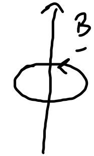
$J = \oint p\, dq$ 	q: coordinata periodica



$p = \frac{\partial \mathcal{L}}{\partial \dot q}$

$\mathcal{L} = \frac{1}{2} m v^2 + q\, \underline{A}\cdot\underline{v} - q\phi$ 	$\underline{B} = \nabla \times \underline{A}$ 	$\underline{E} = -\underline{\nabla}\phi - \partial \underline{A}/\partial t$

$\Rightarrow \quad \underline{F} = q(\underline{E} + \underline{v}\times\underline{B})$ 	$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot x_j} - \frac{\partial\mathcal{L}}{\partial x_j} = 0 \quad j=1,2,3$

$\frac{\partial \mathcal L}{\partial x_i} = q\sum_j \frac{\partial A_j}{\partial x_i}\dot x_j - q\frac{\partial\phi}{\partial x_i}$ 	$\frac{\partial\mathcal L}{\partial \dot x_i} = m\dot x_i + q A_i(\underline{x},t)$

$q(\underline{v}\times\underline{\nabla}\times\underline{A})_i$

$q\sum_j\left[\frac{\partial A_j}{\partial x_i}\dot x_j - \frac{\partial A_i}{\partial x_j}\dot x_j\right]$

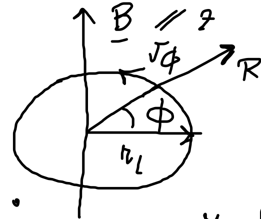
$\frac{d}{dt}\frac{\partial\mathcal L}{\partial\dot x_i} = m\ddot x_i + q\sum_j \frac{\partial A_i}{\partial x_j}\dot x_j + q\frac{\partial A_i}{\partial t}$

$\underbrace{m\ddot x_i}_{F_i} = -q\sum_j \frac{\partial A_i}{\partial x_j}\dot x_j - q\frac{\partial A_i}{\partial t} + q\sum_j\frac{\partial A_j}{\partial x_i}\dot x_j - q\frac{\partial\phi}{\partial x_i} = q\left[\underbrace{-\frac{\partial\phi}{\partial x_i} - \frac{\partial A_i}{\partial t}}_{E_i}\right] +$

$$\mathcal{L} = \frac{1}{2} m v^2 + q\,\underline{A}\cdot\underline{v}$$

$$= \frac{1}{2} m \left(v_R^2 + v_z^2 + v_\phi^2\right) + q A_R v_R + q A_z v_z + q A_\phi v_\phi$$

$$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot\phi}$$



$$v_\phi = r_L \dot\phi$$

$$J = \int p_\phi \, d\phi$$

$$= \frac{1}{2} m \left(v_R^2 + v_z^2 + r_L^2 \dot\phi^2\right) + q A_R v_R + q A_z v_z + q A_\phi r_L \dot\phi$$

$$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot\phi} = r_L^2 m \dot\phi + q A_\phi r_L$$

$$J = \int \left(m r_L^2 \dot\phi + q A_\phi r_L\right) d\phi = \underbrace{\int m \cancel{r_L^2} \frac{v_L^2}{\cancel{r_L^2}} \, dt}_{m v_L^2 T} + q r_L \int A_\phi \, d\phi$$

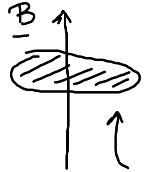
orbita di Larmor

$r_L \approx$ cost in un'orbita

$A_\phi \approx$ cost // // //

$$\dot\phi \, d\phi = (\dot\phi)^2 dt = \frac{v_L^2}{r_L^2} dt$$

$\underline{B} = \underline{\nabla} \times \underline{A}$



Superficie: πr_L^2

$$\int_{\substack{\text{cerchio}\\ \text{di Larmor}}} \underline{B} \cdot d\underline{S} \simeq \pi r_L^2 B$$

$\underline{F} = -\mu\, \underline{\nabla} |\underline{B}|$

$$\int_{\substack{\text{cerchio}\\ \text{di Larmor}}} (\underline{\nabla} \times \underline{A}) \cdot d\underline{S} \underset{\text{Th Stokes}}{=} \oint_{\substack{\text{circonf. di}\\ \text{Larmor}}} \underline{A} \cdot d\underline{l} = \int A_\phi \, dl \simeq A_\phi \oint_{\substack{\text{circonf}\\ \text{di Larmor}}} dl \simeq 2\pi r_L A_\phi \Rightarrow \cancel{\pi} r_L^{\cancel{2}} B = \cancel{2\pi} \cancel{r_L} A_\phi$$

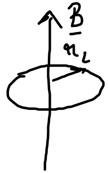
$$A_\phi = \frac{r_L}{2} B$$

$$J = m v_L^2 \frac{2\pi m}{qB} + q r_L \oint_{\substack{\text{orbita}\\ \text{di Larmor}}} \frac{r_L}{2} B \, d\phi = m v_L^2 \frac{2\pi m}{qB} + \frac{\cancel{q}}{2} \cancel{B}\, 2\pi \frac{m^2 v_L^2}{q^{\cancel{2}} B^{\cancel{2}}} = \left(\frac{4\pi m}{q} + \frac{\pi m}{q}\right) \frac{m v_L^2}{2B} \to \mu$$

$$= \left(\frac{6\pi m}{q}\right) \cdot \left(\frac{m v_L^2}{2B}\right)$$

$\mu = I A$

$$\Phi(B) = B\pi r_L^2 = \pi \frac{m^2 v_L^2}{q^2 B^{\cancel{2}}} \cancel{B} = \frac{2\pi \cdot m}{q^2} \frac{m v_L^2}{2B}$$



$\Rightarrow \Phi$ inv. adiabatico

$B \approx 1\,T$

$$T = \frac{2\pi m}{qB} \approx \frac{6}{1.6\cdot 10^{-19}\cdot 1} \cdot m \approx 4\cdot 10^{19}\ \text{massa}$$

$B \uparrow$ Plasma si strizza

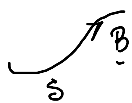
$B \downarrow$ Plasma si allunga

$\Phi = \text{const}$

$\rightarrow 4\cdot 10^{-11} \approx$ frazione ns elettroni

$\rightarrow \approx$ frazione di μs ioni

$$v_\parallel \cdot \quad m\frac{dv_\parallel}{dt} = \cancel{qE_\parallel} - (\mu \nabla\cdot|B|)_\parallel = -\mu\frac{\partial B}{\partial s}\cdot v_\parallel$$



$$\frac{d}{dt}\left(\frac{1}{2} m v_\parallel^2\right) = -\mu \underbrace{\frac{\partial B}{\partial s}\frac{\partial s}{\partial t}}_{-\mu\frac{dB}{dt} = -\frac{d}{dt}(\mu B)}$$

$$\frac{d}{dt}\left(\frac{1}{2}mv_{\parallel}^2 + \mu B\right) = 0 \Rightarrow \underbrace{\frac{1}{2}mv_{\parallel}^2}\; + \mu B = \text{cost} =$$

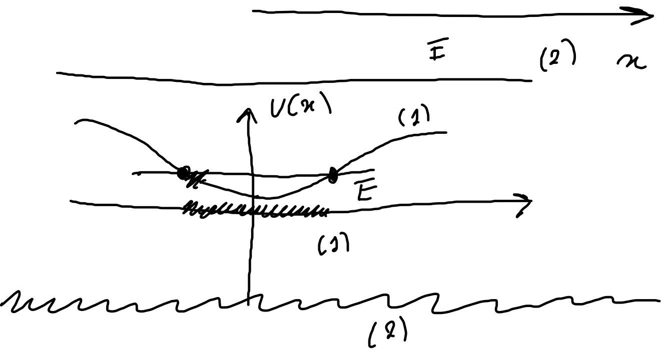
$$= \frac{1}{2}mv_{\parallel}^2 + \frac{mv_\perp^2}{2\cancel{B}}\cancel{B} = W$$

in tutti i pti della traiettoria

1D $\quad \frac{1}{2}mv_{\parallel}^2 + U(x) = E \;; \quad v_{\parallel}^2 = \frac{2}{m}\left(E - U(x)\right)$

Se $E > U(x)$ allora

$$v_{\parallel} = \sqrt{\frac{2}{m}\left(E - U(x)\right)}$$



Se $E \geq U(x)$ in qualche punto c'è una regione limitata dove vale $E > U(x)$,

$$v_{\parallel} = \sqrt{\frac{2}{m}\left(E - U(x)\right)}$$

Ai bordi (dove $E = U(x)$) $v_{\parallel} = 0$

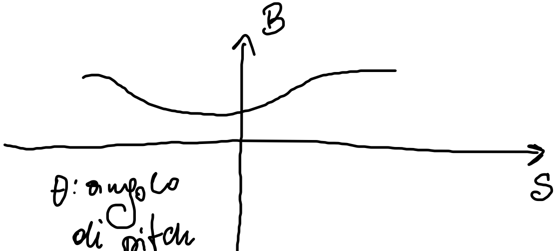
Fuori, dove $E < U(x)$, non c'è traiettoria

Lungo la linea di campo

Particella confinata se $E < \mu B_{max}$

// deconfinata se $E > \mu B_{max}$

μB ha il ruolo di potenziale efficace



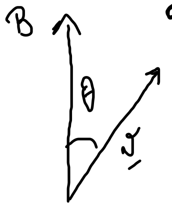
θ: angolo di pitch

$\mu = \frac{m v_\perp^2}{2B}$

$\frac{1}{2} m v_\parallel^2 + \mu B = W$

Se una particella nasce solo con $v_\parallel$, $v_\perp = 0$

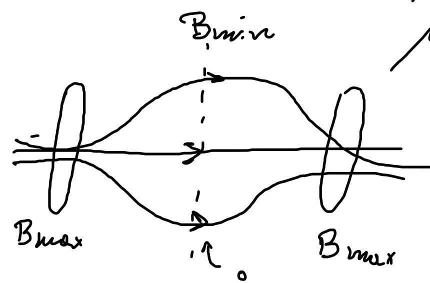
$\mu = 0$

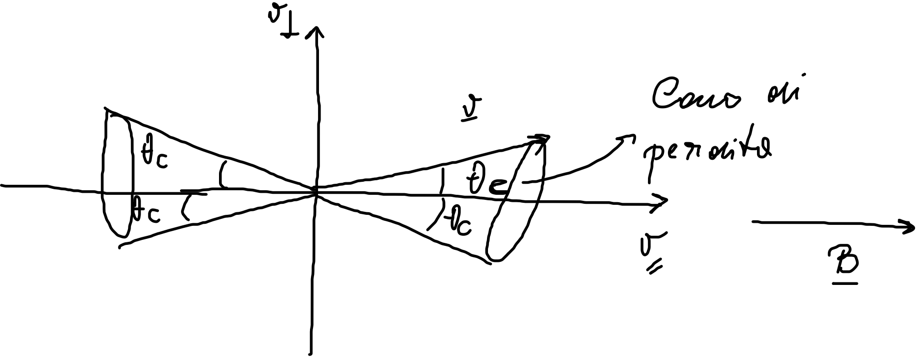


$v_\perp = v \sin\theta$

$\cancel{\frac{1}{2}} \cancel{m v_0^2} < \frac{\cancel{m v_0^2} \sin^2\theta_0}{\cancel{2} B_{min}} B_{max}$

$\sin^2\theta_0 > \frac{B_{min}}{B_{max}}$





Def θ_c angolo critico

$$\operatorname{sen}\theta_c = \pm\sqrt{\frac{B_{min}}{B_{max}}}$$

Se $\theta > \theta_c$: confinamento

se $\theta < \theta_c$: $\underline{\underline{\text{no}}}$ "