_{rel} \times \hat{\underline{e}}_z \right)$$

Average over ξ:

$$\langle \Delta \underline{r}_{gcm} \rangle \Big|_{\text{over } \chi, \xi} = 0$$



$$\left(\Delta r_{gcm} \right)^2 = \frac{v_{rel}^2}{w_{cr}^2} \left[(\cos\chi - 1)^2 + \sin^2\chi \right]$$

$$= \frac{v_{rel}^2}{w_{cr}^2} \left[2 - 2\cos\chi \right] = \frac{2 v_{rel}^2}{w_{cr}^2} (1 - \cos\chi)$$

$$\langle (\Delta r_{gcm})^2 \rangle = \frac{\int_0^{2\pi} d\xi \int_0^{2\pi} d\chi \, \frac{2 v_{rel}^2}{\omega_{cr}^2} (1-\cos\chi)}{\int_0^{2\pi} d\xi \int_0^{2\pi} d\chi}$$

$$= \frac{2 v_{rel}^2}{\omega_{cr}^2}$$

$$D \sim (\Delta r)^2 \cdot \nu$$

$$\omega_{cr} = \frac{qB}{\mu} \qquad \mu = \frac{m_e m_i}{m_e + m_i} \approx m_e$$

$$\frac{qB}{m_e} = \omega_{ce}$$

$$v_{rel} = |\underline{v}_e - \underline{v}_i| \approx v_e$$

Average over the distr. function of e and i : $\langle v_{rel}^2 \rangle \approx v_{the}^2$

$$\langle (\Delta r_{gcm}^2) \rangle \Big|_{\chi, \xi, pdf} \approx \frac{2 v_{the}^2}{\omega_{ce}^2}$$

$$D_{ei} \sim \frac{2v^2_{the}}{\omega^2_{ce}} \cdot \nu_{ei} \sim \frac{T_e}{B^2}\frac{n_e}{T_e^{3/2}} \sim \frac{n_e}{B^2}\frac{1}{T_e^{1/2}}$$

$$\nu_{ei} \propto \frac{n_e}{T_e^{3/2}}$$

$$v^2_{th} \sim T/m$$

$$T_e \sim T_i$$

$$\frac{D_{ei}\big|_{right}}{D_{ii}\big|_{wrong}} \sim \frac{r^2_{Le} \cdot \nu_{ei}}{r^2_{Li} \cdot \nu_{ii}} = \frac{\frac{m^2_e v^2_{the}}{\cancel{e^2 B^2}} \cdot \nu_{ei}}{\frac{m^2_i v^2_{thi}}{\cancel{e^2 B^2}} \nu_{ii}} \sim \frac{m_e}{m_i}\sqrt{\frac{m_i}{m_e}} \sim \sqrt{\frac{m_e}{m_i}}$$

$$\ll 1$$

Braginskii

$$D = 2\cdot 10^{-3} \frac{n_{20}}{B_0^2 T_K^{\frac{1}{2}}} \left[m^2/s \right]$$

Joint European Torus

$n_{20} = n_e / 10^{20}\, m^{-3}$

B_0 in Tesla

$n_{20} \sim 1/2$ $B_0 \sim 3$

T_K temp. in keV

$T_K = 5$

$L \sim 1\, m$

$D \sim 5\cdot 10^{-5}\, m^2/s$

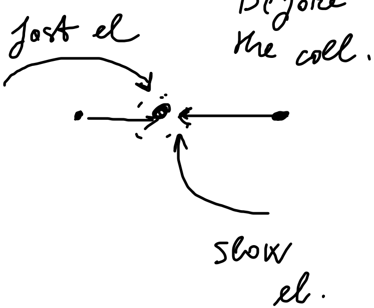
$\tau \sim L^2 / D \sim 10^4\, s$ ~ few hours

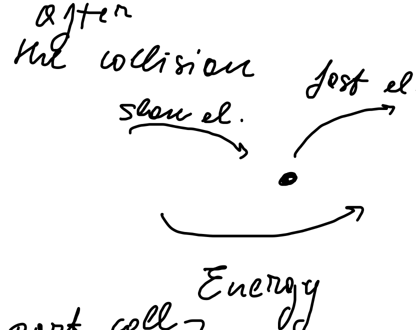
$D_{exp} \sim 1\, m^2/s$

Energy diffusion coefficient (diffusivity)

If particles diffuse, then energy diffuses
but:

Energy diffusion w/o net particle diffusion





Unlike part coll
Like " coll

Both contribute to diffusion of energy

Like particle

$$\underline{r}_{CE} = \frac{\underline{r}_{g_1} v_1^2 + \underline{r}_{g_2} v_2^2}{v_1^2 + v_2^2}$$

center of mass
of energy

$$\Delta \underline{r}_{CE} = \underline{r}'_{CE} - \underline{r}_{CE}$$

$$\left\langle \left(\Delta \underline{r}_{CE}\right)^2 \right\rangle \Big|_{\xi, \chi} = \dots = \frac{2 v_{rel}^4 V_{cm}^2}{\omega_c^2 \left(4V_{cm}^2 + v_{rel}^2\right)^2}$$

$$\left\langle \left(\Delta \underline{r}_{CE}\right)^2 \right\rangle \Big|_{\xi, \chi, \text{pdf } V_{cm}, v_{rel} \sim v_{th}} \sim v_{th}^6 / v_{th}^4 \; \frac{1}{\omega_c^2} \sim v_{th}^2 / \omega_c^2$$

$$\chi \sim \langle \Delta r_{CE} \rangle^2 \cdot \nu$$

$$\left.\begin{matrix} ii \\ ee \end{matrix}\right\} \quad \chi_i \sim \langle \Delta r_{CE} \rangle_i^2 \cdot \nu_{ii}$$

$$\chi_e \sim \langle \Delta r_{CE} \rangle_e^2 \, \nu_{ee}$$

$$\chi \sim \frac{v_{th}^2}{\omega_c^2} \cdot \nu \sim \frac{T}{B^2} \, \frac{n}{T^{3/2}}$$

$$\sim \frac{n}{B^2} \cdot \frac{1}{T^{\frac{1}{2}}}$$

$$\frac{\chi_i}{\chi_e} \sim \frac{\dfrac{v^2_{mi}}{e^2 B^2 / m_i^2} \, \nu_{ii}}{\dfrac{v^2_{me}}{e^2 B^2 / m_e^2} \, \nu_{ee}} \sim \frac{\dfrac{1}{\cancel{m_i}} \, m_i^{\cancel{2}} \, \nu_{ii}}{\dfrac{1}{\cancel{m_e}} \, m_e^{\cancel{2}} \, \nu_{ee}} \sim \frac{m_i}{m_e} \frac{\nu_{ii}}{\nu_{ee}} \sim \frac{m_i}{m_e} \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \sim \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} >> 1$$

$\Rightarrow$ Ion-Ion coll. dominate energy diff.

$$D \sim r_L^2 \cdot \nu \qquad \langle \Delta r \rangle^2 \sim \frac{v_{th}^2}{\omega_c^2} = \frac{m^2 v_{th}^2}{q^2 B^2} \sim r_L^2$$

Braginskii

$$\chi_i = 0.10 \frac{n_{20}}{B_0^2 T_k^{\frac{1}{2}}} \; m^2/s$$

$$\chi_e = 4.8 \cdot 10^{-3} \frac{n_{20}}{B_0^2 T_k^{\frac{1}{2}}} \; m^2/s$$

Joint European Torus $\quad T_k = 5 \quad n_{20} = 0.5 \quad B_0 = 3$

$$\chi_i \approx 2.5 \cdot 10^{-3} \; m^2/s \qquad \chi_{exp} \sim m^2/s$$

Collisional transport + toroidal geometry } Neoclassical transport

Geometry → passing particles
Geometry → trapped particles

Δr = (orbit width)

Passing part: $\Delta r_{nc} \sim 2 \underbrace{q}_{\text{safety factor}} r_L$

$D_{nc} \sim (\Delta r_{nc})^2 \cdot \nu \sim 4q^2 \cdot \underbrace{r_L^2 \cdot \nu}_{D_c} \sim \times 10$

Trapped particles

& for trapped particle

$\Delta r \sim$ (orbit width) $\qquad \Delta r \sim \frac{q}{\sqrt{\varepsilon}} r$

$D^{(nc)} \sim (\Delta r)^2 \cdot \bar{\nu} \cdot$ (fraction of trapped part.) $\sim \varepsilon^{-\frac{3}{2}} q^2 D^{(c)}$

detrapping, $\rightarrow$ larger than ν_{coll}

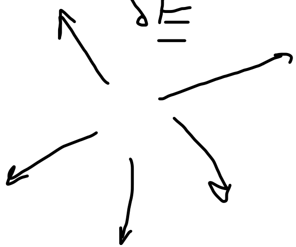
$\sim \times 100 \, D^{(c)}$

Bootstrap current : steady state current

$\propto \frac{\partial p}{\partial r}$

Turbulence

$\delta \underline{E}$



$$\delta \underline{v}_{drift} = \frac{\delta \underline{E} \times \underline{B}}{B^2}$$

Random walk process

Diffusion