$\underline{B} = \overrightarrow{\text{const}}$

$\parallel$ moto rett. uniforme

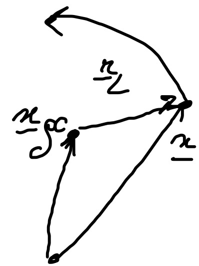
$\perp$ = circ. = $r_L = \dfrac{m v_\perp}{qB}$ $\quad \omega_L = \dfrac{qB}{m}$

$+ \ \underline{F} = \overrightarrow{\text{const}}$

$\parallel$ moto rett. unif. acc. $\quad a_\parallel = F_\parallel / m$

$\perp$

$$\underline{v}_D = \frac{\underline{F}_\perp \times \underline{B}}{qB^2}$$



$$\underline{x}(t) = \underline{x}_{gc}(t) + \underline{r}_L$$

$$\underline{B}(\underline{x}) \simeq \underline{B}(\underline{x}_{gc}) + (\underline{r}_L \cdot \underline{\nabla})\underline{B}$$

$$\underline{E}(\underline{x}) \simeq \underline{E}(\underline{x}_{gc}) + (\underline{r}_L \cdot \underline{\nabla})\underline{E}_0$$

$$m \frac{d\underline{v}}{dt} = q(\underline{E} + \underline{v} \times \underline{B})$$

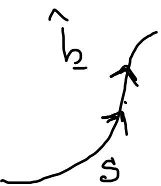
$$m \frac{d\underline{v}_{gc}}{dt} = q\left[\underline{E}(\underline{x}_{gc}) + \langle \underline{v}_L \times (\underline{r}_L\cdot\underline{\nabla})\,\underline{B}(\underline{x}_{gc})\rangle + \underline{v}_{gc}\times\underline{B}(\underline{x}_{gc})\right]$$

$\langle\ \rangle$: media temporale su T_L

$$\underline{v}_{gc} = v_{gc\parallel}\,\hat{\underline{b}} + \underline{v}_{gc\perp}$$

$\hat{\underline{b}}$ = versore della linea di campo

$$\frac{d\underline{v}_{gc}}{dt} = \frac{d\underline{v}_{gc\perp}}{dt} + \frac{d}{dt}\left(v_{gc\parallel}\,\hat{\underline{b}}\right) = \frac{d\underline{v}_{gc\perp}}{dt} + \frac{dv_{gc\parallel}}{dt}\,\hat{\underline{b}} + v_{gc\parallel}\frac{d\hat{\underline{b}}}{dt}$$



s: coordinata curvilinea della linea di campo

$$\hat{\underline{b}} = \hat{\underline{b}}(s) \qquad \frac{d\hat{\underline{b}}(s)}{dt} = \frac{d\hat{\underline{b}}}{ds}\cdot\overbrace{\frac{ds}{dt}}^{v_{gc\parallel}} = \overbrace{v_{gc}\,(\underbrace{\hat{\underline{b}}\cdot\underline{\nabla}}_{\frac{d}{ds}})\,\hat{\underline{b}}}^{\perp}$$

$$\frac{d\underline{v}_{gc}}{dt} = \underbrace{\frac{d\underline{v}_{gc\perp}}{dt}}_{\perp} + \underbrace{\frac{dv_{\parallel gc}}{dt}\hat{\underline{b}}}_{\parallel} + \underbrace{v_{\parallel gc}^{2}(\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}}}_{\perp}$$

$$\underbrace{m\frac{d\underline{v}_{gc\perp}}{dt}}_{\perp} + \underbrace{m\frac{dv_{\parallel gc}}{dt}\hat{\underline{b}}}_{\parallel} = \overbrace{q\underline{E}(\underline{x}_{gc})}^{\parallel,\perp} + \overbrace{q\langle \underline{v}_{\perp}\times(\underline{r}_{\perp}\cdot\underline{\nabla})\underline{B}(\underline{x}_{gc})\rangle}^{\parallel,\perp} + \overbrace{q\underline{v}_{gc}\times\underline{B}(\underline{x}_{gc})}^{\perp}$$

$$\underbrace{- m v_{\parallel gc}^{2}(\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}}}_{\perp}$$

$\parallel$ $$m\frac{dv_{\parallel gc}}{dt} = qE_{\parallel}(\underline{x}_{gc}) + q\langle \underline{v}_{\perp}\times(\underline{r}_{\perp}\cdot\underline{\nabla})\underline{B}(\underline{x}_{gc})\rangle_{\parallel}$$

$\perp$ $$m\frac{d\underline{v}_{\perp gc}}{dt} = q\underline{v}_{\perp gc}\times\underline{B}(\underline{x}_{gc}) + \underline{F}_{\perp}$$

$$\underline{F}_{\perp} = q\underline{E}_{\perp}(\underline{x}_{gc}) + q\langle \underline{v}_{\perp}\times(\underline{r}_{\perp}\cdot\underline{\nabla})\underline{B}(\underline{x}_{gc})\rangle - m v_{\parallel gc}^{2}(\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}}$$

Ci sono delle velocità di deriva

Se $\underline{F}_\perp = \overrightarrow{const}$ $\qquad \underline{v}_D^{(0)} = \dfrac{\underline{F}_\perp \times \underline{B}}{qB^2}$

$$\underline{v}_D = \underline{v}_D^{(0)} + \underline{v}_D^{(1)} \qquad \frac{d\underline{v}_D^{(1)}}{dt} \ll \frac{d\underline{v}_D^{(0)}}{dt}$$

$$m\frac{d}{dt}\left(\underline{v}_D^{(0)} + \cancel{\underline{v}_D^{(1)}}\right) = q\left[\left(\cancel{\underline{v}_D^{(0)}} + \underline{v}_D^{(1)}\right)\times\underline{B}\right] + \cancel{\underline{F}_\perp}$$

Ordine 0: $\qquad q\,\underline{v}_D^{(0)}\times\underline{B} + \underline{F}_\perp = 0$

$\times\underline{B} \qquad \times\underline{B}$

$$m\frac{d\underline{v}_D^{(0)}}{dt} = q\,\underline{v}_D^{(1)}\times\underline{B}$$

$$(\underline{C}\times\underline{B})\times\underline{A} = (\underline{A}\cdot\underline{C})\,\underline{B} - (\underline{A}\cdot\underline{B})$$

$$\underline{v}_D^{(1)} = -\frac{m}{qB^2}\frac{d}{dt}\underline{v}_D^{(0)}\times\underline{B}$$

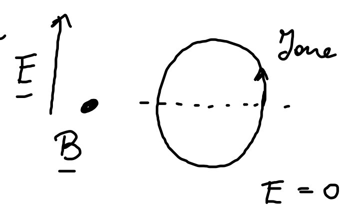
Es $\qquad \underline{F}_\perp = q\,\underline{E}_\perp(t)$

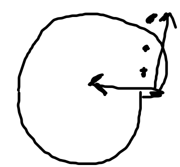
$$\underline{v}_D^{(0)} = \frac{\underline{E}_\perp \times \underline{B}}{B^2}$$

$$\underline{v}_D^{(1)} = -\frac{m}{qB^2}\underbrace{\left[\frac{d\underline{E}_\perp}{dt}\times\frac{\underline{B}}{B^2}\right]}_{\frac{d\underline{v}_D^{(0)}}{dt}} \times \underline{B} = \dots = \frac{m}{qB^2}\frac{d\underline{E}_\perp}{dt}$$

$$(\underline{A}\times\underline{B})\times\underline{C} = \dots$$

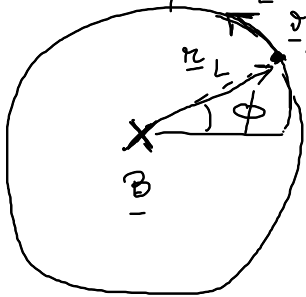
Deriva di polarizzazione

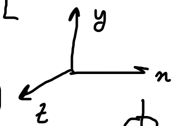




Analizziamo il termine

$$q\left\langle \underline{v}_L \times (\underline{r}_L \cdot \underline{\nabla})\,\underline{B}(\underline{x}) \right\rangle\Big|_{\underline{x}=\underline{x}_{gc}}$$





$\underline{B}$ nel girocentro è $\underline{B} = B\hat{\underline{k}}$

$$\phi = \omega_L t \qquad \underline{r}_L = \left[\cos(\omega_L t)\,\hat{\underline{i}} + \text{sen}(\omega_L t)\,\hat{\underline{j}}\right] r_L$$

$$\underline{v}_L = \frac{d\underline{r}_L}{dt} = r_L\omega_L\left[-\text{sen}(\omega_L t)\,\hat{\underline{i}} + \cos(\omega_L t)\,\hat{\underline{j}}\right]$$

$$q r_L^2 \omega_L \left\langle \left(-\text{sen}(\omega_L t)\,\hat{\underline{i}} + \cos(\omega_L t)\,\hat{\underline{j}}\right) \times \left[\overbrace{\left(\cos(\omega_L t)\,\hat{\underline{i}} + \text{sen}(\omega_L t)\,\hat{\underline{j}}\right) \cdot \left[\frac{\partial}{\partial x}\hat{\underline{i}} + \frac{\partial}{\partial y}\hat{\underline{j}} + \frac{\partial}{\partial z}\hat{\underline{k}}\right]}^{\cos(\omega_L t)\frac{\partial}{\partial x} + \text{sen}(\omega_L t)\frac{\partial}{\partial y}} \left(B_x\hat{\underline{i}} + B_y\hat{\underline{j}} + B_z\hat{\underline{k}}\right)\right] \right\rangle$$

$$\langle \cos^2 \rangle = \langle \sin^2 \rangle = \frac{1}{2}$$

$$\langle \text{sen} \cdot \cos \rangle = 0$$

$$= q r_L^2 \omega_L \Big\langle \Big(\left(-\text{sen}(\omega_L t)\,\hat{\underline{i}} + \cos(\omega_L t)\,\hat{\underline{j}} \right) \times$$

$$\left(\cos(\omega_L t) \left(\frac{\partial B_x}{\partial x}\hat{\underline{i}} + \frac{\partial B_y}{\partial x}\hat{\underline{j}} + \frac{\partial B_z}{\partial x}\hat{\underline{k}} \right) + \text{sen}(\omega_L t) \left(\frac{\partial B_x}{\partial y}\hat{\underline{i}} + \frac{\partial B_y}{\partial y}\hat{\underline{j}} + \frac{\partial B_z}{\partial y}\hat{\underline{k}} \right) \right) \Big) \Big\rangle$$

$$= \frac{q r_L^2 \omega_L}{2} \left[-\frac{\partial B_y}{\partial y}\hat{\underline{k}} + \frac{\partial B_z}{\partial y}\hat{\underline{j}} - \frac{\partial B_x}{\partial x}\hat{\underline{k}} + \frac{\partial B_z}{\partial x}\hat{\underline{i}} \right]$$

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \Rightarrow \frac{\partial B_z}{\partial z} = -\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}$$

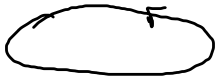
momento magnetico

$$= \frac{q r_L^2 \omega_L}{2} \left[\underbrace{\frac{\partial}{\partial x} B_z\,\hat{\underline{i}} + \frac{\partial B_z}{\partial y}\hat{\underline{j}} + \frac{\partial B_z}{\partial z}\hat{\underline{k}}}_{\underline{\nabla} B_z} \right] = -\frac{q r_L^2 \omega_L}{2} \underline{\nabla} \left| \underline{B}(\underline{x}_g) \right|$$

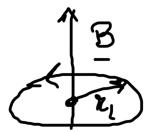
μ

$$\frac{q r_L^2 \omega_L}{2} = \frac{\cancel{q}\, m^2 v_\perp^2}{2 \cancel{q^2} B^{\cancel{2}}} \frac{\cancel{q} \cancel{B}}{\cancel{m}} = \frac{m v_\perp^2}{2B}$$

Momento magnetico (di una spira percorsa da I)

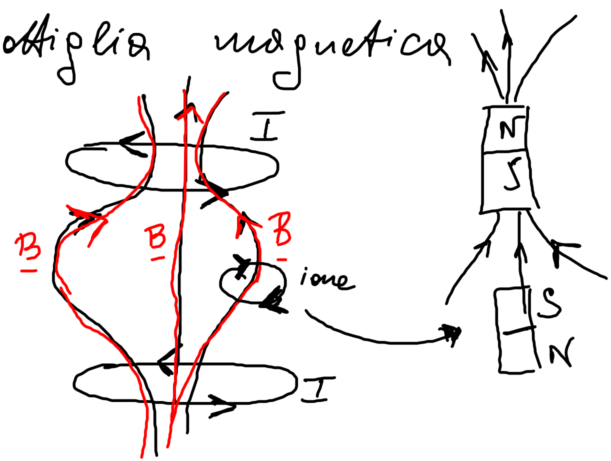


$$\mu = I A = \frac{\cancel{q}}{\cancel{2\pi}} \frac{\cancel{qB}}{\cancel{m}} \cdot \cancel{\pi} \frac{m^{\cancel{2}} v_\perp^2}{\cancel{q^2} B^{\cancel{2}}} = \frac{m v_\perp^2}{2B}$$



$$I = \frac{q}{T_L} \qquad A = \pi r_L^2$$

Bottiglia magnetica

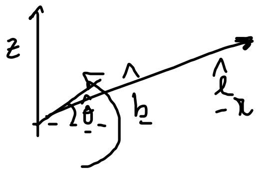


$$-m v_{gc}^2 (\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}} \quad ?$$

$$\hat{\underline{b}} = \hat{\underline{\theta}} = -\mathrm{sen}\,\theta\,\hat{\underline{i}} + \cos\theta\,\hat{\underline{j}}$$

Ipotesi: $\hat{\underline{b}} = \hat{\underline{\theta}}$

$$\hat{\underline{b}}\cdot\underline{\nabla} = \underbrace{\hat{\underline{\theta}}\cdot\underline{\nabla}}_{\text{comp. lungo } \hat{\theta} \text{ di } \underline{\nabla}} = \frac{1}{r}\frac{\partial}{\partial\theta}$$



$$(\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}} = \frac{1}{r}\frac{\partial}{\partial\theta}\left[-\mathrm{sen}\,\theta\,\hat{\underline{i}} + \cos\theta\,\hat{\underline{j}}\right] =$$

$$= -\frac{1}{r}\left[\cos\theta\,\hat{\underline{i}} + \mathrm{sen}\,\theta\,\hat{\underline{j}}\right] = -\frac{\hat{\underline{e}}_r}{r}$$

$$-m v_{gc}^2 (\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}} = \frac{m v_{gc}^2}{r}\,\hat{\underline{e}}_r$$

raggio locale di curvatura di $\hat{\underline{b}}$

Riassunto

Ordine 0: moto di Larmor

+ derive

In direzione $\perp$ $\leftarrow$ $\underline{v}_D = \dfrac{\underline{F} \times \underline{B}}{qB^2}$ (+ deriva polarizzazione se necessaria)

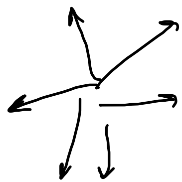
$$\underline{F} = q\underline{E} + \underline{\underline{\left(-\mu \underline{\nabla}|\underline{B}|\right)}} - \frac{m v_{\parallel gc}^2}{r} \underline{\hat{e}}_r$$

Lungo il $\underline{B}$

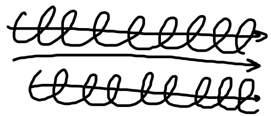
$$F_{\parallel} = q E_{\parallel} - \mu \nabla_{\parallel} |\underline{B}|$$

Sistema toroidale

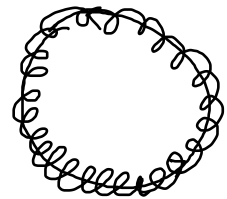
$B = 0$



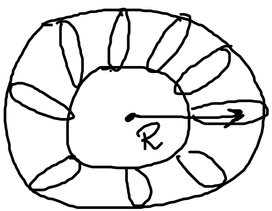
$B \neq 0$



B toroidale



I



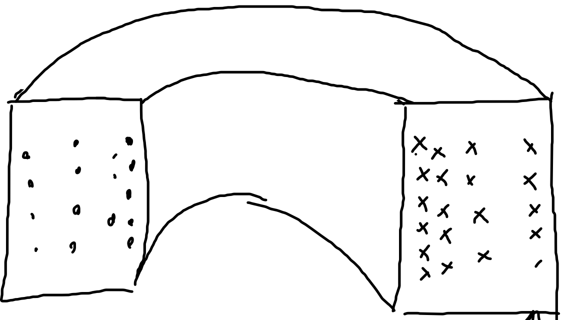
Th Ampere

$$\oint \underline{B} \cdot d\underline{l} = \mu_0 N \underline{I}$$

N: # bobine

$\underline{I}$: corrente in ciascuna bobina

$$B = \frac{\mu_0 N \underline{I}}{2\pi R} \qquad B \propto \frac{1}{R}$$



$\underline{B}$

$$\underline{v}_D = -\mu \frac{\underline{\nabla}|\underline{B}| \times \underline{B}}{qB^2}$$

$\underline{v}_D$ verticale in direzione opposta per ioni ed elettroni

$\underline{\nabla}|\underline{B}|$

ioni

elettroni

$\underline{E}$

$$\underline{v}_D = \frac{\underline{E} \times \underline{B}}{B^2}$$

$\left\{ \begin{array}{l} \text{Tokamak} \\ \text{Stellarator} \end{array} \right.$

$\underline{v}_{D,E}$