$$\frac{dN}{dt} = 0 \Rightarrow \frac{\partial n}{\partial t} + \underline{\nabla} \cdot (n \underline{u}) = 0$$

$n(\underline{x}, t)$
$\underline{u}(\underline{x}, t)$

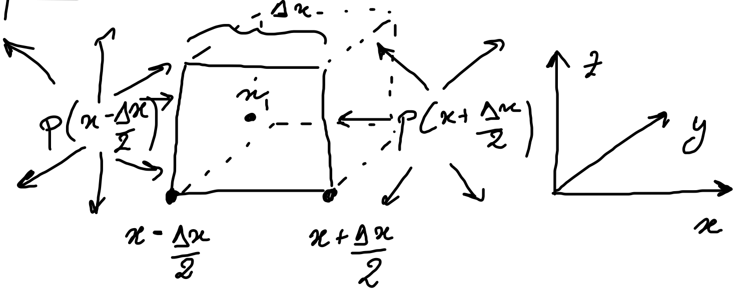
$$\underbrace{\frac{d}{dt}\left(\text{momento lineare}\right)}_{} = \sum \text{Forze}$$

$$mn \frac{d\underline{u}}{dt} = nq(\underline{E} + \underline{u} \times \underline{B}) \overbrace{- m_e \bar{\nu}_{ei} (\underline{u}_i - \underline{u}_e) n_e}^{\text{collisioni su ioni}} + (\text{effetti di pressione})$$

$$\underline{F}_e = - m_e \bar{\nu}_{ei} (\underline{u}_e - \underline{u}_i) n_e$$

$$\underline{F}_i = - \underline{F}_e$$

Effetti di pressione



$$F_x = p\left(x - \frac{\Delta x}{2}\right)\Delta y \Delta z - p\left(x + \frac{\Delta x}{2}\right)\Delta y \Delta z$$

$$\simeq \left(\cancel{p(x)} - \frac{\partial p}{\partial x}\frac{\Delta x}{2} - \cancel{p(x)} - \frac{\partial p}{\partial x}\frac{\Delta x}{2}\right)\Delta y \Delta z = -\frac{\partial p}{\partial x}\Delta x \Delta y \Delta z =$$

$$= -\frac{\partial p}{\partial x}\Delta V$$

$$F_y = -\frac{\partial p}{\partial y}\Delta V \qquad F_z = -\frac{\partial p}{\partial z}\Delta V$$

$$\underline{F} = \left(-\underline{\nabla} p\right)\Delta V$$

Ioni

$$m_i n_i \frac{d\underline{u}_i}{dt} = q n_i (\underline{E} + \underline{u}_i \times \underline{B}) - \underline{\nabla} p_i - m_e \nu_{ei} (\underline{u}_i - \underline{u}_e) n_e$$

$$\frac{\partial n_i}{\partial t} + \underline{\nabla} \cdot (n_i \underline{u}_i) = 0$$

$$n_\alpha \quad \underline{u}_\alpha \quad p_\alpha$$

Elettroni

$$m_e n_e \frac{d\underline{u}_e}{dt} = q n_e (\underline{E} + \underline{u}_e \times \underline{B}) - \underline{\nabla} p_e - m_e \nu_{ei} (\underline{u}_e - \underline{u}_i) n_e$$

$$\frac{\partial n_e}{\partial t} + \underline{\nabla} \cdot (n_e \underline{u}_e) = 0$$

$$\rho = \sum_\alpha n_\alpha q_\alpha \qquad \underline{j} = \sum_\alpha \underbrace{n_\alpha q_\alpha \underline{u}_\alpha}_{\underline{j}_\alpha} \longrightarrow \underline{\nabla} \cdot \underline{E} = \rho \qquad \underline{\nabla} \cdot \underline{B} = 0$$

$$\underline{\nabla} \times \underline{E} = -\partial \underline{B}/\partial t \qquad \underline{\nabla} \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \partial \underline{E}/\partial t$$

Per la chiusura aggiungiamo un'equazione di stato

$$p = (\text{const}) \cdot \rho_m^{\gamma} \qquad \rho_m = m n$$

$$\gamma = \frac{C_p}{C_V}$$

$$\underline{\nabla} p = (\text{const})\, \gamma\, \rho_m^{\gamma-1}\, \underline{\nabla} \rho_m = \frac{(\text{const})\, \gamma\, \rho_m^{\gamma}\, \underline{\nabla} \rho_m}{\rho_m}$$

$$\boxed{\frac{\underline{\nabla} p}{p} = \gamma \frac{\underline{\nabla} n}{n}}$$

Trasformazione isoterma $\quad pV = \text{const} \qquad T = \text{const} \qquad pV = N k_B T$

$$\frac{\underline{\nabla} p}{p} = \frac{\underline{\nabla} n}{n} \;\Leftarrow\; \gamma = 1 \qquad \underline{\nabla} p = (\underline{\nabla} n)\, k_B T = p \frac{\underline{\nabla} n}{n} \qquad p = n k_B T$$

Trasformazione adiabatica

$$dU = \delta Q + \delta L \qquad \text{II pr. termodinamica}$$

$$\delta Q = 0$$

$$dU = \delta L \qquad \text{Se il sistema fa lavoro } dU < 0$$

in media per part. f = # gradi di libertà

$$dU = -p\,dV \qquad U = f\left(\frac{1}{2} k_B T\right) N$$

gas ideale

$$\frac{f}{2} k_B N\, dT = -p\,dV \qquad pV = N k_B T$$

$$dT = \frac{d(pV)}{N k_B}$$

$$\frac{f}{2} \cancel{k_B N} \frac{d(pV)}{\cancel{N k_B}} = -p\,dV$$

$$N = nV$$

$$\frac{f}{2}\left[dp\,V + p\,dV\right] = -p\,dV; \qquad dN = 0 \Rightarrow d(nV) = 0; \quad dn\,V + n\,dV = 0$$

$$dp\,V = -p\,dV\left(1 + f/2\right) \cdot 2/f \qquad \frac{dn}{n} = -\frac{dV}{V}$$

$$d_p V = -p\,dV \quad \frac{2+f}{f} \xrightarrow{def} \gamma$$

$$\frac{dn}{n} = -\frac{dV}{V}$$

$$\frac{dp}{p} = -\gamma \frac{dV}{V} \; ; \qquad \frac{dp}{p} = \gamma \frac{dn}{n}$$

$$\frac{\underline{\nabla} p}{p} = \gamma \frac{\underline{\nabla} n}{n}$$

adiabatica

$$\gamma = \frac{2+f}{f}$$

isoterma

$$\gamma = 1$$

Velocità diamagnetica (plasma non collisionale)

$$mn \frac{d\underline{u}}{dt} = qn(\underline{E} + \underline{u} \times \underline{B}) - \underline{\nabla} p$$

$$\hookrightarrow \left(\frac{\partial \underline{u}}{\partial t}(\underline{x},t) + (\underline{u}\cdot\underline{\nabla})\,\underline{u}(\underline{x},t)\right)$$

$\underline{B} = \overrightarrow{const}$

Caso stazionario: $\frac{d\underline{u}}{dt} = 0$

$\underline{u} = \overrightarrow{const}$

$$0 = qn(\underline{E} + \underline{u} \times \underline{B}) - \underline{\nabla} p$$

$\times \underline{B}$ $\qquad \times \underline{B}$

$$0 = qn\,\underline{E}\times\underline{B} + qn\,(\underline{u}_\perp \times \underline{B})\times\underline{B} - \underline{\nabla}p \times \underline{B}$$

$\underline{C}$ $\underline{B}$ $\underline{A}$

$$(\underline{C}\times\underline{B})\times\underline{A} = (\underline{A}\cdot\underline{C})\underline{B} - (\underline{A}\cdot\underline{B})\underline{C}$$

$$0 = qn\,\underline{E}\times\underline{B} + qn\,(0 - B^2\underline{u}_\perp) - \underline{\nabla}p\times\underline{B}$$

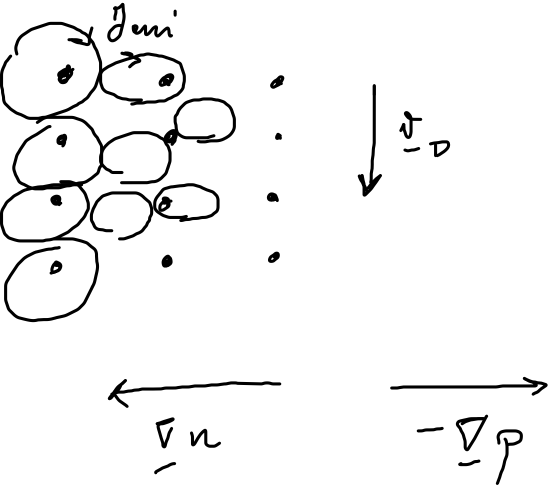
deriva diamagnetica

$$\underline{u}_\perp = \underbrace{\frac{\underline{E}\times\underline{B}}{B^2}}_{\text{deriva anche per particella singola } \propto \underline{E}\times\underline{B}} - \frac{\underline{\nabla}p\times\underline{B}}{qnB^2}$$

$$p = nTk_B$$

Supponiamo $T = \text{const}$

$$\underline{\nabla}p \propto \underline{\nabla}n$$



$(-\underline{\nabla} p \times \underline{B})$ verso il basso