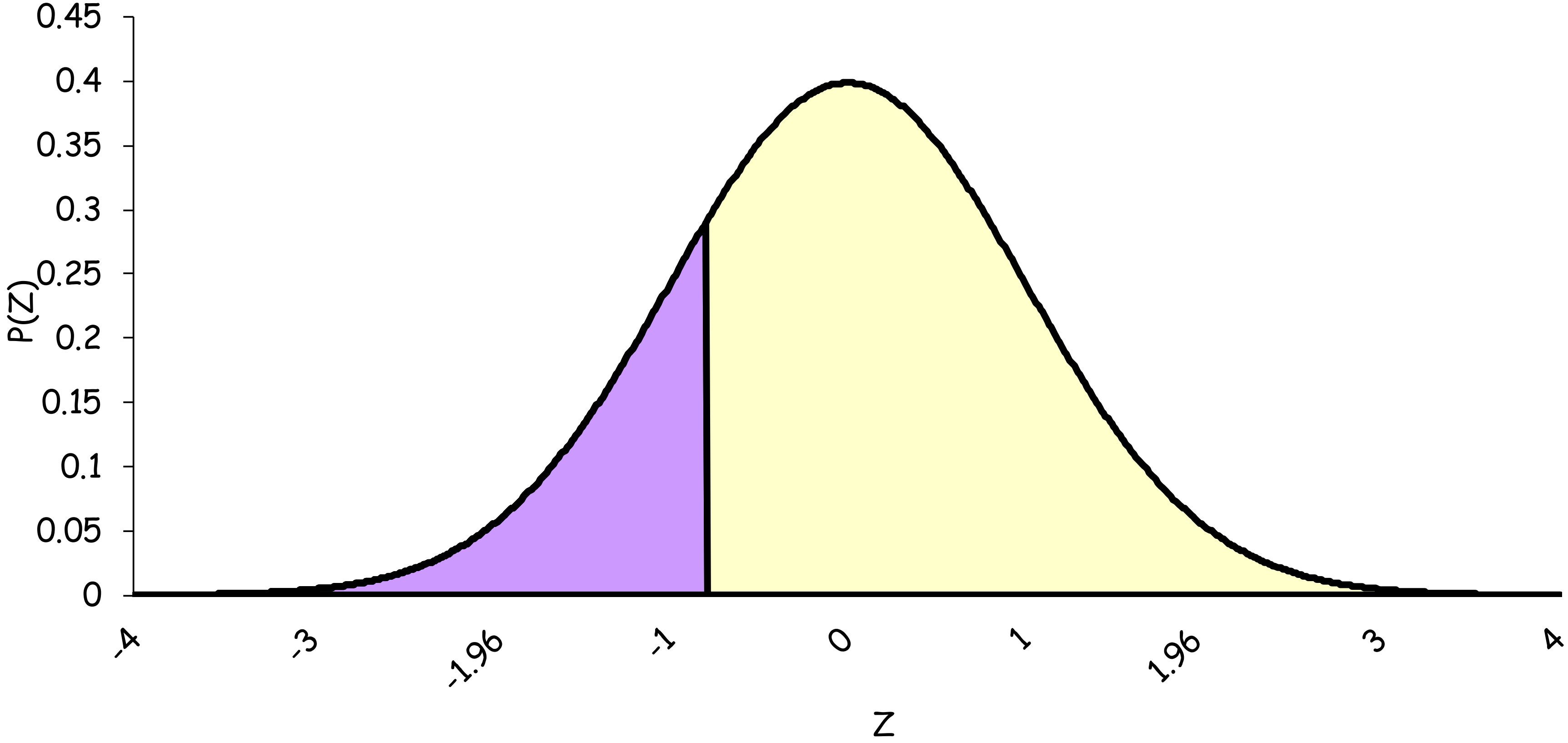
THE NORMAL DISTRIBUTION

EXERCISE

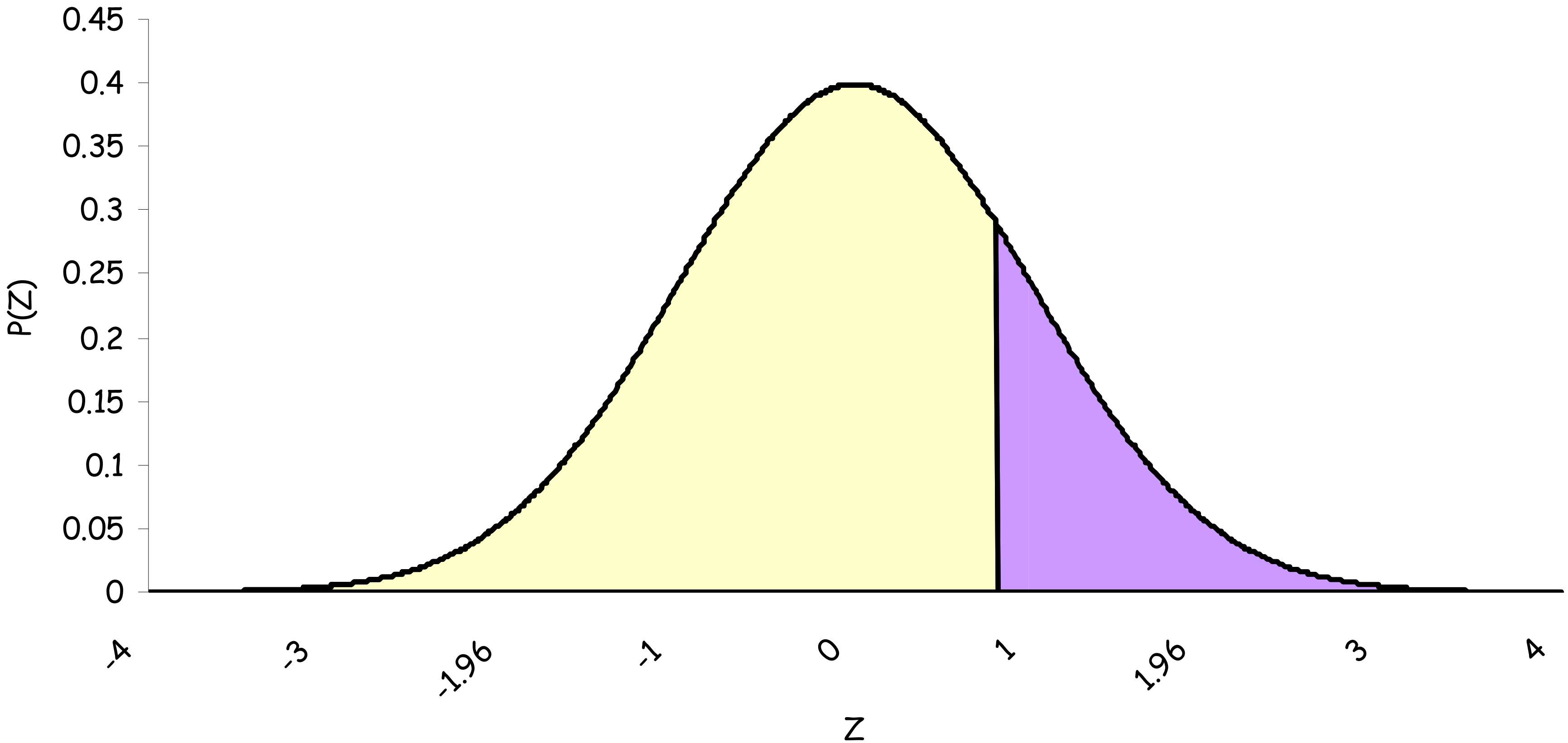
Exercise 1

In a study involving children aged 8 to 15, Eldridge et al. evaluated 529 normally developed children to assess the time spent standing upright. The researchers found that the total time a child spends standing upright follows a normal distribution with a mean of 5.4 hours and a standard deviation of 1.3 hours.

1. Assuming the study applies to all children aged 8-15, find the probability that a randomly chosen child will spend less than 3 hours standing upright in one day (24h).
2. In a population of 10,000 children, how many children do you expect to find that spend more than 8.5h in a standing upright position?



$1. P[x<3] = P[\ (x-\mu)/\sigma\ <\ (3-\mu)/\sigma\ ] = P[z<(3-5.4)/1.3] = P[z<1.85] = 0.0322$

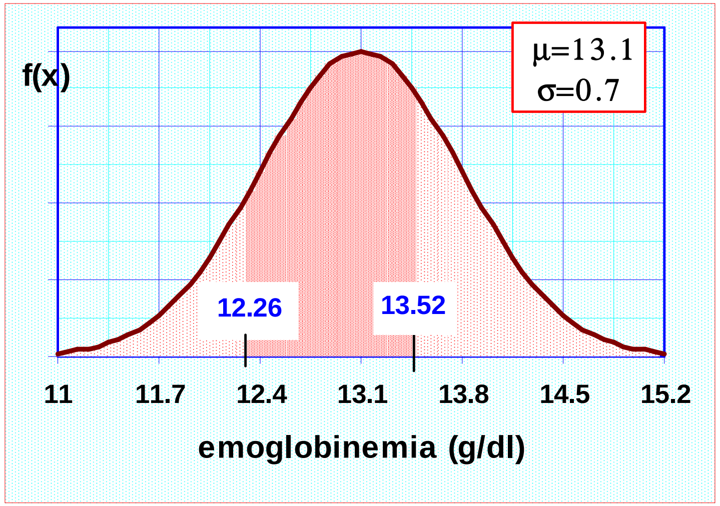


2.P[x>8.5]=P[(x-μ)/σ > (8.5-μ)/σ] =
P[z>(8.5-5.4)/1.3] = P[z>2.38] =
=0.0087
N=10000*0.0087= 87

Exercise 2

In a population of girls aged 18-25 years, the hemoglobin concentration in the blood (x) approximates a Gaussian distribution with mean = 13.1 g/dl and standard deviation = 0.7 g/dl. Based on this information, we can calculate, for example, how many girls have hemoglobin between 12.26 and 13.52 g/dl.

Distribution of hemoglobin in a population of girls aged 18-25.



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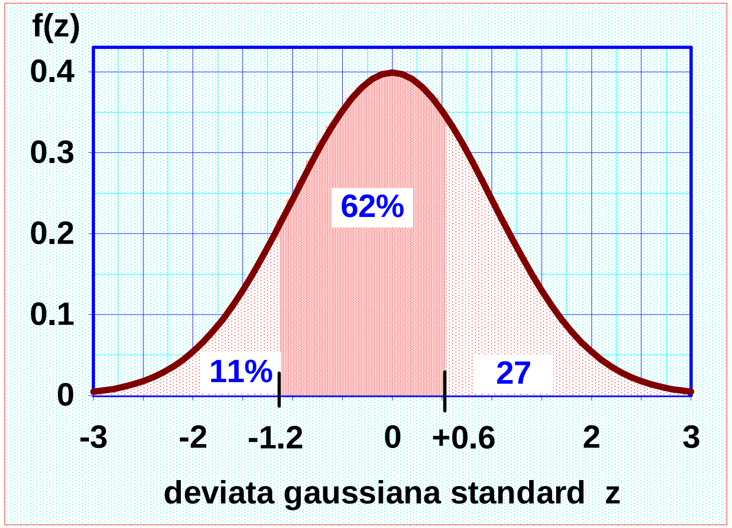
In fact: $z_1 = (12.26-13.10) / 0.7 = -1.2$

$z_2 = (13.52-13.10) / 0.7 = +0.6$



In 11% of girls the Hb values are less than 12.26 g/dl, and in 27% they are greater than 13.52 g/dl.
So 62% of girls have Hb values between 12.26 and 13.52 g/dl.

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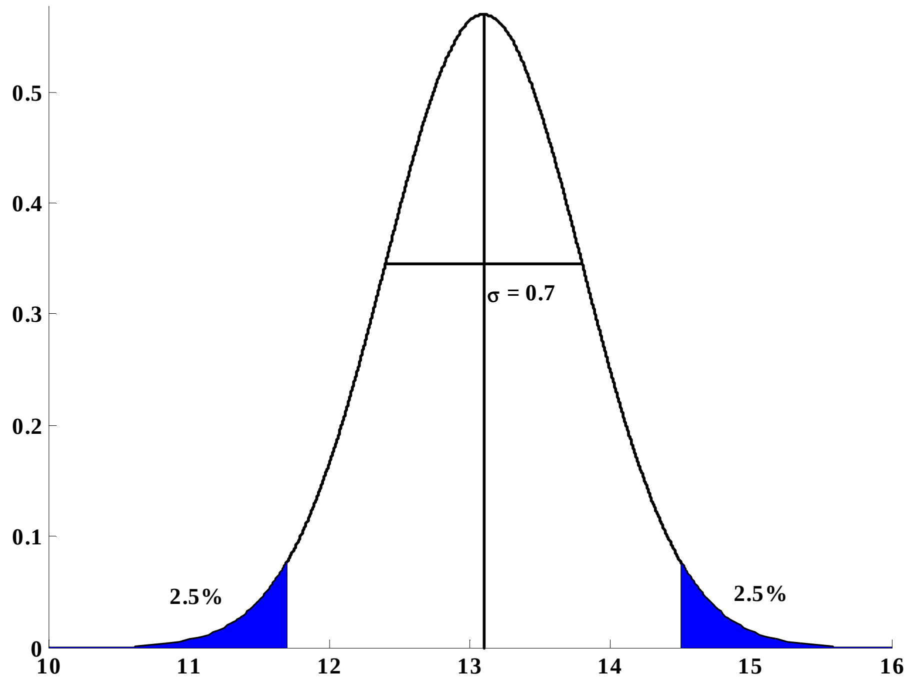
What are the values that enclose 95% of the observations, which I consider as the values within which the normality range is included?

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = z \cdot \sigma + \mu \quad con\ z_{0.025} = 1.96$$

$$x_{1,2} = \mu \pm z_{0.025} \cdot \sigma$$

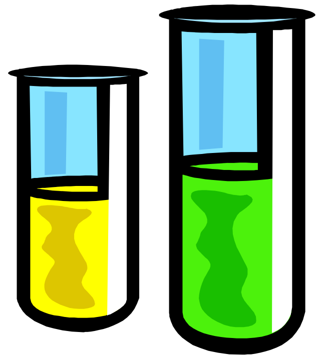
$$x_1 = 13.1 - 1.96 \cdot 0.7 = 11.728$$

$$x_2 = 13.1 + 1.96 \cdot 0.7 = 14.472$$



Exercise 3

If we assume that, in the adult population, the level of uric acid (mg/100 ml) follows a Gaussian distribution with mean and s.d. respectively equal to 5.7 and 1 (mg/100ml), find the probability that a subject chosen at random from this population has a level of uric acid:

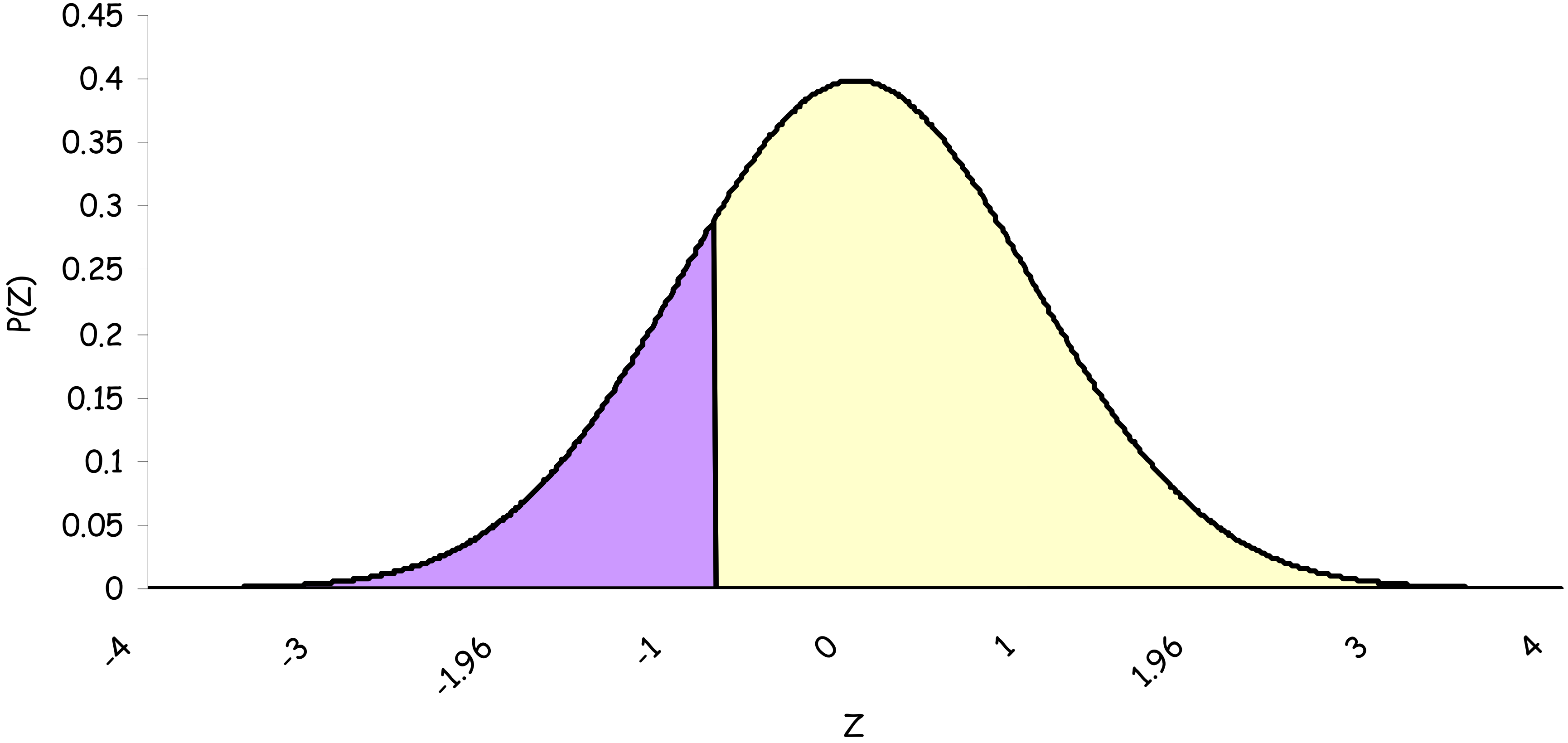


1. Less than 4.9 mg/100ml
2. Between 4.9 and 6.2 mg/100ml
3. Find also the value of uric acid x such that $P(X \geq x)=0.40$

Answers

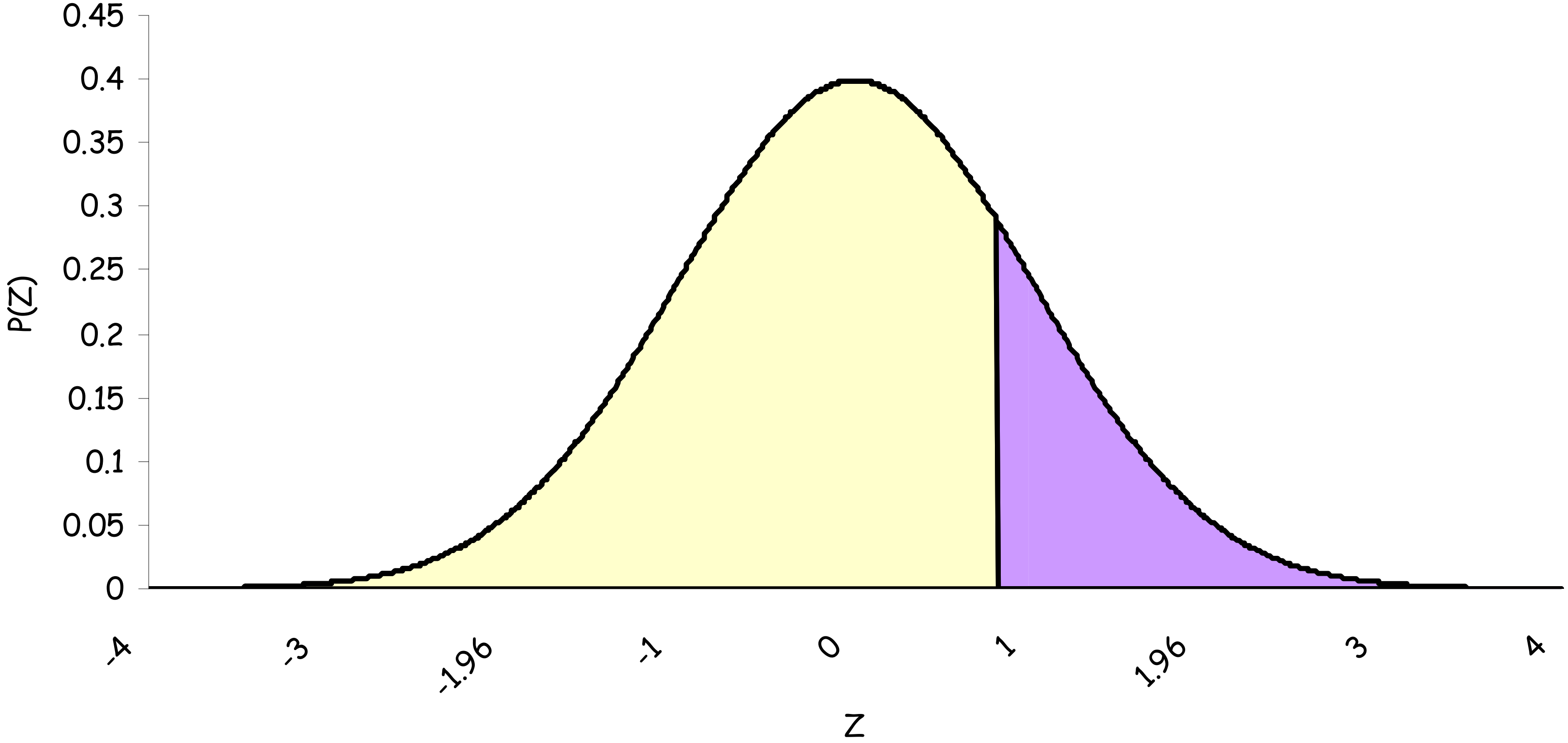
1. $P(X<4.9) = P[(X-5.7)/1 < (4.9-5.7)/1] =$
   $P(Z<-0.8) = \ldots$

   Z



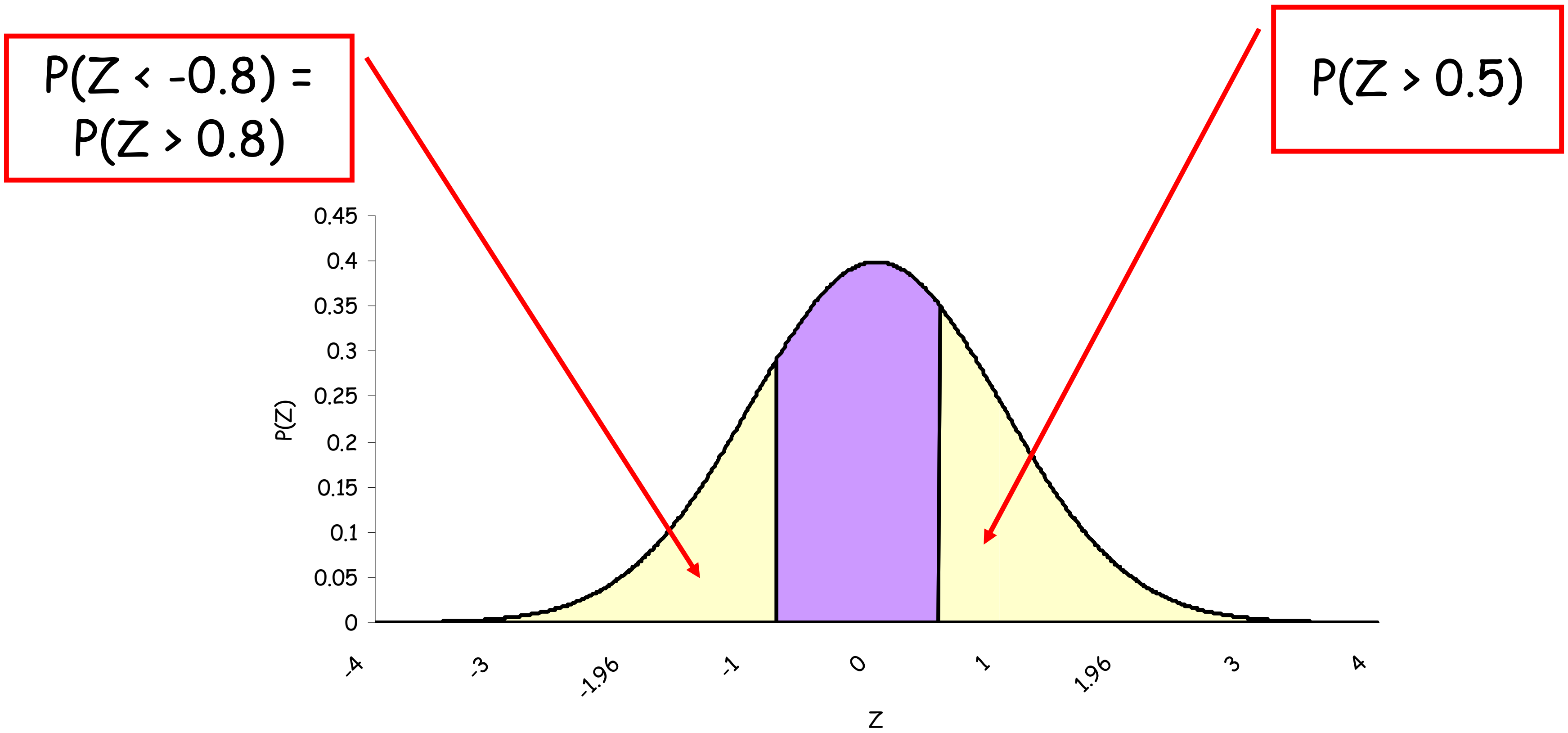
Answers

1. … = P(Z>0.8) = 0.212



Answers

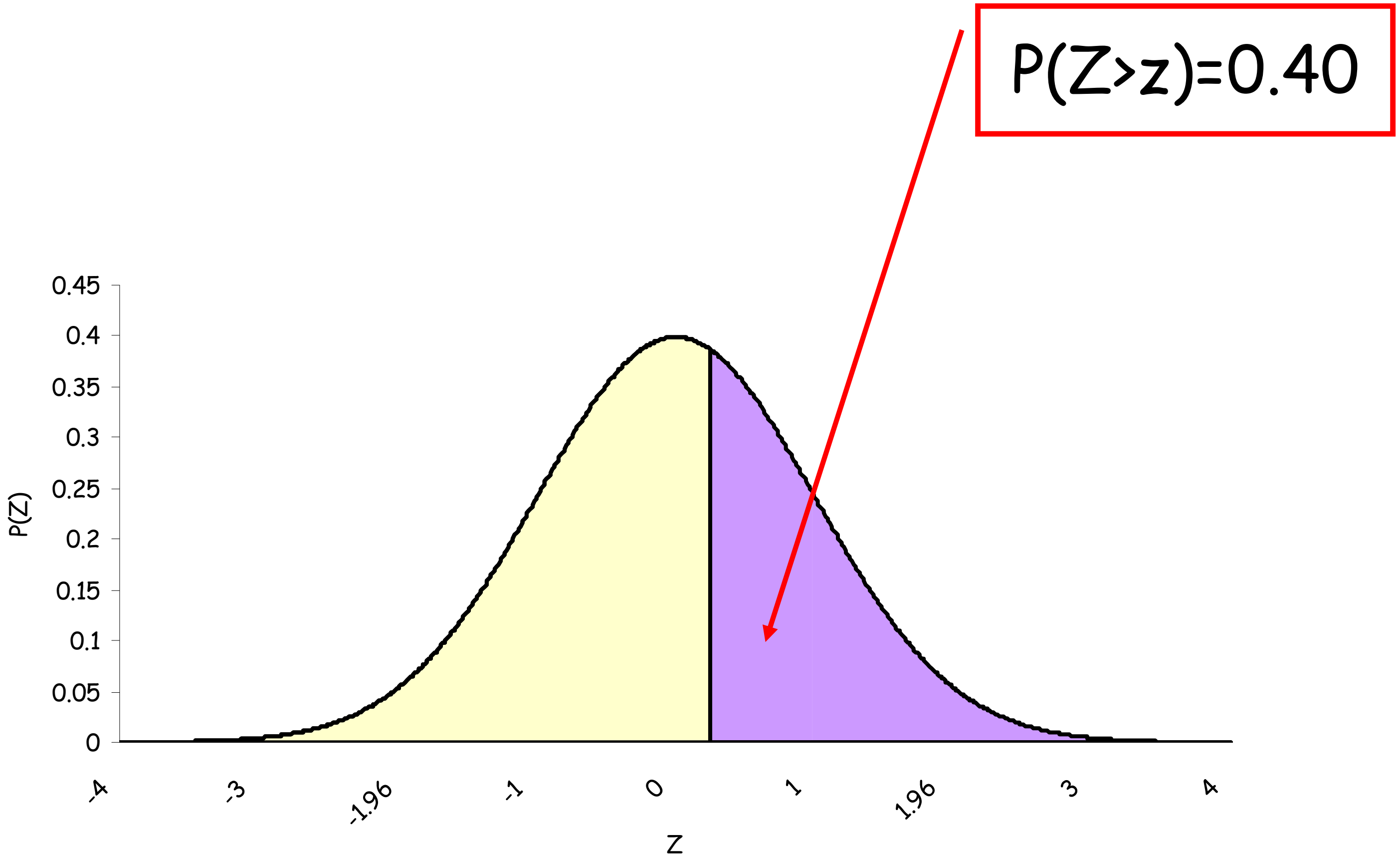
2. $P(4.9<X<6.2) = P(-0.8<Z<0.5) = 1-P(Z>0.8)-P(Z>0.5) =$
$1-0.212-0.308 = 0.479$

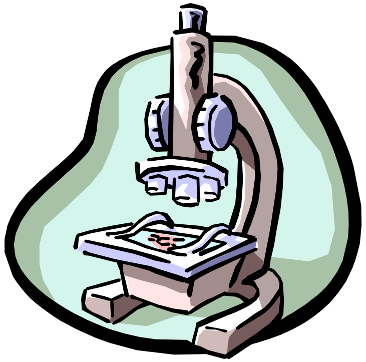


Answers

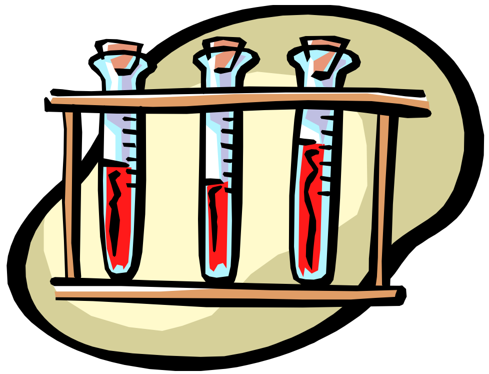
3. $P(Z > z_{0.40}) = 0.40 \Rightarrow z_{0.40} = 0.25$

$$\frac{x - 5.7}{1} = 0.25 \Rightarrow x = 5.95$$





Exercise 4



From the microscopic examination of the red blood cells of a patient with Plasmodium vivax malaria, it was found that the mean and the variance of the measurements of the maximum diameter of an uninfected red blood cell are respectively 7.6 and 0.9 microns, while for an infected red blood cell, the mean and standard deviation of the maximum diameter measurements are 9.6 and 1.0 microns, respectively. Assume that the reported values are equal to the population parameters and that the maximum diameter of red blood cells, infected and not, is Gaussian distributed and calculate:

Questions

a) What proportion of uninfected red blood cells do you expect to find with a measuring diameter between 9.4 and 9.6 microns?

b) What proportion of uninfected red blood cells do you expect to find with a measuring diameter between 7.6 and 9.4 microns?

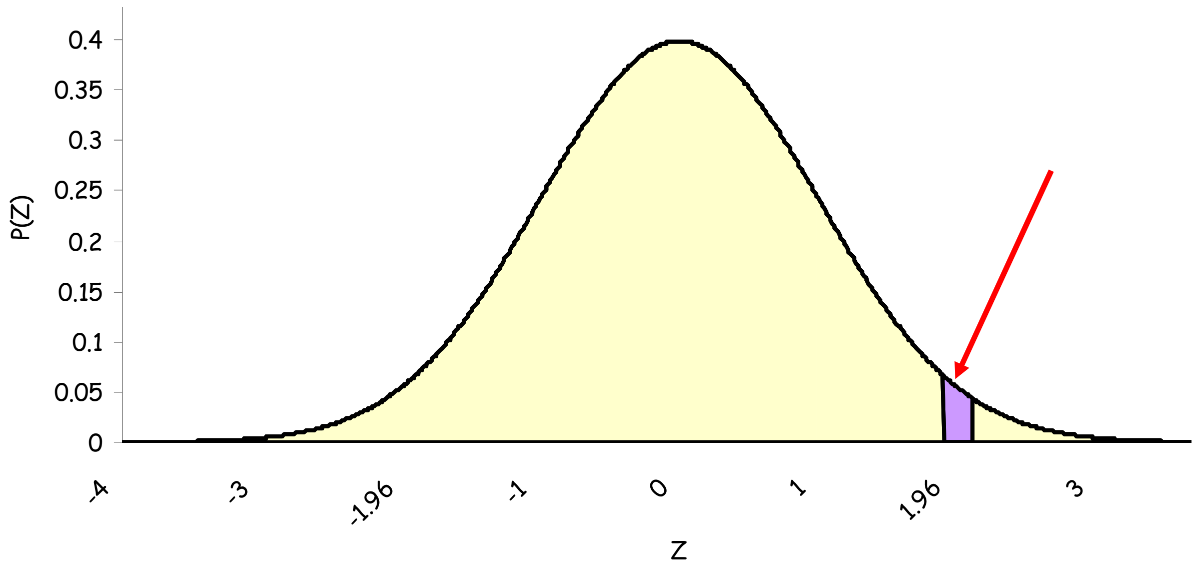
c) Suppose 20% of red blood cells are infected. What percentage of all red blood cells will be greater than 9.0 microns in diameter?

Answers

a) The proportion of uninfected red blood cells with a diameter between 9.4 and 9.6 microns is:

$$P(9.4 < X < 9.6) = P\left[\frac{(9.4-7.6)}{\sqrt{0.9}} < z < \frac{(9.6-7.6)}{\sqrt{0.9}}\right] =$$

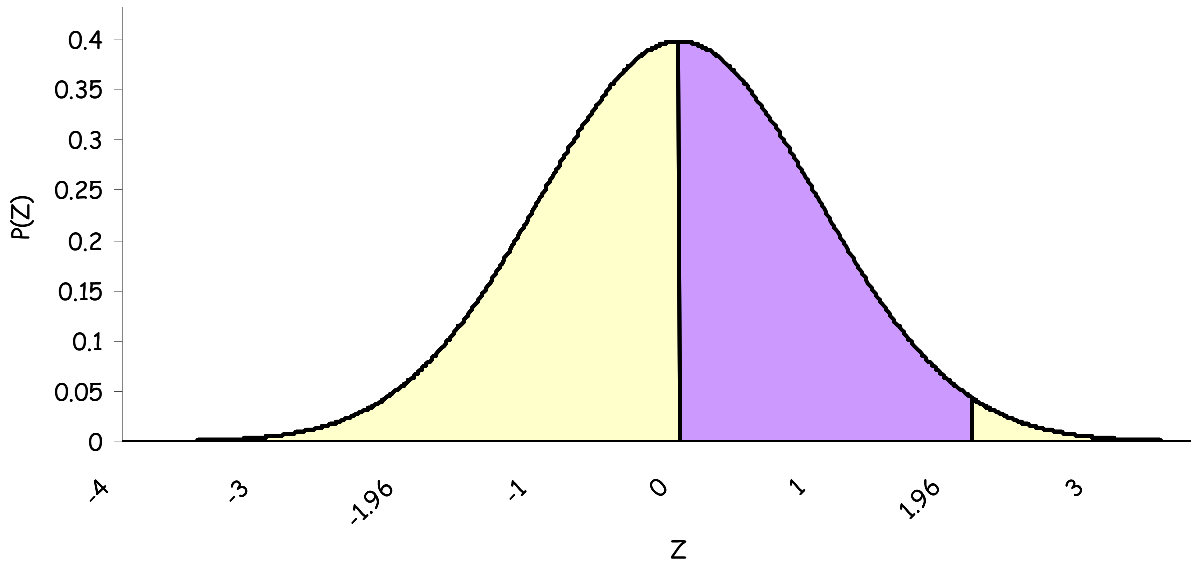
= P(1.897 < Z < 2.108) = P(Z > 1.897) – P(Z > 2.109) =
= 0.028717 - 0.01786 = 0.01086



b) The proportion of uninfected red blood cells with a diameter between 7.6 and 9.4 microns is:

$$P(7.6 < X < 9.4) = P\left[\frac{(7.6-7.6)}{\sqrt{0.9}} < z < \frac{(9.4-7.6)}{\sqrt{0.9}}\right] =$$

$= P(0 < Z < 1.897) = P(Z > 0) - P(Z > 1.897) =$
$= 0.5 - 0.028717 = 0.471$



c) The proportion of uninfected red blood cells (S) with diameter greater than 9 microns is:

$$P(X_S > 9) = P\left[Z > \frac{(9-7.6)}{\sqrt{0.9}}\right] = P(Z > 1.4757) = 0.07078$$

The proportion of infected red blood cells (M) with a diameter greater than 9 microns is:

$$P(X_M > 9) = P\left[Z > \frac{(9-9.6)}{1}\right] = P(Z > -0.6) =$$

$$= 1 - P(Z < -0.6) = 1 - P(Z > 0.6) = 0.72575$$

If 20% infected red blood cells, the proportion of all red blood cells with a diameter greater than 9 microns is:

$$P(X > 9) = 0.8 \times 0.07078 + 0.2 \times \mathbf{0.72575} = 0.201774$$

Exercise 5

Graduates of a certain faculty have an mean grade of 100 with a sd of 4. Suppose the distribution of grades is normal:

a) Calculate the percentage of graduates who obtained a grade between 96 and 104

b) Calculate the percentage of graduates who obtained a grade higher than 108

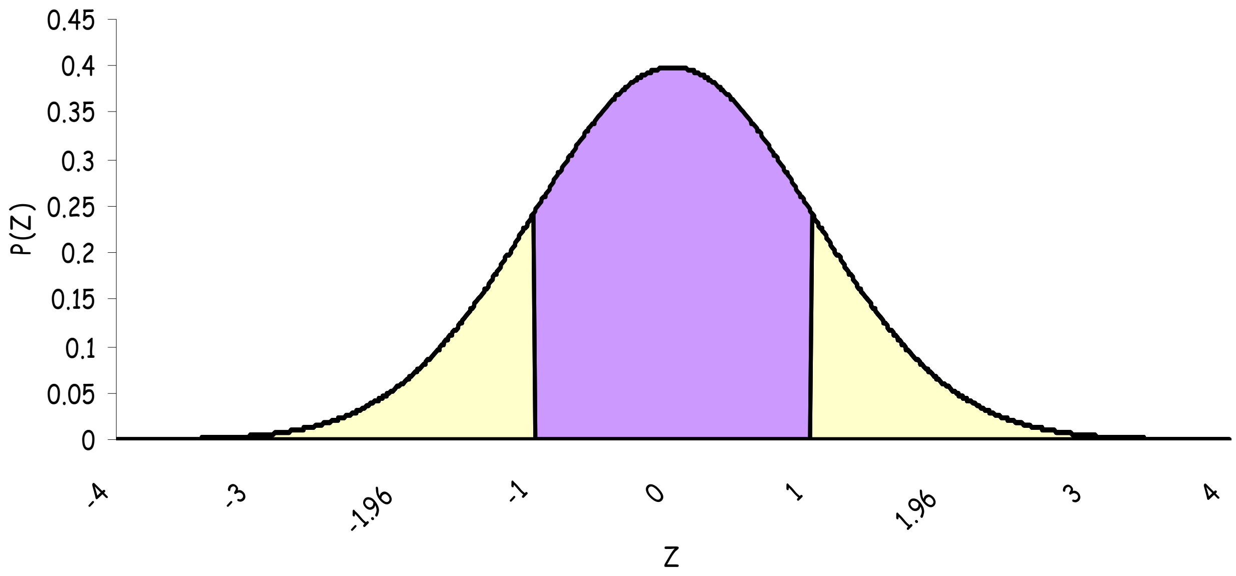
c) Calculate the interquartile range

Answers

a) P (96 < L < 104) =

= P [(96-100)/4) < Z < (104-100) /4] = P (-1 < Z < 1) =

= 1-2*0.15866 = 0.6827 = 68.27% of graduates obtained a grade between 96 and 104



Answers

b) $P (L > 108) =$
$= P (Z > (108-100) /4] = P (Z > 2) = 0.02275 =$
2.275%

c) $P (L < Q_3) = 0.75 =$
$= P [(L - \mu) / \sigma < (Q_3 - \mu) / \sigma] = P (z < z_0) =0.75$
$z_0 = 0.67 \Rightarrow Q_3 = 0.67 * 4 + 100 = 102.68$

$P (L < Q_1) = 0.25 =$
$= P [(L - \mu) / \sigma < (Q_1 - \mu) / \sigma] = P (z < z_0) =0.25$
$z_0 = - 0.67 \Rightarrow Q_1 = - 0.67 * 4 + 100 = 97.32$
$Q_1-Q_3 = 102.68 – 97.32 = 5.36$

Exercise 6

A company packages boxes of coffee with an average content of 1 kg, with a sd of 6 g. If the law prevents packages containing less than 985 g from being put on the market with the declared weight of 1 kg, how many packages on average, every 1000, cannot be marketed?

Answer

$P(X < 985) = P[Z < (985 - 1000) / 6] =$

$= P(Z < -2.5) = 0.00621$

packages = 0.00621*1000 = 6.21

Exercise 7

A machine produces bars whose length is a normal random variable with mean μ = 25 cm and sd σ = 0.3 cm

1) Calculate the probability that the length of a bar differs from its average value by at least 0.5 cm.

Answers

1) $P\ (X \leq \mu - 0.5;\ X \geq \ \mu + 0.5) =$

$= 1 - P\ (\mu - 0.5 \leq X \leq \mu + 0.5) =$

$= 1 - P(\ -0.5\ /\ 0.3 \leq Z \leq 0.5\ /\ 0.3) = 1 - P(-1.67 \leq Z \leq 1.67) =$

$= 1-(1-(2*0.04746)) = 0.09492$