Thermonuclear Fusion

1939: Bethe   Sun is powered by fusion reactions

$$p + p \to d + e^+ + \nu_e$$

$d + d \to n + {}^3He$ $\simeq 50\%$ $Q = 3.27$ MeV

$ p + t$ $\simeq 50\%$ $Q = 4.03$ MeV

$d + t \to \alpha + n$ $Q = 17.6$ MeV

$d + {}^3He \to p + \alpha$ $Q = 18.3$ MeV

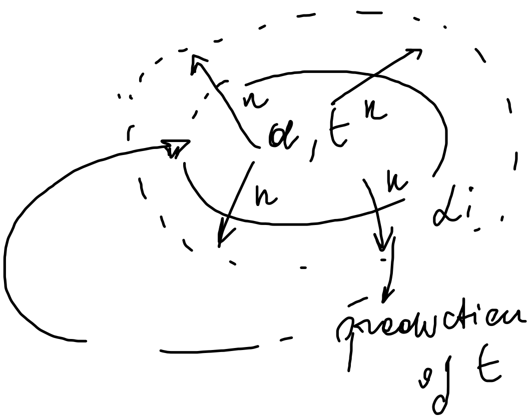
Fuel: d from water $\simeq 10^6$ years

t rare element

t breeding

$${}^{1}_{0}n + {}^{6}_{3}Li \longrightarrow {}^{3}_{1}t + \alpha \qquad Q = 4.8 \text{ MeV}$$

$${}^{1}_{0}n + {}^{7}_{3}Li \longrightarrow {}^{3}_{1}t + \alpha + n \qquad Q = -2.5 \text{ MeV}$$



heat the plasma

extract energy

$\underbrace{d + t}_{\text{keV}} \rightarrow \alpha + n$

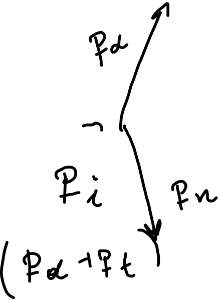
$Q = 17.6$ MeV

n kinetic energy converted into heat.

Li blanket

Cons. energy and momentum

$$Q \approx \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_n v_n^2$$

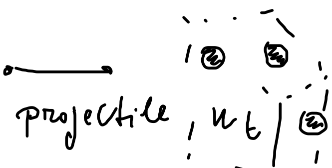


$$p_\alpha \approx p_n \quad ; \quad m_\alpha v_\alpha = m_n v_n$$

$$Q \approx \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_n v_\alpha^2$$

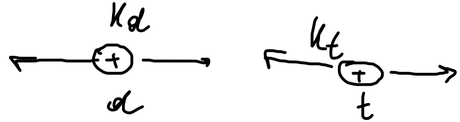
$$E_\alpha \approx Q \cdot \frac{m_n}{m_\alpha + m_n} \approx \frac{1}{5} Q \approx 3.5 \text{ MeV}$$

$$E_n \approx Q \cdot \frac{m_\alpha}{m_\alpha + m_n} \approx \frac{4}{5} Q \approx 14.1 \text{ MeV}$$

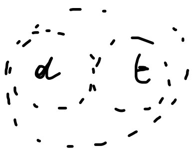


$$w = n_{\text{target}} \cdot \sigma \cdot v_{rel}$$

Classical estimate of σ



We want ~~touch~~ to "touch" each other



$$\sigma \approx \pi \cdot (r_d + r_t)^2 \qquad r \approx r_0 A^{1/3} \qquad r_0 \approx 1.2\ \text{fm} \to 10^{-15}\ \text{m}$$

$$\approx 0.28 \cdot 10^{-28}\ \text{m}^2 \qquad A_d = 2,\quad A_t = 3$$

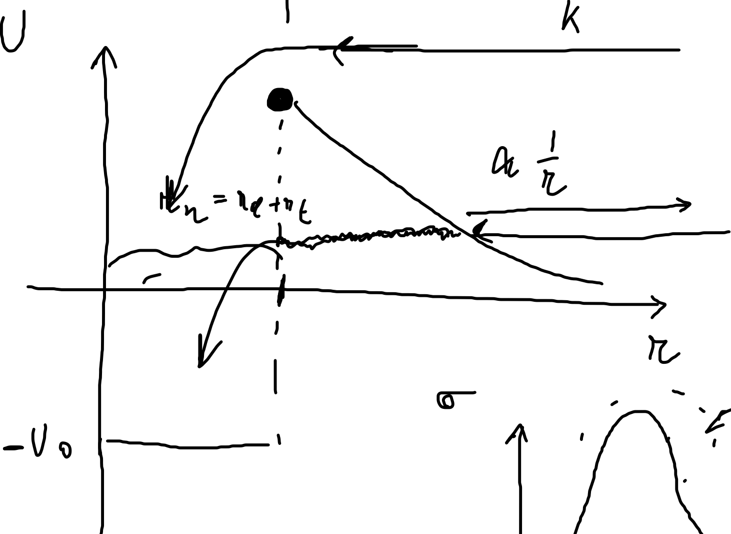
$$\approx \text{barn} \qquad 1\ \text{b} = 10^{-28}\ \text{m}^2$$

$$\frac{1}{2}\mu v_{rel}^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{(r_d + r_t)} \approx 480\ \text{keV}$$

$$1\ \text{eV} \to 12000\ \text{K}$$

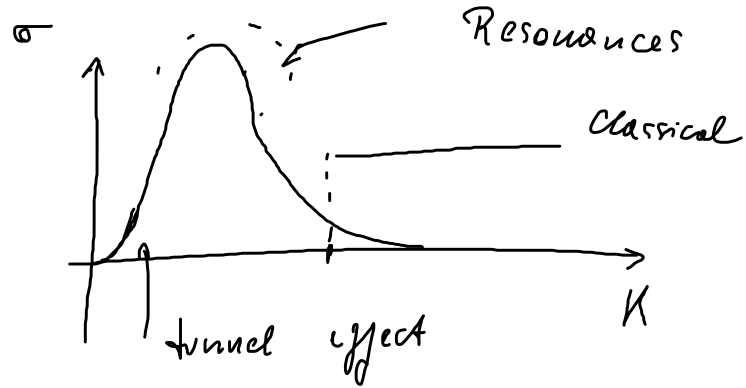
$$500 \cdot 10^3 \cdot 10^4 \sim \underline{5\ \text{billion K}}\,!$$

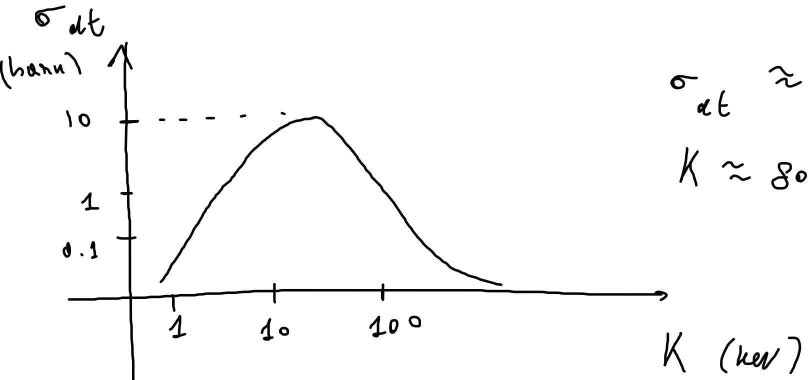
"Fusion potential"



classical picture

quantum mechanics





$\sigma_{dt} \approx 10$ barn

$K \approx 80$ keV

Reaction rate: reaction/s

Plasma

$f_d(\underline{v}_d)\, d^3\underline{v}_d$

1 projectile (d)

$\text{prob}/s = n_t \cdot \sigma \cdot v_{rel} \cdot$ (fraction of deuterons that have $\underline{v}_d$) $\cdot$ (fraction of tritons that have $\underline{v}_t$)

$f_t(\underline{v}_t)\, d^3\underline{v}_t$

$\underline{v}_d \qquad \underline{v}_t$

$$= n_t\, \sigma\, v_{rel}\, f_t(\underline{v}_t)\, f_d(\underline{v}_d)\, d^3\underline{v}_d\, d^3\underline{v}_t$$

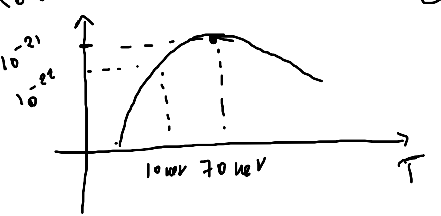
1 projectile d $= n_t \int_{\underline{v}_d, \underline{v}_t} \sigma(v_{rel})\, v_{rel}\, f_d(\underline{v}_d) f_t(\underline{v}_t)\, d^3\underline{v}_d\, d^3\underline{v}_t$

'it doesn't matter which $\underline{v}_d$, $\underline{v}_t$ you take'

at equilibrium: T

$$\frac{\text{Prob. to get fusion}}{\text{volume time}} = n_d n_t \int \sigma(v_{rel})\, v_{rel}\, f_d(\underline{v}_d) f_t(\underline{v}_t)\, d^3\underline{v}_d\, d^3\underline{v}_t = \frac{\text{reactions}}{m^3 \cdot s} = r$$

$\langle\sigma v\rangle [m^3/s]$ $= n_d n_t \underbrace{\langle \sigma v \rangle}_{\text{reactivity: } m^3/s} (T)$



$$\frac{\text{fusion react}}{s} = \int_{\text{Volume}} r\, dV \approx r \cdot V \quad r \text{ uniform}$$

$n_d \sim 10^{20}\, m^{-3}$

$10^{40} \cdot 10^{-22} \sim 10^{18} \frac{\text{reactions}}{m^3 \cdot s}$

1 m^3 of d,t plasma

$\sim 10^{18}$ react/s

$\sim$ 14 MeV energy/react

$$P_{fusion} \sim 10^{18} \cdot 14 \cdot 1.6 \cdot 10^{-13} \, \underbrace{J/s}_{W} \sim 2 \cdot 10^{6} \, W \sim 2 \, MW$$

Gains	Losses
1) α particle heating	1) Emission of radiation (bremsstrahlung)
2) External heating	2) Finite confinement τ_E (diffusion)

If

(gain) = (losses) then the system goes on

α particle heating W/m^3 S_α

$n_d = \frac{n_e}{2} = n_t$

$$S_\alpha = \left(\frac{\#\text{reactions}}{s\,m^3}\right) \cdot \underbrace{E_\alpha}_{3.5\ \text{MeV}} = n_d n_t \langle\sigma v\rangle(T) \cdot E_\alpha = \frac{n_e^2}{4} E_\alpha \langle\sigma v\rangle(T)$$

set by temp.

$d + t$ Optimal mixture?

$n_e = n_d + n_t$ Quasi neutrality n_e fixed

$n_d \rightarrow n_t = n_e - n_d$ Maximize: $n_d n_t = (n_e - n_d) \cdot n_d$

$$\frac{\partial}{\partial n_d}\left[(n_e - n_d)\, n_d\right] = 0 \Rightarrow \left.\begin{aligned} n_d &= n_e/2 \\ n_t &= n_e/2 \end{aligned}\right\} n_d = n_t$$

S_h does not depend on pl. parameters

<u>Losses</u>

$$S_B = \left(n_e^2 \cdot T^{\frac{1}{2}}\right) \cdot \text{coeff}$$

$$\underbrace{S_K}_{\text{Transport losses}} = ? = \frac{3}{2}\frac{p}{\tau_E} \rightarrow \text{work out from experiment}$$

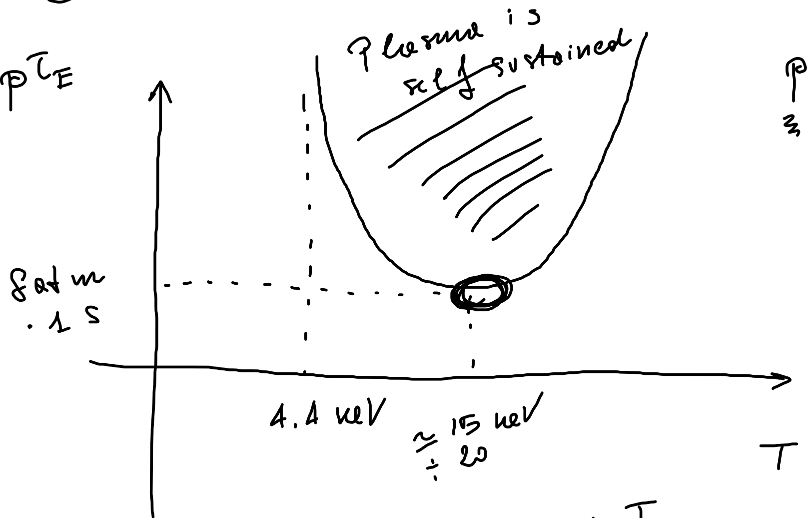
$$S_\alpha + S_h = S_B + S_K$$

Super optimistic: $\tau_E = +\infty$ $\quad S_h = 0$

Ideal ignition $\quad S_\alpha = S_B \; ; \; \cancel{n_e^2}\langle\sigma v\rangle(T)\,(\text{coeff}) = \cancel{n_e^2}\, T^{\frac{1}{2}}\,(\text{coeff})$

Equation for T $\qquad T = 4.4\ \text{keV}$

Ignition $S_\alpha = S_B + S_K$



$\underset{3}{p}, \underset{3}{\tau_E}, \underset{2}{T}$

If you keep a plasma confined for $\begin{cases} 1\ s \\ 8\ atm \\ 15 \div 20\ keV \end{cases}$

$p = n \cdot T$

$n \sim 10^{20}\ m^{-3}$ $t_E \sim s$

$p\tau_E \cdot T$ Lawson's criterion

$n\tau_E T \simeq 3 \cdot 10^{21}\ m^{-3}\, s\, keV$

Target : energy amplification

$S_n \neq 0$ $\quad Q = \dfrac{P_{fus}}{P_n} = \begin{cases} 10 \text{ for ITER} \\ 40 \div 50 \text{ for reactor (DEMO)} \end{cases}$

target

Start of ITER $\simeq 2027$ Full power demostr $\simeq 2035$

" " DEMO $\simeq 2040$ " " " in 2050s