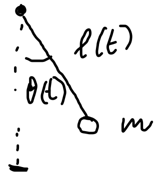


$\theta \ll 1$

$$\ddot{\theta} = -\omega^2(t)\,\theta \qquad \omega^2(t) = g/l(t)$$

Se $l = \text{const}$

$$\theta = \mathrm{Re}\left[\theta_0\, e^{i\int_0^t \omega\, dt'}\right] = \theta_0 \cos(\omega t)$$

Se $l = l(t)$ cerchiamo una soluzione

$$\theta(t) = \mathrm{Re}\left[\theta_0(t)\, e^{i\int_0^t \omega(t')\, dt'}\right]$$

Calcoliamo

$$\frac{d\theta}{dt} = \mathrm{Re}\left[\dot{\theta}_0 \exp[\quad] + \theta_0 \exp[\quad]\cdot i\omega(t)\right]$$

$$\frac{d^2\theta}{dt^2} = \mathrm{Re}\left[\ddot{\theta}_0 \exp[\quad] + \dot{\theta}_0\, i\omega \exp[\quad] + \dot{\theta}_0 \exp[\quad]\, i\omega(t) + \theta_0 \overbrace{i\omega \exp[\quad]\, i\omega}^{-\omega^2} + \theta_0 \exp[\quad]\, i\dot{\omega}\right] =$$

$$= \mathrm{Re}\left[\exp\left(i\int_0^t \omega(t')\,dt'\right)\cdot\left(\ddot{\theta}_0 + 2i\omega\dot{\theta}_0 - \omega^2\theta_0 + i\dot{\omega}\theta_0\right)\right]$$

Nell' equazione di partenza:

$$\mathrm{Re}\Big[\underbrace{\exp(\quad)\left(\ddot{\theta}_0 + 2i\omega\dot{\theta}_0 - \cancel{\omega^2\theta_0} + i\dot{\omega}\theta_0\right)}_{\ddot{\theta}}\Big] = \underbrace{\mathrm{Re}\left[\cancel{-\omega^2\theta_0}\exp(\quad)\right]}_{-\omega^2\theta}$$

$$\cancel{\ddot{\theta}_0} + 2i\omega\dot{\theta}_0 + i\dot{\omega}\theta_0 = 0$$

trascurabile

$$2\cancel{i}\omega\frac{d\theta_0}{\cancel{dt}} = -\cancel{i}\theta_0\frac{d\omega}{\cancel{dt}}$$

$$\omega = \sqrt{g/l(t)}$$

$$\int_{\theta_0(0)}^{\theta_0(t)}\frac{d\theta_0}{\theta_0} = -\frac{1}{2}\int\frac{d\omega}{\omega}$$

$$\omega_0 = \sqrt{g/l(0)}$$

$$\ln\frac{\theta_0(t)}{\theta_0(0)} = \ln\left(\sqrt{\frac{\omega_0}{\omega(t)}}\right)$$

$$\theta_0(t) = \theta_0(0)\sqrt{\frac{\omega_0}{\omega(t)}}$$

$$\theta(t) = \mathrm{Re}\left[\theta_0(0)\sqrt{\frac{\omega_0}{\omega(t)}}\exp\left[i\int_0^t \omega(t')\,dt'\right]\right] =$$

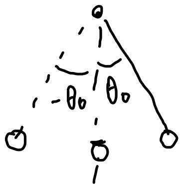
$$= \theta_0(0)\sqrt{\frac{\omega_0}{\omega(t)}}\cos\left(\int_0^t \omega(t')\,dt'\right) \qquad \omega \approx \omega_0 \quad \frac{\omega}{\omega_0} \simeq 1$$

1) Significato di $\frac{d^2\theta_0}{dt^2} \ll \omega\frac{d\theta_0}{dt}$ $\qquad \frac{d\theta_0}{dt} = -\frac{\theta_0\sqrt{\omega_0}}{2}\frac{1}{\omega^{3/2}}\frac{d\omega}{dt}$

2) Invariante?

$$\frac{d^2\theta_0}{dt^2} \simeq +\frac{\theta_0}{2}\frac{1}{\omega^2}\left(\frac{d\omega}{dt}\right)^2 + O\left(\frac{d^2\omega}{dt^2}\right) \simeq -\frac{\theta_0}{2}\frac{1}{\omega}\frac{d\omega}{dt}$$

$$\cancel{\frac{\theta_0}{2}}\,\frac{1}{\omega^2}\left(\frac{d\omega}{dt}\right)^2 \ll \cancel{\frac{\theta_0}{2}}\,\cancel{\frac{d\omega}{dt}}$$



$$\frac{d\omega}{dt} \ll \omega^2$$

$$\frac{1}{\omega}\frac{d\omega}{dt} \ll \omega$$

$$\frac{d\omega}{dt} \approx \frac{\Delta\omega}{\Delta t}$$

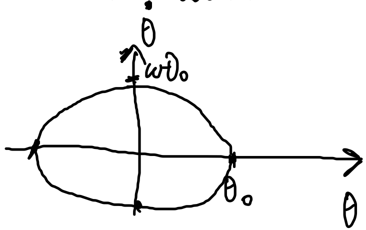
$$\frac{1}{\omega}\frac{\Delta\omega}{\Delta t} \ll \omega \qquad \omega \approx \frac{1}{T}$$

$$\underbrace{\frac{\Delta\omega}{\omega}}_{\text{Variazione relativa della frequenza}} \ll \frac{\Delta t}{T} \ll 1$$

T: periodo dell'oscillazione

frequenza

Se $l =$ const



$$\theta(t) = \theta_0 \cos(\omega t)$$

$$\dot\theta(t) = -\theta_0\omega\sin(\omega t)$$

$$\text{Area} = \pi\theta_0^2\omega = \text{cost}$$

$$\left(\frac{\theta}{\theta_0}\right)^2 + \left(\frac{\dot\theta}{\omega\theta_0}\right)^2 = 1$$

Ellisse nel piano $(\theta, \dot\theta)$

$$\text{Area} = \int_{\substack{\text{ciclo} \\ \text{o} \\ \text{periodo}}} \theta \frac{d\theta}{dt} \, \cancel{d}t = \int_{\text{periodo}} (\dot{\theta})^2 \, dt$$

Trucco:

$$\frac{d}{dt}\left(\theta \frac{d\theta}{dt}\right) = (\dot{\theta})^2 + \theta \frac{d^2\theta}{dt^2}$$

$$\dot{\theta}^2 = \frac{d}{dt}\left(\theta \frac{d\theta}{dt}\right) - \theta \underbrace{\frac{d^2\theta}{dt^2}}_{-\omega^2\theta}$$

$$\underbrace{\qquad}_{+\omega^2\theta^2}$$

$$= \underbrace{\int_{\text{periodo}} \frac{d}{dt}\left(\theta \frac{d\theta}{dt}\right) dt}_{} + \int_{\text{periodo}} \omega^2 \theta^2 \, dt$$

$$\cancel{\theta \frac{d\theta}{dt}\Big|_{\text{inizio dell'oscillazione}}^{\text{fine dell'oscillazione}}} + \int_0^T \omega^2 \theta_0^2 \frac{\omega_0}{\cancel{\omega}} \cos^2\left(\int_0^t \omega(t')\,dt'\right) dt$$

$$\overset{\shortparallel}{0}$$

Cambio di variabile

$$\xi = \int_0^t \omega(t')\,dt'$$

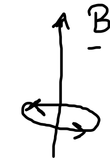
$$t = 0 \Rightarrow \xi = 0$$

$$t = T \Rightarrow \int_0^T \frac{d\theta}{\cancel{dt'}} \cancel{dt'} = 2\pi$$

$$\xi = \int_0^t \omega(t')\,dt'$$

$$\frac{d\xi}{dt} = \omega(t) \Rightarrow d\xi = \omega\, dt$$

$$\text{Area} = \omega_0 \theta_0^2 \underbrace{\int_0^{2\pi} \cos^2(\xi)\; d\xi}_{\pi} = \pi \omega_0 \theta_0^2$$



In un moto adiabatico e periodico nella coordinata q

si conserva l'integrale di azione $\int_{\text{periodo}} p\, dq$ $\qquad p = \frac{\partial L}{\partial \dot{q}}$

$$\mathcal{L} = \frac{1}{2} m v^2 + q \underline{A}^{(\underline{x},t)} \cdot \underline{v} - q\phi(\underline{x},t)$$

$$\underline{B}^{(\underline{x},t)} = \underline{\nabla} \times \underline{A}(\underline{x},t)$$

$$\underline{E}(\underline{x},t) = -\underline{\nabla}\phi - \partial \underline{A}/\partial t$$

$$\downarrow$$

$$m \frac{d\underline{x}^2}{dt^2} = q(\underline{E} + \underline{v} \times \underline{B})$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{q_j} = 0 \qquad q_j \quad j = 1 \ldots N$$

$$q_j \longrightarrow \underline{x}, y, z$$

x_i $\qquad x_j \quad j = 1 \ldots 3$

$$\mathcal{L} = \frac{1}{2} m \sum_{j=1}^{3} (\dot{x}_j)^2 + q \sum_{j=1}^{3} A_j(\underline{x},t)\dot{x}_j - q\phi(\underline{x},t)$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = q \sum_{j=1}^{3} \frac{\partial A_j}{\partial x_i} \dot{x}_j - q \frac{\partial \phi}{\partial x_i}$$

Formula per Derivata di funzione composta

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_i} = m \dot{x}_i + q A_i(\underline{x}, t)$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = m \ddot{x}_i + q \sum_{j=1}^{3} \frac{\partial A_i}{\partial x_j} \dot{x}_j + q \frac{\partial A_i}{\partial t}$$