TEST: Stone-Geary utility function

The Stone-Geary function is often used to model problems involving subsistence levels of consumption. In these cases, a certain minimal level of some good has to be consumed, irrespective of its price or the consumer's income.

The Stone-Geary uses the natural log function to model utility. The sum of all the proportions of the goods consumed must equal 1. In the problem below, the subsistence levels of X and Y are $\overline{X}$ and $\overline{Y}$. The term R is income, and p_k $\{k=X,Y\}$ are the prices of X and Y.

The Lagrangean and the stationarity conditions of the constrained (turned into an unconstrained) problem are:

$$\Lambda = \gamma \ln(X - \overline{X}) + (1-\gamma)\ln(Y - \overline{Y}) + \lambda(R - p_X X - p_Y Y)$$

$$\Lambda_X = \frac{\gamma}{X - \overline{X}} - p_X \lambda = 0$$

$$\Lambda_Y = \frac{1-\gamma}{Y - \overline{Y}} - p_Y \lambda = 0$$

$$\Lambda_\lambda = R - p_X X - p_Y Y = 0$$

Use the first two conditions to eliminate the Lagrangean Multiplier.

$$X = \frac{p_Y}{p_X}\frac{\gamma}{1-\gamma}(Y - \overline{Y}) + \overline{X}$$

or

$$Y = \frac{p_X}{p_Y}\frac{1-\gamma}{\gamma}(X - \overline{X}) + \overline{Y}$$

Substituting into the third and solving for X and Y we get the ordinary demand functions

$$X^M = \frac{\gamma}{p_X}\left(R - p_X \overline{X} - p_Y \overline{Y}\right) + \overline{X}$$

$$Y^M = \frac{1-\gamma}{p_Y}\left(R - p_X \overline{X} - p_Y \overline{Y}\right) + \overline{Y}$$

Notes

1. Each of the functions is the **Marshallian** or **Ordinary** demand functions for the *Stone-Geary* utility. Do you know other kind of demand functions?
2. The last term on the right-hand-side of the equality, is the **subsistence** consumption. A consumer will always consume this amount irrespective of their budgets or the price. Rewrite the demand and sum the terms including $\overline{X}$. Interpret.
3. The term $R - p_X \overline{X} - p_Y \overline{Y}$ is the income the consumer has left over, after the subsistence levels are met. It is in effect, the residual/disposable income.
4. The amount of X and Y that this residual income is used to buy, is now *negatively* influenced by price, and *positively* influenced by the good's importance (value of γ). For instance, if γ increases, it implies that good X is relatively more important than Y.

According to these demand functions, our consumer will purchase less of Y and more of X, all other things equal.

Questions

a) Derive the main properties of the two demand functions (derivative wrt p_x, degree of homogeneity in R and prices; etc.)
b) What happens to two functions when subsistence is zero? Do they look like Cobb-Douglas demands?
c) Evaluate if the sum of the two elasticities wrt prices and Income elasticity is zero. Interpret.
d) Evaluate if Cournot aggregation applies.
e) Evaluate if Engel aggregation applies.
f) Are you able to derive the optimal λ and to interpret it?

You are expected to follow the derivation of the demands without difficulties and to answer questions a) and b) only.