Un plasma è un gas ionizzato che contiene un certo numero di particelle cariche che dominano la dinamica del sistema esibendo un comportamento collettivo

<u>Equazione di Saha-Boltzmann</u>

$$N \propto \exp\left(-\frac{E}{k_B T}\right) \cdot \overbrace{\left(\begin{array}{l}\text{peso statistico} \\ \text{dello stato all'energia } E\end{array}\right)}^{g \;\text{(degenerazione)}}$$

part. all'energia E

<u>es</u> atomo idrogeno $E_n = -R_y/n^2$ $n = \#$ livello $R_y = 13.6$ eV

$l = 0 \dots n-1$ $s = +\frac{1}{2}, -\frac{1}{2}$

$m = -l \dots l$

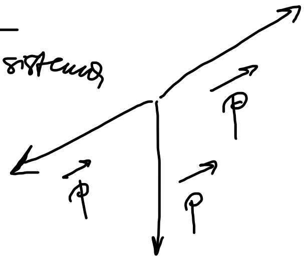
$E = \chi + \frac{p^2}{2m}$

$\vec{p} = m\vec{v}$

$N_{\text{non ioniz}} \rightarrow E = \chi$

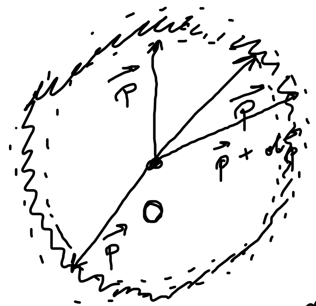
$n = \frac{N}{\text{Vol sistema}}$

$dV = \frac{\text{Vol sistema}}{N} = \frac{1}{n}$



$N_{\text{ioniz}} \rightarrow E = \chi + \frac{p^2}{2m}$

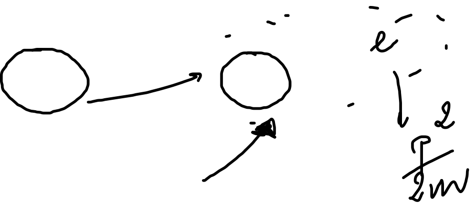
$\Delta x \, \Delta p_x \sim h$



$(4\pi p^2 dp) \rightarrow$ in questo volume $\frac{p^2}{2m}$

$dV_{TOT} = (4\pi p^2 dp)\, dV$

$\text{degen} = \frac{dV_{TOT}}{h^3} = \frac{4\pi p^2 dp \, dV_{\text{spazio}}}{h^3} = \frac{4\pi p^2 dp}{h^3 n}$



$$\frac{N_A}{N_B} = \frac{g_A}{g_B} \exp\left(-\frac{(E_A - E_B)}{k_B T}\right)$$

A = stato ionizzato

B = stato fondamentale dell'atomo $\Big\}\ E_B$

$k = O(1)$ $\qquad E_A = E_B + \chi + \frac{p^2}{2m}$

$$\frac{N_A}{N_B} = K \cdot \int_0^{+\infty} \frac{4\pi p^2\, dp}{n h^3} \exp\left(-\frac{\chi}{k_B T}\right) \exp\left(-\frac{p^2}{2 m k_B T}\right) = K \exp\left(-\frac{\chi}{k_B T}\right) \frac{4\pi}{n h^3} \cdot \int_0^{+\infty} dp\, p^2 \exp\left(-\frac{p^2}{2 m k_B T}\right)$$

$$\int_0^{+\infty} dp\, p^2 \exp\left(-\frac{p^2}{2mTk_B}\right) = (2mk_BT)^{3/2}\int_0^{+\infty} dy\, y^2 \exp(-y^2) =$$

$$y = \frac{p}{\sqrt{2mk_BT}}$$

$$-\frac{y}{2}\frac{d}{dy}\exp(-y^2) = -\frac{y}{2}(-2y)\exp(-y^2)$$

$$= y^2\exp(-y^2)$$

$$= -\frac{(2mk_BT)^{3/2}}{2}\int_0^{+\infty} dy\, y\frac{d}{dy}\exp(-y^2) = -\frac{(2mk_BT)^{3/2}}{2}\left[\cancel{y\exp(-y^2)\Big|_0^{+\infty}} - \overbrace{\int_0^{+\infty}\exp(-y^2)}^{\sqrt{\pi}/2}\right]$$

$$= \frac{(2mk_BT)^{3/2}\sqrt{\pi}}{4}$$

$$\frac{n_i}{n} = \frac{N_{ion}}{N_{OS}} = K \frac{(2\pi m_e T k_B)^{3/2}}{n_e h^3} \exp\left(-\frac{J}{k_B T}\right)$$

$n_i \rightarrow$ densità $= \frac{part}{vol}$

$$pV = n_{mol} R T$$

$$pV = N k_B T \Rightarrow \frac{N}{V} = \frac{p}{k_B T} = n$$

part.

$n_i \sim n_e$

$$\frac{n_i^2}{n^2} = \frac{K}{n} \cdot \left[\qquad \right] \Rightarrow \frac{n_i}{n} = \left[\frac{K}{(p/k_B T)} \cdot \frac{(2\pi m_e T k_B)^{3/2}}{h^3} \cdot \exp\left(-\frac{J}{k_B T}\right)\right]^{\frac{1}{2}}$$

N_2 $\quad J = 14.5\ eV \quad T \sim 300\ K \quad p \sim 1\ atm \quad \frac{n_i}{n} \sim 10^{-12}$

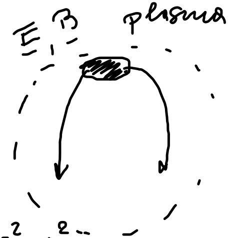
$k_B T \sim meV \quad k_B = 1.38 \cdot 10^{-23}\ J/K \quad T \approx 300\ K \quad k_B T = 1\ eV \rightarrow T = 12000\ K$

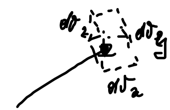
$$\exp\left(-\frac{14.5}{10^{-3}}\right) \sim \exp(-10^4)$$

$T \sim 10^4\ K \uparrow$

$T \sim 10^4$ K $\quad \frac{n_i}{n} \sim 6\%$

$T \sim 10^6$ K $\quad \frac{n_i}{n} \sim 100\%$





Eq. termico

$v^2 = v_x^2 + v_y^2 + v_z^2$

$$f(\underline{v}) = N \exp\left(-\frac{mv^2}{2k_B T}\right) = N \exp\left(-\frac{m}{2k_B T}(v_x^2+v_y^2+v_z^2)\right)$$

$f(\underline{v})\, dv_x\, dv_y\, dv_z$

Imponiamo: $\int_{-\infty}^{+\infty} f(\underline{v})\, d^3\underline{v} = n$ $\quad \frac{\#\text{particelle}}{\text{volume}}$

$$\int d^3\underline{v}\, f(\underline{v}) = n = N \int_{-\infty}^{+\infty} dv_x \exp\left(-\frac{mv_x^2}{2k_B T}\right) \int_{-\infty}^{+\infty} dv_y \exp\left(-\frac{mv_y^2}{2k_B T}\right) \cdot \int_{-\infty}^{+\infty} dv_z \exp\left(-\frac{mv_z^2}{2k_B T}\right)$$

$$\int_{-\infty}^{+\infty} dv_x \exp\left(-\frac{m v_x^2}{2k_BT}\right) = \left(\frac{2k_BT}{m}\right)^{\frac{1}{2}} \int_{-\infty}^{+\infty} dy \exp(-y^2) = \left(\frac{2\pi k_BT}{m}\right)^{\frac{1}{2}}$$

$$y = \sqrt{\frac{m}{2k_BT}}\, v_x$$

$$n = N \cdot \left(\frac{2\pi k_BT}{m}\right)^{3/2} \Rightarrow N = \left(\frac{m}{2\pi k_BT}\right)^{3/2} \cdot n$$

$$f(\underline{v}) = n \left(\frac{m}{2\pi k_BT}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_BT}\right)$$

$$\langle E \rangle = \frac{\int_v \frac{1}{2} m v^2 f(\underline{v})\, d^3\underline{v}}{\int f(\underline{v})\, d\underline{v}} = \frac{1}{2} m \underbrace{\overbrace{\int d\varphi \sin\theta\, d\theta}^{4\pi} \overbrace{\int dv\, v^2 \cdot v^2 \exp\left(-\frac{mv^2}{2T}\right)}^{\int dv\, v^4 \exp\left(\frac{-mv^2}{2T}\right)}}_{n} \cdot \left(\frac{m}{2\pi k_BT}\right)^{3/2}$$

$T_e \neq T_i$

$T_e = T_i \qquad T_\parallel \neq T_\perp$

$$= \ldots = \frac{3}{2} k_B T$$