SCALE FACTORS

"STRUCTURAL AND FUNCTIONAL CONSEQUENCES
OF THE SIZE CHANGES AMONG OTHERWISE
SIMILAR ORGANISMS "

(Schmidt-Nielsen)

- ISOMETRIC

- ALLOMETRIC

IMPLICATIONS:
- STRUCTURAL
- METABOLIC
- PHYSIOLOGICAL
- PHARMACOLOGICAL

ECOLOGICAL
EVOLUTION.

Surface - Volume RELATION for
ISOMETRIC SIZE VARIATIONS

$$A \propto \ell^2 \longrightarrow \ell \propto \sqrt{A}$$

$$m \propto Vol \propto \ell^3 \longrightarrow \ell \propto \sqrt[3]{m}$$

$$\Downarrow$$

$$A \propto \left(\sqrt[3]{m}\right)^2 = m^{2/3} = m^{0.67}$$
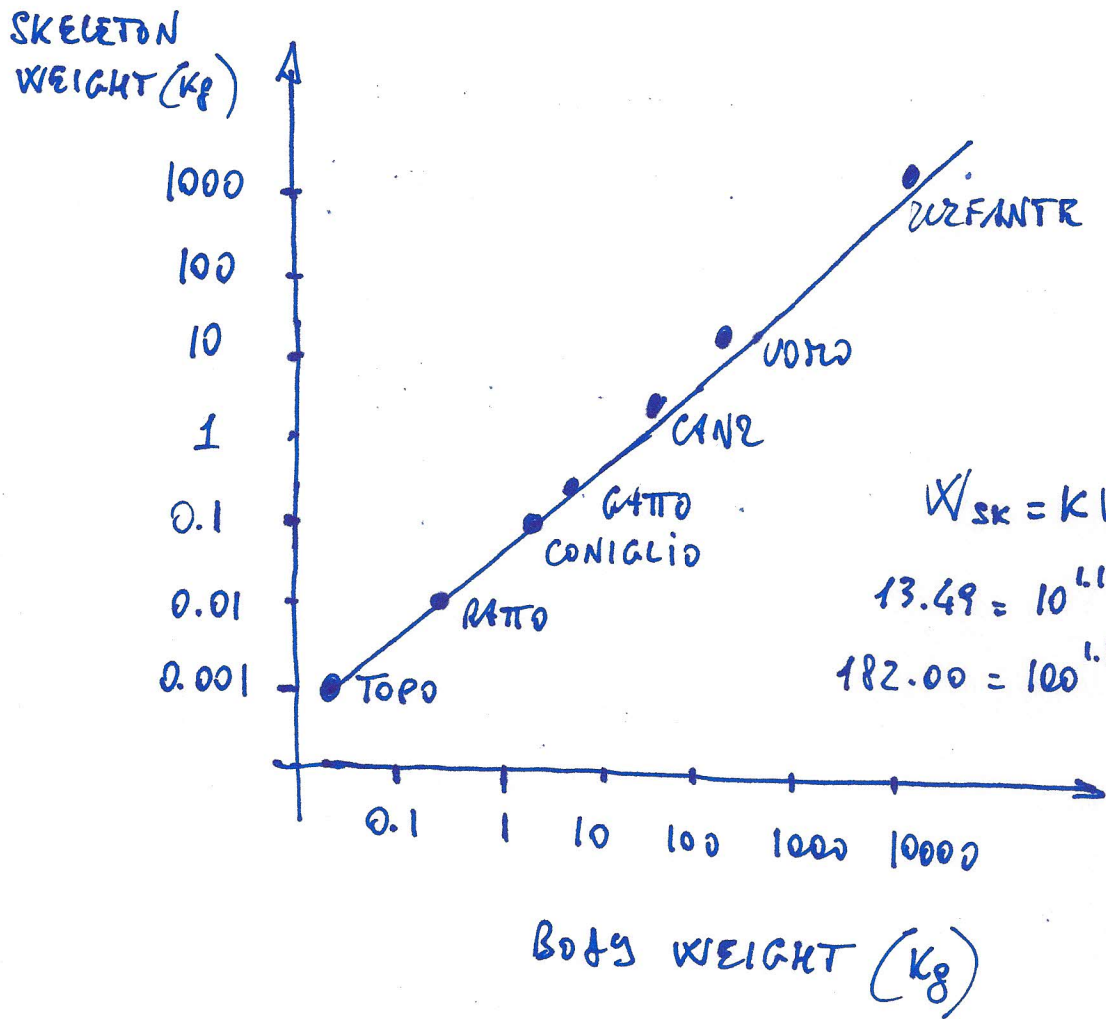
For instance, if m doubles, A does not.
If m becomes 10m, A becomes 4.7A.

IN CASE OF ALLOMETRIC VARIATIONS,
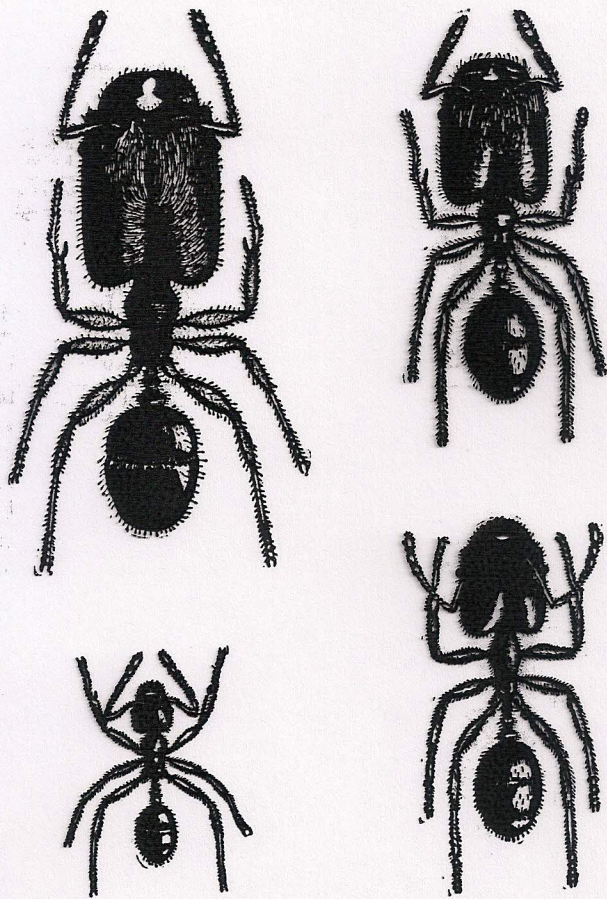THE EXPONENTS (also of linear dimensions)
ARE NOT THOSE EXPECTED FROM THE
ISOMETRIC CASE.

SKELETON WEIGHT (Kg)

1000

100

10

1

0.1

0.01

0.001

ELEFANTE

UOMO

CANE

GATTO

CONIGLIO

RATTO

TOPO

0.1   1   10   100   1000   10000

BODY WEIGHT (Kg)

$$W_{SK} = K \, W_{BODY}^{1.13}$$

$$13.49 = 10^{1.13}$$
$$182.00 = 100^{1.13} \quad (\text{if } K = 1)$$

SLOPE OF THE REGRESSION LINE:

$$1.13$$

i.e., bigger animals have comparatively heavier skeletons.

FIG. 36.—Increase of relative size of head with absolute size of body in the neuters of the ant, *Pheidole instabilis*.
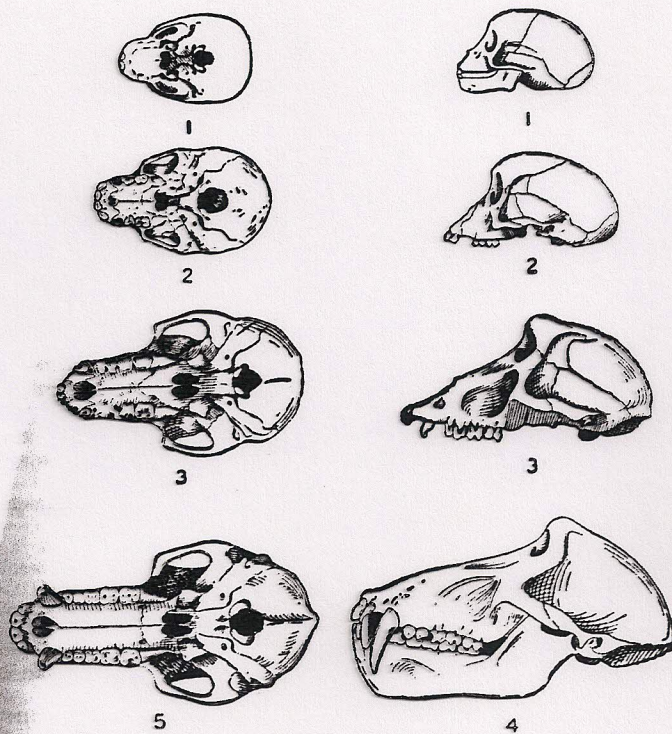
J. Huxley 1932
PROBLEMS OF RELATIVE GROWTH

FIG. 10.—Baboon skulls of various sizes, to show the increase in relative size of facial region with absolute size of skull.

1, new-born ; 2, juvenile (with milk dentition) ; 3, adult female ; 4, adult male.

$$L = bA^K \qquad \text{POWER LAW}$$

L = ant head length

A = abdomen length

$K = 2.06$ = slope of the line
in log-log

$$\ln L = \ln b + k \ln A$$

$$y = q + mx \qquad \text{STRAIGHT LINE}$$

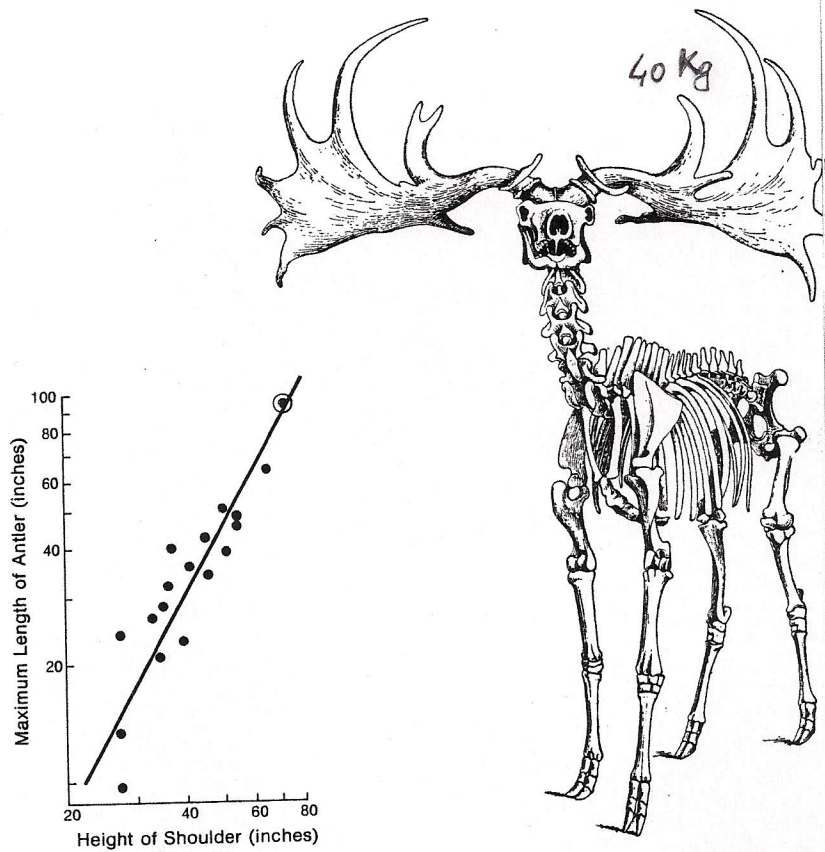REGRESSION : LEAST SQUARES

ORTHOGENESIS ?

40 Kg

FIGURE 8-7.
Relation between body size and antler size in cervine deer. On the whole, the antlers of cervine deer become proportionately larger as body size increases. The largest known ratio of antler to body size belonged to the Pleistocene Irish "elk" *Megaloceros* (pictured). Although the antlers are quite large, they are not unexpectedly large, and in fact are precisely the size (circled dot in the graph) that could be predicted from the observed trend (the straight line). [After Gould, 1973.]

COPE'S RULE

S.J. GOULD   ONTOGENY AND PHYLOGENY

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