

Exercise

In one of Mendel's famous hybridization experiments, he expected that among 580 offspring peas, 145 of them (or 25%) would be yellow, but he actually got 152 yellow peas. Assuming that Mendel's rate of 25% is correct, find the probability of getting 152 or more yellow peas by random chance. That is, given $n = 580$ and $p = 0.25$, find $P(\text{at least } 152 \text{ yellow peas})$. Is 152 yellow peas significantly high ?

Let's find the probability of getting 152 or more yellow peas by random chance:

$$n=580 \quad p=0.25 \quad P(X \geq 152)=?$$

$$P(X = 152) = \frac{580!}{152! (580 - 152)!} 0.25^{152} 0.75^{580-152} = \dots$$

$$P(X = 153) = \frac{580!}{153! (580 - 153)!} 0.25^{153} 0.75^{580-153} = \dots$$

...

Exact value from binomial distribution 0.2348
(manually very long procedure)

Normal as approximation of a Binomial?

$X \sim \text{Bi}(\pi=0.25, n=580)$

Expected value $E(X) = n * \pi = 580 * 0.25 = 145$

Variance: $\text{VAR}(X) = n * \pi * (1 - \pi) = 580 * 0.25 * 0.75 = 108.75$

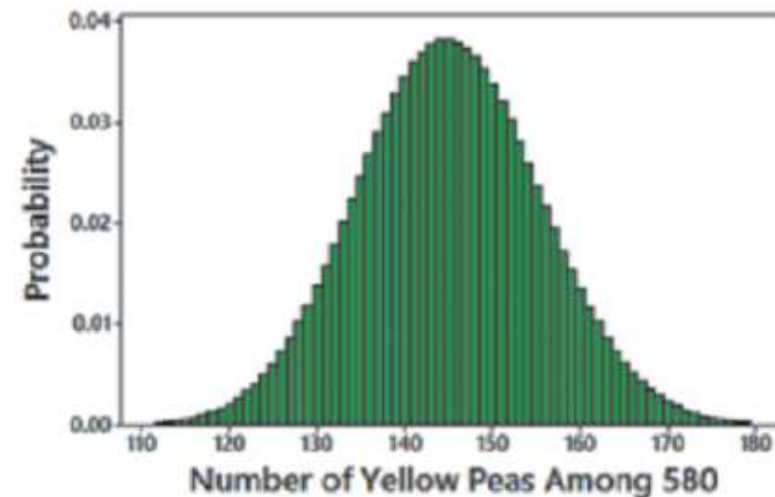
If the conditions **$n\pi \geq 5$** and **$n(1 - \pi) \geq 5$** are both satisfied, then probabilities from a binomial probability distribution can be approximated reasonably well by using a normal distribution having these parameters:

$$\mu = n * \pi$$
$$\sigma = \sqrt{n\pi(1 - \pi)}$$

Exercise

In one of Mendel's famous hybridization experiments, he expected that among 580 offspring peas, 145 of them (or 25%) would be yellow, but he actually got 152 yellow peas. Assuming that Mendel's rate of 25% is correct, find the probability of getting 152 or more yellow peas by random chance. That is, given $n = 580$ and $p = 0.25$, find $P(\text{at least } 152 \text{ yellow peas})$. Is 152 yellow peas significantly high?

The number of yellow peas can be approximated by a Gaussian distribution with mean $580 * 0.25 = 145$ and standard deviation $\sqrt{580 * 0.25 * 0.75} = 10.43$



Exercise

The number of yellow peas can be approximated by a Gaussian distribution with mean

$$580 * 0.25 = 145$$

and standard deviation

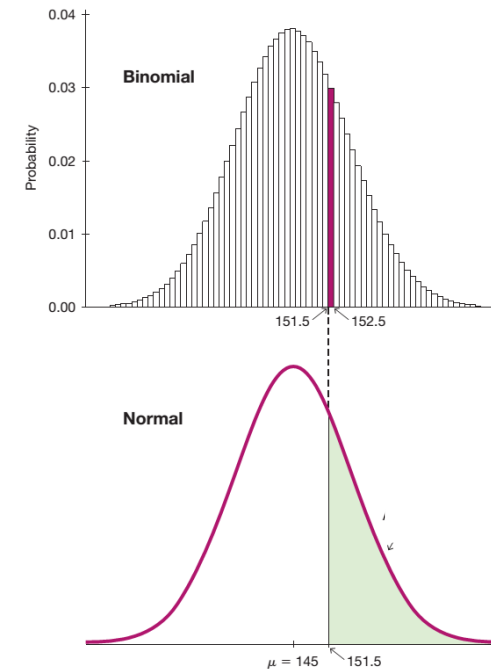
$$\sqrt{580 * 0.25 * 0.75} = 10.43$$

Find the probability of getting 152 or more yellow peas by random chance:

from Gaussian approximation:

$$z = (152 - 145) / 10.43 = 0.67 \quad P(z > 0.67) = \mathbf{0.2514}$$

Exact value from binomial distribution 0.2348 (manually very long procedure)



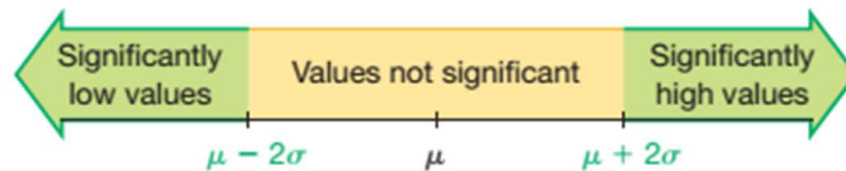
Exercise

Mendel's result of 152 yellow peas is greater than the 145 yellow peas he expected with his theory of hybrids, but with $P(152 \text{ or more yellow peas}) = 0.2514$.

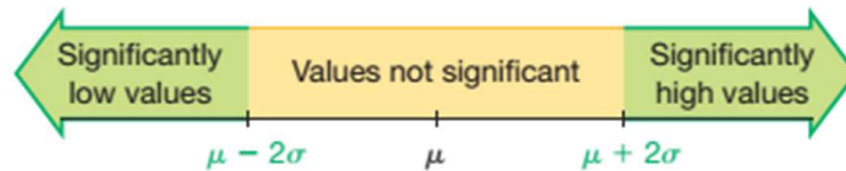
Is 152 yellow peas significantly high?

Range Rule of Thumb

It is based on the principle that for many data sets, the vast majority (such as 95%) of sample values lie within 2 standard deviations of the mean.



Exercise



Let's compute the "threshold" values that separate significant and not significant values according with the Range Rule of Thumb:

$$145 \pm 2 * 10.43$$

$$[124.14; 165.86]$$

We see that 152 yellow peas is not significantly high. That is a result that could easily occur with a true rate of 25% for yellow peas. This experiment does not contradict Mendel's theory.