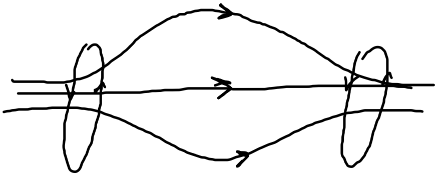
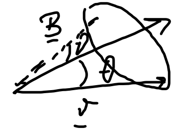
μ è invariante adiabatico

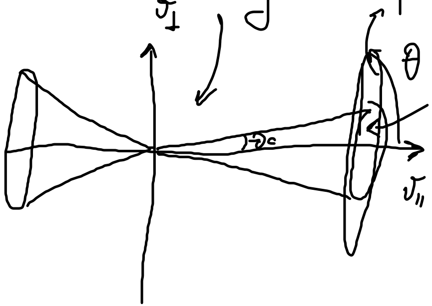


$$m \frac{dv_\parallel}{dt} = -\mu \frac{\partial B}{\partial s}$$



$$\frac{1}{2} m v_\parallel^2 + \underbrace{\mu B}_{\Xi} = W$$

Confinate φ



$$\text{sen}\,\theta_c = \sqrt{\frac{B_{min}}{B_{max}}}$$

$\text{sen}\,\theta < \text{sen}\,\theta_c$ <u>non</u> confinate

$\text{sen}\,\theta > \text{sen}\,\theta_c$ confinate

$f(\underline{v})$ è Maxwelliana $= f_M$

$$\text{frazione parti. confinate} = \frac{\sim\!\sim}{\underbrace{\int f_M(\underline{v})\, d^3 \underline{v}}}$$

Coordinate sferiche

$$\int_0^{\pi} \text{sen}\,\theta\, d\theta \int_0^{2\pi} d\varphi \int_0^{+\infty} dv\, v^2 f_M(v)$$

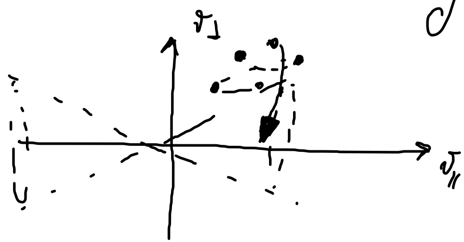
$$\text{frazione} = \frac{\int_{\theta_C}^{\pi-\theta_C} d\theta \sin\theta \cancel{\int_0^{2\pi} d\varphi} \cancel{\int_0^{+\infty} dv\, v^2} f_M(v)}{\int_0^{\pi} d\theta \sin\theta \cancel{\int_0^{2\pi} d\varphi} \cancel{\int_0^{+\infty} dv\, v^2} f_M(v)} = \frac{-\cos\theta \Big|_{\theta_C}^{\pi-\theta_C}}{2} = \frac{\cos\theta_C + \cos\theta_C}{2}$$

$$= \cos\theta_C = \sqrt{1 - \operatorname{sen}^2\theta_C}$$

$$= \sqrt{1 - \frac{B_{min}}{B_{max}}}$$

$$\frac{B_{min}}{B_{max}} \simeq \frac{1}{2} \quad (\text{ragionevole})$$

$$\text{frazione} \simeq \frac{1}{\sqrt{2}} \simeq 70\%$$



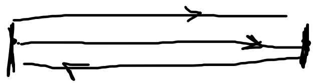
$$\tau_{coll} \simeq ms \qquad \tau_{perdita} \simeq \frac{L}{v_\parallel} \simeq \frac{L}{v}$$

$$T \simeq 1 \text{ keV} \qquad v_{ione} \sim \sqrt{\frac{2T}{m}} \sim 4 \cdot 10^5 \text{ m/s}$$

$$L \sim 1 \text{ m} \qquad \tau_{perdite} \sim 10\ \mu s$$

Particelle confinate

$$\frac{1}{2} m v_{\parallel}^2 + \mu B = W$$



$W = \mu B_{max}$ $\qquad$ $W = \mu B_{max}$

$$J = \int P_{\parallel} \, ds \qquad P_{\parallel} = m v_{\parallel} + q A_{\parallel}$$

Legge di Ampere

$$(\underline{\nabla} \times \underline{B})_{\parallel} = \mu_0 \underline{j}_{\parallel} \qquad \underline{B} = \underline{\nabla} \times \underline{A}$$

$$\mathcal{L} = \frac{1}{2} m v_{\parallel}^2 + \frac{1}{2} m v_{\perp}^2 + q A_{\parallel} v_{\parallel} + q \underline{A}_{\perp} \cdot \underline{v}_{\perp}$$

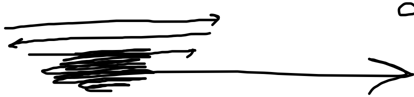
$$(\underline{\nabla} \times (\underline{\nabla} \times \underline{A}))_{\parallel} = \mu_0 \underline{j}_{\parallel}$$

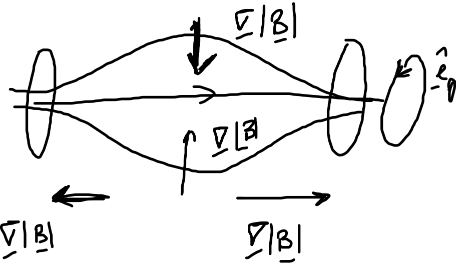
$$p = \frac{\partial \mathcal{L}}{\partial \dot{s}} = \frac{\partial \mathcal{L}}{\partial v_{\parallel}}$$

$$(\underline{\nabla} \underbrace{(\underline{\nabla} \cdot \underline{A})}_{=0} - \nabla^2 \underline{A})_{\parallel} = \mu_0 j_{\parallel} \qquad (\nabla^2 \underline{A})_{\parallel} = \mu_0 j_{\parallel} \qquad \text{se } j_{\parallel} = 0 \Rightarrow A_{\parallel} = 0$$

$I \uparrow$ Avvicinando bobine i p.ti di inversione del moto

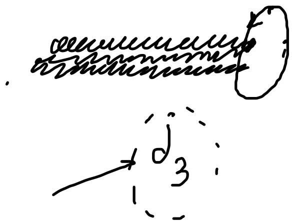
$$J = \int m v_{\parallel} \, ds \simeq m v_{\parallel} \cdot L \qquad L \downarrow \quad v_{\parallel} \uparrow$$





$$\underline{v}_D \propto \underbrace{\nabla|\underline{B}|}_{\hat{e}_r} \times \underbrace{\underline{B}}_{\hat{z}} \propto \hat{e}_\theta$$

$$\underline{F}_c \propto \hat{e}_r \qquad \underline{v}_{centr} \propto \underline{F}_c \times \underline{B} \propto \hat{e}_\theta$$



particelle che fanno giri completi attorno all'asse

= = "rimbalzano" poloidalmente

$\underline{F} \uparrow\downarrow$

$$J(\mu) > J(j_2) > J(j_1)$$

Plasma come insieme di cariche

$n \sim 10^{20}\ m^{-3}$

$V \sim 1\ m^{3}$ $\quad 10^{20}$

Risolvo $\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$

per tutte le particelle del plasma $\xrightarrow{\text{calcolare}}$ $\rho, \underline{j}$

$$\underline{\nabla} \cdot \underline{E} = \rho / \varepsilon_0 \qquad \underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad \underline{\nabla} \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \, \partial \underline{E} / \partial t$$

$f_e(\underline{x}, \underline{v}, t) \qquad f_i(\underline{x}, \underline{v}, t)$ $\qquad$ Plasmi 1

$n(\underline{x}, t) \qquad \underline{u}(\underline{x}, t)$ $\qquad$ Descrizione per quantità medie

Plasma per fusione nucleare

$L \sim m$ $\qquad n \sim 10^{20}\, m^{-3}$

$\Delta x \sim 10^{-5}\, m$

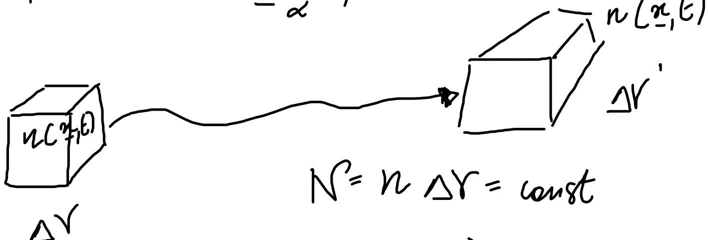
$N \sim n(\Delta x)^3 \sim 10^5$

Vento solare $\qquad L \sim$ qualche raggio solare $\qquad R \simeq 7 \cdot 10^8\, m \qquad n \sim 5 \cdot 10^6\, m^{-3}$

$\Delta x \sim m$

$N \sim n(\Delta x)^3 \sim 10^{18}$

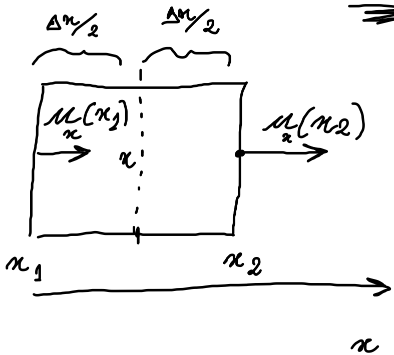
$n(\underline{x}, t)$ $\qquad \underline{u}(\underline{x}, t)$



$N = n\,\Delta V = \text{const}$ $\qquad \frac{dN}{dt} = 0$

$$\frac{d}{dt}(n\Delta V) = \frac{dn}{dt}\Delta V + n\frac{d\Delta V}{dt} = 0$$

$$\frac{d}{dt}(\Delta V) = \frac{d}{dt}(\Delta x \Delta y \Delta z) = \underline{\underline{\frac{d\Delta x}{dt}}}\,\Delta y \Delta z + \Delta x \Delta z \frac{d}{dt}\Delta y + \Delta x \Delta y \frac{d}{dt}\Delta z$$



$$x_2 = x + \frac{\Delta x}{2} \qquad x_1 = x - \frac{\Delta x}{2} \qquad \Delta x = x_2 - x_1$$

$$x_2' = x_2 + u_x(x_2)\Delta t = x_2 + u_x\left(x + \frac{\Delta x}{2}\right)\Delta t = x_2 + u_x(x)\Delta t + \frac{\partial u_x}{\partial x}\frac{\Delta x}{2}\Delta t$$

$$x_1' = x_1 + u_x(x_1)\Delta t = x_1 + u_x\left(x - \frac{\Delta x}{2}\right)\Delta t \simeq x_1 + u_x(x)\Delta t - \frac{\partial u_x}{\partial x}\frac{\Delta x}{2}\Delta t$$

$$\Delta x' = x_2' - x_1' = x_2 - x_1 + \frac{\partial u_x}{\partial x}\Delta x \Delta t = \Delta x\left(1 + \frac{\partial u_x}{\partial x}\Delta t\right)$$

$$\frac{d}{dt}(\Delta x) = \frac{\Delta x' - \Delta x}{\Delta t} = \frac{\partial u_x}{\partial x}\Delta x$$

$$\frac{d}{dt}(\Delta V) = \Delta x \Delta y \Delta z \frac{\partial u_x}{\partial x} + \Delta x \Delta y \Delta z \frac{\partial u_y}{\partial y} + \Delta x \Delta y \Delta z \frac{\partial u_z}{\partial z} = \Delta V \cdot (\underline{\nabla} \cdot \underline{u})$$

$$\frac{d}{dt} n(\underline{x}, t) = \frac{\partial n}{\partial x} \overbrace{\frac{\partial x}{\partial t}}^{u_x} + \frac{\partial n}{\partial y} \overbrace{\frac{\partial y}{\partial t}}^{u_y} + \frac{\partial n}{\partial z} \overbrace{\frac{\partial z}{\partial t}}^{u_z} + \frac{\partial n}{\partial t} = (\underline{u} \cdot \underline{\nabla}) n + \frac{\partial n}{\partial t}$$

$$\frac{dn}{dt} \Delta V + n \frac{d\Delta V}{dt} = \cancel{\Delta V} \underbrace{\left((\underline{u} \cdot \underline{\nabla}) n + \frac{\partial n}{\partial t} \right) + n \cancel{\Delta V} (\underline{\nabla} \cdot \underline{u})} = 0$$

$$\boxed{\underline{\nabla} \cdot (n \underline{u}) + \frac{\partial n}{\partial t} = 0}$$

Conservazione della qtà moto

$$\frac{d}{dt} (\text{qtà di moto}) = \sum \text{Forze}$$

qtà di moto $= \underbrace{m\underline{u}}_{\text{qtà moto di 1 particella}} \underbrace{n\Delta V}_{\text{numero di particelle}}$

$$\frac{d}{dt}\left(m\underline{u}\underbrace{n\Delta V}_{N}\right) = n\Delta V m \frac{d\underline{u}}{dt}\underline{u} \qquad \underline{u}(\underline{x},t)$$

$$= mn\Delta V\left((\underline{u}\cdot\underline{\nabla})\underline{u} + \frac{\partial\underline{u}}{\partial t}\right)$$

Forze

1) Interazioni con $\underline{E}$ e $\underline{B}$ $\qquad q(\underline{E}+\underline{u}\times\underline{B})\underbrace{n\Delta V}_{N}$

2) Gradienti di pressione

3) Collisioni interspecie

$$\underline{F}_{coll,e} = \overbrace{n\Delta V}^{N} m_e\bar{\nu}_{ei}(\underline{u}_e-\underline{u}_i)$$

$$\underline{F}_{coll,i} = \overbrace{n\Delta V}^{N} m_e\bar{\nu}_{ei}(\underline{u}_i-\underline{u}_e)$$