The Binomial Distribution

Exercise

Exercise 1

Marco is an enterprising boy. Suppose he asks 10 girls for their phone number, and has a probability of 0.2 of getting it.

Calculate:

1. The probability that the three girls give their phone number
2. The probability that at least one girl gives her phone number
3. The probability that at least two girls give their phone number

1.The probability that the three girls give their phone number

- n = 10 (number of girls to which he asks for the phone number)
- x = 3 (number of girls who give him the phone number)
- p = 0.20 (probability that a girl gives him the phone number)
- q = 1-p = 0.8

Ai = {Probability that i girls give him the phone number}

$$P(A_3) = \binom{10}{3} \cdot 0.2^3 \cdot 0.8^{(10-3)} =$$

$$= \frac{10!}{7! \cdot 3!} 0.2^3 \cdot 0.8^{(10-3)} = 120 \cdot 0.008 \cdot 0.21 = 0.20$$

2. The probability that at least one girl gives her phone number

A_0 = {No girl gives him her phone number}

$$P(A) = 1 - P(A_0)$$

$$P(A_0) = \binom{10}{0} \cdot 0.2^0 \cdot 0.8^{10} = 0.11$$

$$P(A) = 1 - 0.11 = 0.89$$

3. The probability that at least two girls give their phone number

P(B)= {Probability that at least two girls give their phone number}

$$P(B)=1-P(\bar{B})$$

$$P(\bar{B})=P(A_0)+P(A_1)$$

$$P(A_1)=\binom{10}{1}\cdot 0.2^1\cdot 0.8^9=0.27$$

$$P(\bar{B})=0.11+0.27=0.38$$

$$P(B)=0.62$$

$P(\bar{B})$= {probability that at most 1 girl gives him her phone number}

Exercise 2

Paolo and Jonathan have an urn each containing 4 white and 5 red balls. Everyone draws 4 balls, putting the ball back into their urn after each draw.

Calculate:

1. the probability that Paolo draws 2 white and 2 red balls;
2. the probability that Paolo draws 2 white and 2 red balls and that Jonathan draws 4 white balls.

1. P(P) = {Paolo's event, 2 white and 2 red balls}
2. P(J) = {Jonathan's event, 4 white balls}

1. <u>the probability that Paolo draws 2 white and 2 red balls</u>;

$$\boxed{P(P)}$$

$$P(P)=\binom{4}{2}\left(\frac{4}{9}\right)^{2}\left(\frac{5}{9}\right)^{(4-2)}=\frac{4!}{2!2!}\left(\frac{4}{9}\right)^{2}\left(\frac{5}{9}\right)^{(4-2)}=$$

$$=\frac{4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 2\cdot 1}\cdot\left(\frac{4}{9}\right)^{2}\left(\frac{5}{9}\right)^{(4-2)}=0.3658$$

1. P(P) = {Paolo's event, 2 white and 2 red balls}
2. P(J) = {Jonathan's event, 4 white balls}

2. the probability that Paolo draws 2 white and 2 red balls and that Jonathan draws 4 white balls.

$$P(P \cap J) = P(P) \cdot P(J)$$

$$\boxed{P(P \cap J)}$$

$$P(J) = \binom{4}{4}\left(\frac{4}{9}\right)^4 \left(\frac{5}{9}\right)^{(4-4)} = \frac{4!}{4!0!}\left(\frac{4}{9}\right)^4 \left(\frac{5}{9}\right)^0 =$$

$$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} \cdot \left(\frac{4}{9}\right)^4 = 0.039$$

$$P(P \cap J) = 0.3658 \cdot 0.039 = 0.0143$$

Exercise 3

Among 12 patients summoned for a visit, 2 belong to blood group AB.

Calculate:

1. **What is the probability that, among the first 4 who came for the visit, only one has group AB?**

1. Probability that, among the first 4 who came for the visit, only one has group AB

The probability of success (finding a patient of group AB) is constant in each attempt

$$p = \frac{2}{12} = 0.1667$$

We want to calculate the probability of observing x = 1 successes in n attempts;
n=4 and p=0.1667

$$P(x=1) = \binom{4}{1} \cdot 0.1667^{1} \cdot 0.83333^{3} = 0.385833$$

Exercise 4

Giorgio, who statistically has a 20% success rate in target shooting, makes five shots.
Calculate the probability that:
(1) He never hits the target,
(2) He hits it once,
(3) He hits it more than once

Choose the correct answer:

A) the first two events have the same probability, lower than the probability of the third

B) the first two events have the same probability, higher than the probability of the third

C) the first event is more probable than the others

D) the second event is more probable than the others

E) the third event is more probable than the other two, which have different probabilities among themselves

Answer

$$1)\ P(A_0 \mid 5) = \binom{5}{0} \cdot 0.2^0 \cdot (1-0.2)^5 = \frac{5!}{0!5!} \cdot 1 \cdot 0.8^5 = 0.8^5 = 0.33$$

$$2)\ P(A_1 \mid 5) = \binom{5}{1} \cdot 0.2^1 \cdot (1-0.2)^4 = \frac{5!}{1!4!} \cdot 0.2 \cdot 0.8^4 =$$

$$= 5 \cdot 0.2 \cdot 0.8^4 = 1 \cdot 0.8^4 = 0.41$$

3) Hitting more than once means hitting the target 2, 3, 4 or 5 times. The probability of this event E can be calculated as 1-P($\bar{E}$) = 1-P(0 ∪ 1 hit)

$P(\bar{E}) = P(A_0)+P(A_1) = 0.8^5+0.8^4 = 0.74$

$P(E) = 1- P(\bar{E}) = 1-0.74 =0.26$

The second event is more probable than the others, so D is the correct answer

Exercise 5

A quiz consists of 5 questions with three possible answers. One person randomly answers all 5.

Determine the probability that you get it right:

a) 2 correct answers;

b) 3 correct answers.

a) 0.329; b) 0.165

Exercise 6

Determine the probability that in a family with 4 children there are:

1) At least one male;
2) At least one male and one female
3) Out of 2,000 families with 4 children each, how many families have on average at least one male child? And how many families have two boys on average?

Assume that the probabilities of birth of boys and girls are equal

Answers

The random variable X indicates the number of males and p is the probability of a boy being born

1) n=4 x=0.5 P(X≥1)=P(X=1)+ P(X=2)+ P(X=3)+ P(X=4)

$$P(X=1)=\binom{4}{1}\cdot(0.5)^1\cdot(1-0.5)^{4-1}=\frac{1}{4}$$

$$P(X=2)=\binom{4}{2}\cdot(0.5)^2\cdot(1-0.5)^{4-2}=\frac{3}{8}$$

$$P(X=3)=\binom{4}{3}\cdot(0.5)^3\cdot(1-0.5)^{4-3}=\frac{1}{4}$$

$$P(X=4)=\binom{4}{4}\cdot(0.5)^4\cdot(1-0.5)^{4-4}=\frac{1}{16}$$

$$P(X\geq 1)=\frac{1}{4}+\frac{3}{8}+\frac{1}{4}+\frac{1}{16}=\frac{15}{16}$$

or …

… **or**

P(X≥1)=1-P(X=0)

$$P(X=0)=\binom{4}{0}\cdot(0.5)^0\cdot(1-0.5)^{4-0}=\frac{1}{16} \qquad P(X\geq 1)=1-\frac{1}{16}=\frac{15}{16}$$

2) P(at least one boy and one girl)=

<u>universe</u>

MMMM

MMMF MMFM MFMM FMMM

MMFF MFFM FFMM FMMF FMFM MFMF

FFFM FFMF FMFF MFFF

FFFF

$$P=\frac{14}{16}=\frac{7}{8}$$

or …

… or

P(at leat one boy and one girl)=

=1-[P(no boy) ∪ P(no girl)]=

$$=1-\frac{1}{16}-\frac{1}{16}=1-\frac{2}{16}=\frac{14}{16}=\frac{7}{8}$$

3) Remembering the results obtained at 1):

$$P(X \geq 1)=\frac{15}{16} \qquad P(X=2)=\frac{3}{8}$$

The average number of families with at least one boy is:

$$N_1 = 2000 \times \frac{15}{16} = 1875$$

The average number of families with two boys is:

$$N_2 = 2000 \times \frac{3}{8} = 750$$

Exercise 7

A completely unprepared student must answer 13 quizzes for each of which 3 answers are suggested, only one of which is correct. The student is guessing.

1) With what probability he will answer exactly 12 times?

2) And with what probability he will answer wrong to all the answers?

Answers:

1) P(x=12) = $$\binom{13}{12}\left(\frac{1}{3}\right)^{12}\left(\frac{2}{3}\right)^{1} = \frac{26}{3^{13}}$$

2) P(x=0) = $$\binom{13}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{13} = \left(\frac{2}{3}\right)^{13}$$