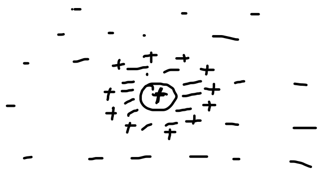
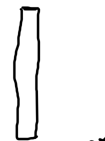
Quasi neutralità

$$\Sigma \text{ cariche positive} = \Sigma \text{ cariche negative}$$

Schermo di Debye



$\sigma \rightarrow C/m^2$



Potenziale prodotto da una carica in $x=0$ in 1D

C/m^3

$$\underline{\nabla} \cdot \underline{E} = \rho/\varepsilon_0 \qquad \frac{dE}{dx} = \frac{\sigma\,\delta(x)}{\varepsilon_0} \qquad \underline{E} = -\underline{\nabla}\phi$$

pot. e.s.

$$\frac{d^2\phi}{dx^2} = -\frac{\sigma\,\delta(x)}{\varepsilon_0}$$

In $x>0$ e $x<0$

$$\frac{d^2\phi}{dx^2} = 0$$

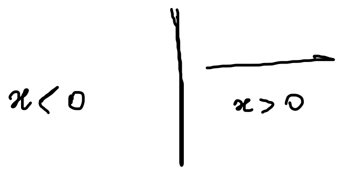
In $x=0$ $\quad \phi(x) = \phi_0$

$x>0 \qquad \phi(x) = A_+ x + B_+$

$x<0 \qquad \phi(x) = A_- x + B_-$

$$\lim_{x\to 0^+} \phi(x) = \lim_{x\to 0^-} \phi(x) = \phi_0$$

$$B_+ = B_- = \phi_0$$



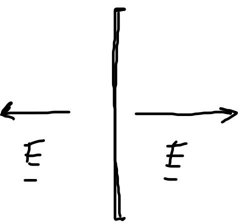
Simmetria: $\phi(x) = \phi(-x)$

$$A_+ x + \cancel{\phi_0} = -A_- x + \cancel{\phi_0}$$

$$A_+ = -A_-$$

$$\lim_{\varepsilon \to 0} \int_{-\varepsilon}^{\varepsilon} \frac{d^2\phi}{dx^2}\,dx = \lim_{\varepsilon \to 0} -\int_{-\varepsilon}^{\varepsilon} \frac{\sigma\,\delta(x)\,dx}{\varepsilon_0} \qquad \varepsilon > 0 \quad \text{"piccola"}$$

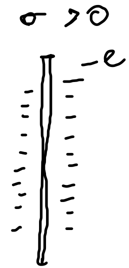
$$\left[\underbrace{\left.\frac{d\phi}{dx}\right|_{0^+}} - \underbrace{\left.\frac{d\phi}{dx}\right|_{0^-}} = -\frac{\sigma}{\varepsilon_0} \cdot 1 \right]$$



$$A_+ - A_- = -\frac{\sigma}{\varepsilon_0} \;;\quad -2A_- = -\frac{\sigma}{\varepsilon_0} \;;\quad A_- = \frac{\sigma}{2\varepsilon_0}$$

$$\phi(x) = \begin{cases} -\dfrac{\sigma}{2\varepsilon_0}x + \phi_0 & x > 0 \\[2ex] \dfrac{\sigma}{2\varepsilon_0}x + \phi_0 & x < 0 \end{cases} \qquad \underline{E}(x) = -\frac{d\phi}{dx}\hat{\underline{i}} = \begin{cases} \dfrac{\sigma}{2\varepsilon_0}\hat{\underline{i}} \\[2ex] -\dfrac{\sigma}{2\varepsilon_0}\hat{\underline{i}} \end{cases}$$

E in un plasma?



$\underline{E}$

$\underline{F} = q\underline{E} \qquad \underline{a} = \frac{q\underline{E}}{m}$

$m_{ioni} \sim 1.67 \cdot 10^{-27}$ kg

$m_e \sim 9.1 \cdot 10^{-31}$ kg

$$\rho = \underbrace{e n_\infty}_{\text{densità ioni}} - \underbrace{e n_e(x)}_{\text{densità elettroni}}$$

$\phi(x)$

$$f_e(\underline{v}) = N \exp\left(-\frac{\frac{1}{2}mv^2 - e\phi(x)}{k_B T_e}\right) = f_e(\underline{v}, x)$$

$$n_e = \int f_e(\underline{v})\, d^3\underline{v} = \overbrace{\int N \exp\left(\frac{-mv^2}{2k_B T_e}\right) d^3\underline{v}}^{n_{e\infty}} \cdot \exp\left(\frac{e\phi(x)}{k_B T_e}\right)$$

$\to dv_x\, dv_y\, dv_z$

$$n_e(x) = n_{e\infty} \exp\left(\frac{e\phi(x)}{k_B T_e}\right)$$

Plasma idrogeno

$$n_{e\infty} = n_{i\infty} = n_0$$

$$\rho(x) = e n_0 - e n_0 \exp\left(\frac{e\phi(x)}{k_B T_e}\right) = e n_0 \left(1 - \exp\left(\frac{e\phi(x)}{k_B T_e}\right)\right)$$

$$\frac{d^2\phi}{dx^2} = \underbrace{-\frac{\sigma\delta(x)}{\varepsilon_0}}_{\text{piano}} \underbrace{-\frac{e n_0}{\varepsilon_0}\left(1 - \exp\left(\frac{e\phi(x)}{k_B T_e}\right)\right)}_{\text{risposta del plasma}}$$

$$\frac{e\phi}{k_B T_e} \ll 1$$

$$1 - \exp\left(\frac{e\phi(x)}{k_B T_e}\right) \approx -\frac{e\phi(x)}{k_B T_e}$$

$$\frac{d^2\phi}{dx^2} = -\frac{\sigma\delta(x)}{\varepsilon_0} + \frac{e^2 n_0}{\varepsilon_0 k_B T_e}\phi(x)$$

lungh. di Debye

$$\frac{1}{\lambda_D^2} = \frac{e^2 n_0}{\varepsilon_0 k_B T_e}$$

$$\frac{d^2\phi}{dx^2} = -\frac{\sigma\delta(x)}{\varepsilon_0} + \frac{\phi(x)}{\lambda_D^2}$$

In $x > 0$

$x < 0$

$$\frac{d^2\phi}{dx^2} = \frac{\phi(x)}{\lambda_D^2}$$

$$\left(\frac{d^2x}{dt^2} = k\,x(t) \right)$$

$x > 0$ $\quad \phi_+(x) = A_+ e^{x/\lambda_D} + B_+ e^{-x/\lambda_D}$ $\quad A_+ = 0$

$x < 0$ $\quad \phi_-(x) = A_- e^{x/\lambda_D} + B_- e^{-x/\lambda_D}$ $\quad B_- = 0$

$$\phi_+(x) = B_+ e^{-x/\lambda_D}$$

In $x = 0$ $\quad \phi(x) = \phi_0$

$$\phi_-(x) = A_- e^{x/\lambda_D} \quad \Longrightarrow \quad \phi(x) = \phi_0\, e^{-|x|/\lambda_D}$$

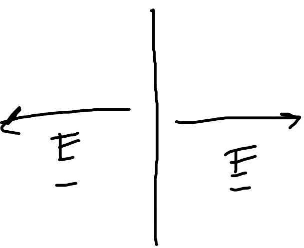
$$E(x) = -\frac{d\phi}{dx} = \frac{\phi_0}{\lambda_D} e^{-|x|/\lambda_D}\,\hat{\underline{i}}$$

$$\underline{E}(x) = \frac{\phi_0}{\lambda_D} e^{-|x|/\lambda_D}\,\hat{\underline{i}} = \frac{\sigma}{2\varepsilon_0} e^{-|x|/\lambda_D}\,\hat{\underline{i}}$$

$$\lim_{\varepsilon \to 0} \int_{-\varepsilon}^{\varepsilon} \frac{d^2\phi}{dx^2}\,dx = \lim_{\varepsilon \to 0} \int \left[-\frac{\sigma}{\varepsilon_0}\delta(x) + \frac{\phi(x)}{\lambda_D^2} \right] dx$$

$$\left.\frac{d\phi}{dx}\right|_{0^+} - \left.\frac{d\phi}{dx}\right|_{0^-} = -\frac{\sigma}{\varepsilon_0} \Rightarrow -\frac{\phi_0}{\lambda_D} - \frac{\phi_0}{\lambda_D} = -\frac{\sigma}{\varepsilon_0} ;$$

$$\frac{\phi_0}{\lambda_D} = \frac{\sigma}{2\varepsilon_0}$$



Criterio per un buon plasma

$$L \gg \lambda_D$$

$$\frac{4}{3}\pi \lambda_D^3 n_e \gg 1$$

τ : tempo medio tra collisioni

ω freq. di una perturbazione

$\tau \gg \omega^{-1}$

$$\omega_{pe}^2 = \frac{ne^2}{m_e \varepsilon_0}$$