

DESCRIPTIVE STATISTICS

Individual data

Aggregated data

MEAN

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^k x_i f(x_i)}{\sum_{i=1}^k f(x_i)}$$

$k = \# \text{ class}$
 $i = 1, \dots, k$
 $f(x_i) = \text{absolute freq.}$

VARIANCE

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^k (x_i - \bar{x})^2 f(x_i)$$

$$s^2 = \frac{\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i\right)^2}{n}}{n-1}$$

$$s^2 = \frac{\sum_{i=1}^k x_i^2 f(x_i) - \frac{\left(\sum_{i=1}^k x_i f(x_i)\right)^2}{n}}{n-1}$$

COEFFICIENT OF VARIATION

$$CV = \frac{s}{\bar{x}} \cdot 100$$

PROBABILITY

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A)P(B|A)$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

DIAGNOSTIC TEST

$$\mathbf{Sn} = \Pr(T+ | D+)$$

$$\mathbf{Sp} = \Pr(T- | D-)$$

BAYES THEOREM

$$PPV = \frac{P(D+)P(T+|D+)}{P(D+)P(T+|D+) + P(D-)P(T+|D-)} \quad NPV = \frac{P(D-)P(T-|D-)}{P(D-)P(T-|D-) + P(D+)P(T-|D+)}$$

BINOMIAL DISTRIBUTION

$$\begin{aligned} P(x|n) &= \frac{n!}{x!(n-x)!} \cdot \pi^x \cdot (1-\pi)^{n-x} = \\ &= \binom{n}{x} \cdot \pi^x \cdot (1-\pi)^{n-x} \end{aligned}$$

GAUSSIAN DISTRIBUTION

Given $X \sim N(\mu, \sigma^2)$, can be standardised in $Z \sim N(0, 1)$ by:

$$Z = \frac{(x - \mu)}{\sigma}$$

SAMPLE SIZE

...needed to get a confidence interval $(1-\alpha)$ of a proportion with length $2E$

$$n = \frac{z_{\alpha/2}^2 p(1-p)}{E^2}$$

...needed to get a confidence interval $(1-\alpha)$ of a mean with length $2E$

$$n = \left[\frac{z_{\alpha/2} \sigma}{E} \right]^2$$

...needed to compare two means

$$n = 2(z_{\alpha/2} + z_{\beta})^2 \frac{\sigma^2}{\Delta^2}$$

TEST t
one sample

$$\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 \end{cases} \quad t_{n-1} = \frac{\bar{x} - \mu_0}{ES(\bar{x})} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \quad IC : \bar{x} \pm t_{n-1, \alpha/2} \frac{s}{\sqrt{n}}$$

two samples

$$\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 \neq \mu_2 \end{cases}$$

Assuming equal variances

$$t_{n_1+n_2-2} = \frac{\bar{x}_1 - \bar{x}_2}{ES(\bar{x}_1 - \bar{x}_2)} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2 \left[\frac{1}{n_1} + \frac{1}{n_2} \right]}}$$

where $s_p = \sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}}$

$$IC : (\bar{x}_1 - \bar{x}_2) \pm t_{n_1+n_2-2, \alpha/2} \sqrt{s_p^2 \left[\frac{1}{n_1} + \frac{1}{n_2} \right]}$$

Not assuming equal variances

$$t_{\text{MIN}(n_1-1, n_2-1)} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$IC_{1-\alpha} : (\bar{x}_1 - \bar{x}_2) \pm t_{df, \alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$df = \text{MIN}(n_1 - 1, n_2 - 1)$$

TEST on PROPORTION

one sample

$$\begin{cases} H_0 : \pi = \pi_0 \\ H_1 : \pi \neq \pi_0 \end{cases}$$

$$z = \frac{p - \pi_0}{ES(p)} = \frac{p - \pi_0}{\sqrt{\pi_0(1 - \pi_0)/n}}$$

$$IC : p \pm z_{\alpha/2} \sqrt{p(1 - p)/n}$$

two samples

$$\begin{cases} H_0 : \pi_1 = \pi_2 \\ H_1 : \pi_1 \neq \pi_2 \end{cases}$$

$$z = \frac{p_1 - p_2}{\sqrt{\frac{p(1 - p)}{n_1} + \frac{p(1 - p)}{n_2}}}$$

$$p = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2} = \frac{f_1 + f_2}{n_1 + n_2}$$

$$\text{C.I.95\%: } p_1 - p_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}}$$