Chapter 4 - Collisional transport

1 Collisions in a weakly ionized plasma

The cross section for a collision between 2 eV electrons and a neutral particle is approximately $\sigma \approx 6\pi a_0^2$, where a_0 is the Bohr radius. A plasma column made of helium is at the pressure p=1 Torr (at room temperature) and the electrons have a temperature of 2 eV. There is no magnetic field.

- Find the electron diffusion coefficient D_e in m^2/s, assuming that the electron distribution averaged reactivity ($< \sigma v >$) is equal to the reactivity for monoenergetic electrons at the energy of 2 eV.
- If $\mathbf{E}$ is the electric field along the plasma column, show that the average electron velocity $\mathbf{u_e}$ satisfies

$$\mathbf{u_e} = -\frac{e}{m\nu}\mathbf{E} - \frac{T}{m\nu}\frac{\nabla n}{n} \tag{1}$$

  where ∇n is the density gradient along the plasma column and ν is the electron-neutral collision frequency.

- A current density j =2 kA/m^2 and a plasma density of 10^{16} m^{-3} are measured. Verify that

$$\mathbf{E} \approx \frac{m\nu}{ne^2}\mathbf{j} - \frac{D_e m\nu}{ne}\nabla \mathbf{n} \tag{2}$$

  where e is the electron charge and n is the plasma density.

- By neglecting the term proportional to ∇n in the previous equation, determine the magnitude of $\mathbf{E}$. How large can the density variation length be so that the term proportional to ∇n can be neglected?

2 One dimensional diffusion equation

We want to solve the diffusion equation for the density n in the absence of sources

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = 0 \tag{3}$$

in a plane geometry and in one dimension. D_a is the ambipolar diffusion coefficient, which is known and uniform. The plasma extends from $x = -L$ to $x = L$ and satisfies the boundary condition $n(\pm L) = 0$.

- By searching for a solution of the type $n(x,t) = X(x)T(t)$, show that the general solution of the equation can be written as

$$n(x,t) = \sum_{n=0}^{+\infty} A_n \cos\left(\pi\left(n+\frac{1}{2}\right)\frac{x}{L}\right)\exp\left(-t/\tau_n\right) \tag{4}$$

  where $\tau_n = \frac{L^2}{D_a \pi^2}\frac{1}{(n+\frac{1}{2})^2}$ and A_n are coefficients. Each n term is called a "diffusion mode". Note that higher diffusion modes decay more rapidly.

- Assume that there now is a source term $S(x,t) = S_0\delta(x)$ so that the diffusion equation becomes

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = S_0 \delta(x) \tag{5}$$

  Determine the steady state density profile $n(x)$ and verify that

$$n(x) = \frac{S_0 L}{2D_a}\left(1 - \frac{|x|}{L}\right) \tag{6}$$