a : dimensioni del sistema

Plasmi compl. ionizzati

$u \sim v_{th_i}$ $\tau \sim \frac{a}{v_{th_i}}$

$T \sim keV$ $v_{th_i} \sim$ qualche 10^5 m/s

$a \sim m$

$\tau \sim$ qualche μs

Plasmi parz. ionizzati

$T \sim eV$ $\tau \sim$ ms o frazione ms

$B \sim 1T$

$\tau_e \sim \frac{a}{v_{th_e}} \sim \frac{1}{40} \tau_i \Rightarrow m_e \simeq 0$

$\omega_c = \frac{qB}{m}$ $\nu_c = \frac{q_i B}{2\pi m_i} \sim \frac{1.6 \cdot 10^{-19} \cdot 1}{6 \cdot 1.67 \cdot 10^{-27}} \sim 10^7$ Hz $\sim$ decine di MHz

$v_{th_i} << c$

$\tau_c \sim$ frazione del μs

$\underline{\nabla} \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \partial \underline{E} / \partial t$

$a >> \lambda_D \Rightarrow Z n_i \simeq n_e$

$$\frac{\varepsilon_0 \mu_0 \left\| \partial \underline{E} / \partial t \right\|}{\left\| \underline{\nabla} \times \underline{B} \right\|} \sim \frac{\frac{1}{c^2} \frac{E}{\cancel{a}} v_{th_i}}{B / \cancel{a}} = \frac{1}{c^2} \overset{\simeq c}{\left(\frac{E}{B} \right)} v_{th_i} \sim \frac{v_{th_i}}{c} << 1 \rightarrow \underline{\nabla} \times \underline{B} = \mu_0 \underline{j}$$

$$\frac{\partial n_e}{\partial t} + \underline{\nabla}\cdot(n_e \underline{u}_e) = 0$$

$$\frac{\partial n_i}{\partial t} + \underline{\nabla}\cdot(n_i \underline{u}_i) = 0$$

$$0 = -e n_e (\underline{E} + \underline{u}_e \times \underline{B}) - \underline{\nabla} p_e - m_e n_e \overline{\nu}_{ei} (\underline{u}_e - \underline{u}_i)$$

$$m_i n_i \left(\frac{\partial}{\partial t}\underline{u}_i + (\underline{u}_i\cdot\underline{\nabla})\underline{u}_i\right) = Z e n_i (\underline{E} + \underline{u}_i \times \underline{B}) - \underline{\nabla} p_i + m_e n_e \overline{\nu}_{ei} (\underline{u}_e - \underline{u}_i)$$

$$\underline{\nabla}\times\underline{E} = -\partial \underline{B}/\partial t \qquad \underline{\nabla}\times\underline{B} = \mu_0 \underbrace{e n_e (\underline{u}_i - \underline{u}_e)}_{\underline{j}}$$

$$\underline{\nabla}\cdot\underline{B} = 0 \qquad (\underline{\nabla}\cdot\underline{E} = 0) \Rightarrow Z_i n_i = n_e$$

Densità di massa: $\rho = n_i m_i + n_e m_e \approx n_i m_i$

$$\underline{u} = \underbrace{\frac{\underline{E} \times \underline{B}}{B^2}}_{u_{E\times B}} - \cancel{\underbrace{\frac{\nabla p \times \underline{B}}{q n B^2}}_{u_{\nabla p}}}$$

MHD

magneto hydro dynamics

$$\xrightarrow{} p = nT$$

$$\frac{u_{\nabla p}}{u_{E\times B}} \sim \frac{\frac{p}{a} B}{q n \cancel{B^2}} \frac{\cancel{B^2}}{E B} \sim \frac{\cancel{n} T \cancel{B}}{a q \cancel{n} E \cancel{B}}$$

Legge di Faraday

$$\underline{\nabla} \times \underline{E} = -\partial \underline{B}/\partial t$$

$$E/a \sim B/\tau \qquad \tau \sim \frac{a}{v_{th}}$$

$$\sim \frac{T}{a q \frac{B a}{\tau}} \sim \frac{m v_{th}^2 \tau}{a^2 q B} \qquad v_{th}^2 \sim \frac{T}{m}$$

$$\underline{u} = \frac{\underline{E} \times \underline{B}}{B^2}$$

Stesso per ioni ed elettroni

$$\sim T_L \frac{v_{th}^2}{a^2} \tau \sim \frac{T_L \cancel{a} v_{th}}{a^{\cancel{2}}} \sim \frac{T_L}{\tau} << 1$$

$$\underline{u} = \frac{m_e \underline{u}_e + m_i \underline{u}_i}{m_e + m_i} \simeq \frac{m_i \underline{u}_i}{m_i} \approx \underline{u}_i$$

$\rho, \underline{u}, \underline{j}, P$

$$\underline{j} = \underline{j}_e + \underline{j}_i = e n_e (\underline{u}_i - \underline{u}_e)$$

$\underline{j}$ "piccola" perché $\underline{u}_e \sim \underline{u}_i$

$$\underline{j} = -e n_e \underline{u}_e + Z n_i e \underline{u}_i =$$

$$\underline{u}_e = \underline{u}_i - \underline{j}/e n_e$$

$$\approx \underline{u} - \underline{j}/e n_e$$

$\underline{u}_i \sim \underline{u}$

$Z n_i = n_e$

$p = p_e + p_i$

$$\rho = \frac{m_e n_e + m_i n_i}{m_e + m_i}$$

$\sim m_i n_i$

Eq. continuità

$$\left.\begin{aligned} &\frac{\partial n_e}{\partial t} + \underline{\nabla}\cdot(n_e \underline{u}_e) = 0 \quad \times m_e \\ &\frac{\partial n_i}{\partial t} + \underline{\nabla}\cdot(n_i \underline{u}_i) = 0 \quad \times m_i \end{aligned}\right\} + \quad \frac{\partial}{\partial t}\rho + \underline{\nabla}\cdot(m_e n_e \underline{u}_e + m_i n_i \underline{u}_i)$$

$$\wr\wr$$

$$m_i n_i \underline{u}_i \sim \rho \underline{u}$$

$$\frac{\partial \rho}{\partial t} + \underline{\nabla}\cdot(\rho \underline{u}) = 0$$

$$\frac{\partial \rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{u}) = 0$$

$$\underbrace{\frac{\partial \rho}{\partial t} + (\underline{u}\cdot\underline{\nabla})\rho} + \rho\underline{\nabla}\cdot\underline{u} = 0$$

$$\frac{d}{dt}\rho + \rho\underline{\nabla}\cdot\underline{u} = 0$$ eq. continuità

Sottraggo eq. per el − Ze× eq. per ioni

$$\cancel{\frac{e\partial n_e}{\partial t}} - \cancel{\frac{Ze\partial n_i}{\partial t}} + \underline{\nabla}\cdot(\underbrace{e n_e\underline{u}_e - Z e n_i\underline{u}_i}_{\underline{j}}) = 0$$

$$n_e = Z n_i$$

$$\underline{\nabla}\cdot\underline{j} = 0$$

$$\underline{\nabla}\cdot[\underline{\nabla}\times\underline{B}] = \mu_0\underline{\nabla}\cdot\underline{j} \Rightarrow \underline{\nabla}\cdot\underline{j} = 0$$

$$0 \simeq -en_e(\underline{E}+\underline{u}_e\times\underline{B}) - \underline{\nabla}p_e - \cancel{m_e n\nu_{ei}(\underline{u}_i-\underline{u}_e)}$$

$$m_i n_i \frac{d\underline{u}_i}{dt} = Zen_i(\underline{E}+\underline{u}_i\times\underline{B}) - \underline{\nabla}p_i + \cancel{m_e n\nu_{ei}(\underline{u}_i-\underline{u}_e)}$$

$+$

$$\underset{\rho}{\parallel} \qquad \frac{d\underline{u}_i}{dt} \simeq \underline{u}$$

$$\boxed{\rho\frac{d\underline{u}}{dt} = \underline{j}\times\underline{B} - \underline{\nabla}p}$$

all'equilibrio

$$\underline{j}\times\underline{B} = \underline{\nabla}p$$

Se esiste un equilibrio $\quad jB \simeq p/a \Rightarrow j \sim \frac{p}{aB} \sim \frac{nT}{aB} \sim \frac{n m v_{thi}^2}{aB}$

$$p = nT$$

$$j = j_e + j_i$$

$$\frac{j}{j_i} \sim \frac{j}{Zenv_{thi}} \sim \frac{\cancel{n} m v_{thi}^{\cancel{2}}}{aB}\frac{1}{Ze\cancel{n}\cancel{v_{thi}}} \sim \frac{r_{Li}}{a} \ll 1$$

Considero cons. momento per elettroni $\underline{u}_e = \underline{u} - \underline{j}/en_e$

$$0 \simeq -en_e(\underline{E} + \underline{u}_e \times \underline{B}) - \underline{\nabla} p_e - m_e n_e \bar{\nu}_{ei}(\underline{u}_e - \underline{u}_i)$$

$$0 \simeq -en_e\left(\underline{E} + \left(\underline{u} - \frac{\underline{j}}{en_e}\right) \times \underline{B}\right) - \underline{\nabla} p_e - \underbrace{m_e n_e \bar{\nu}_{ei}}(\underline{u}_e - \underline{u}_i)$$

$$\underline{E} + \underline{u} \times \underline{B} \simeq \frac{1}{en_e}(\underline{j} \times \underline{B} - \underline{\nabla} p_e) + \eta \underline{j} \qquad \eta = \frac{m_e \bar{\nu}_{ei}}{n_e e^2}$$

resistività

c. el. nel lab.

c. el.

$$\underbrace{\underline{E} + \underline{u} \times \underline{B}}$$

c. el. nel S.R. del plasma

Trasf. di Lorentz per $\underline{E}$ e $\underline{B}$

$\underline{v}$ vel. del S.R. in movimento

$\parallel, \perp$ sono rispetto a $\underline{v}$

$'$: misurato nel S.R. che si muove con $\underline{v}$

$$\underline{E}'_{\parallel} = \underline{E}_{\parallel} \qquad \underline{B}'_{\parallel} = \underline{B}$$

$$\underline{E}'_{\perp} = \gamma\left(\underline{E}_{\perp} + \underline{v} \times \underline{B}\right) \qquad \underline{B}'_{\perp} = \gamma\left(\underline{B}_{\perp} - \frac{1}{c^2}\underline{v} \times \underline{E}\right)$$

Caso Classico $\frac{v}{c} \ll 1$ $\quad \gamma \simeq 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\underline{E}_{plasma} = \underline{E}_{\parallel pl} + \underline{E}_{\perp pl} = \underline{E}_{\parallel} + \underline{E}_{\perp} + \underline{u} \times \underline{B} = \underline{E} + \underline{u} \times \underline{B}$$

Se $\underline{j} \times \underline{B}$ e $\underline{\nabla} p$ sono trascurabili

$\underline{E}_{pl} = \eta \underline{j}$ legge di Ohm