$\rho \simeq m_i n_i \qquad \underline{u} \qquad |\underline{u}| \sim v_{th_i} \qquad \underline{j} = n e (\underline{u}_i - \underline{u}_e)$

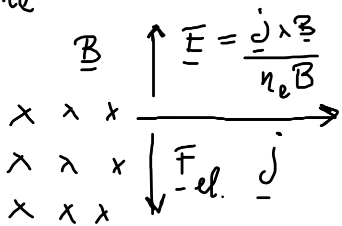
$\rho = \rho_e + \rho_i$

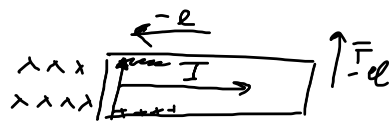
$\frac{d}{dt}\rho + \rho \underline{\nabla} \cdot \underline{u} = 0$ eq. continuità

$\rho \frac{d\underline{u}}{dt} = \underline{j} \times \underline{B} - \underline{\nabla} p$ eq. bilancio delle forze

$$\underbrace{\underline{E} + \underline{u} \times \underline{B}}_{\underline{E} \text{ nel S.R. plasma}} = \frac{1}{e n_e}\left(\underline{j} \times \underline{B} - \underline{\nabla} p\right) + \eta \underline{j}$$

$\eta = \frac{m_e \nu_{ei}}{n_e e^2}$ resistività





Termine Hall

$$\frac{\cancel{j B}}{\frac{e n e_i a B}{\cancel{B}}} \sim \frac{\cancel{n_i} m v_{th_i}^2}{a B} \frac{1}{e_i \cancel{n}\, \cancel{v_{th_i}}}$$

$$\| \underline{u} \times \underline{B} \| \sim \frac{r_{L_i}}{a} \ll 1$$

$$\left\| \frac{\frac{\nabla p_e}{e n_e}}{u B} \right\| \sim \frac{p_e}{e n_{ei} a\, u B} \sim \frac{j}{j_i} \sim \frac{r_{L_i}}{a} \ll 1$$

$$\underline{j} \propto \underline{u}_i - \underline{u}_e$$

$$j \approx \frac{n m v_{th_i}^2}{a B} \qquad \frac{j}{j_i} \sim \frac{r_{L_i}}{a} \ll 1$$

$$\sim \frac{n T}{a B} \sim \frac{p}{a B}$$

$\underline{E} + \underline{u} \times \underline{B} = \underline{0}$ se collisioni sono trascurabili

MHD ideale

$\underline{E} + \underline{u} \times \underline{B} = \eta \underline{j} \Rightarrow E_{\parallel} = \eta j_{\parallel} \neq 0$

MHD resistiva

$\underline{E}_{\parallel} + \underline{E}_{\perp} + (\underline{u} \times \underline{B})_{\perp} = 0 \rightarrow E_{\parallel} = 0$

Legge di stato

$$\frac{p}{\rho^\gamma} = \text{cost} \qquad \frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0$$

$\gamma = 1$ isoterma

$\gamma \neq 1 = \frac{2+f}{f}$ adiabatica

$\rho, \underline{j}, \underline{u}, p$

$\underline{E}, \underline{B}$

$$\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0$$ eq. continuità

$$\rho \, d\underline{u}/dt = \underline{j} \times \underline{B} - \underline{\nabla} p$$ eq. conserv. momento lineare

$$\underline{E} + \underline{u} \times \underline{B} = 0$$ Legge di Ohm (MHD ideale)

$$\underline{E} + \underline{u} \times \underline{B} = \eta \underline{j}$$ " " " (MHD resistiva)

$$\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0$$

$Z n_i = n_e$

$\underline{\nabla} \times \underline{E} = -\partial \underline{B}/\partial t$

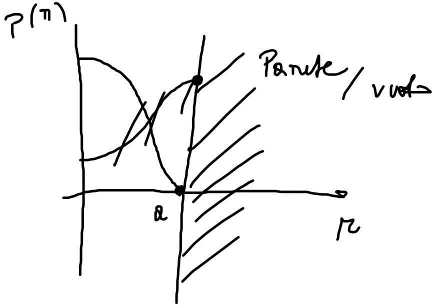
$\underline{\nabla} \cdot \underline{B} = \underline{0}$

$\underline{\nabla} \times \underline{B} = \mu_0 \underline{j}$

Equilibri MHD statici $\quad \underline{u} = \underline{0}$

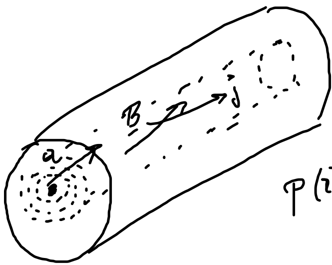
$$\frac{d}{dt} = 0 \qquad \underline{\nabla} p = \underline{j} \times \underline{B} \qquad \underline{\nabla} \times \underline{B} = \mu_0 \underline{j}$$

$$\underline{\nabla} \cdot \underline{B} = \underline{0}$$

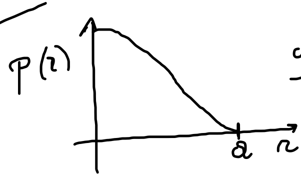


$$\underline{\nabla} p \cdot \underline{B} = (\underline{j} \times \underline{B}) \cdot \underline{B} = 0$$

$\underline{B} \perp \underline{\nabla} p \Rightarrow$ Linee di $\underline{B}$ si trovano nella regione dove p è costante (sulle superfici di livello di p)



$$\underline{\nabla} p \cdot \underline{j} = (\underline{j} \times \underline{B}) \cdot \underline{j} = 0$$



$\underline{j} \perp \underline{\nabla} p \Rightarrow \underline{j}$ è sulle sup. di livello di p

Dal teorema di Ampere

$$\underline{j} = \frac{1}{\mu_0}(\underline{\nabla}\times\underline{B})$$

$$\underline{\nabla} p = \frac{1}{\mu_0}(\underline{\nabla}\times\underline{B})\times\underline{B}$$

$$\underline{\nabla}(B^2) = 2\,\underline{B}\times(\underline{\nabla}\times\underline{B}) + 2(\underline{B}\cdot\underline{\nabla})\underline{B}$$

$$\underline{B}\times(\underline{\nabla}\times\underline{B}) = \underline{\nabla}\left(\frac{B^2}{2}\right) - (\underline{B}\cdot\underline{\nabla})\underline{B}$$

$s \quad \underline{B} \qquad \hat{\underline{b}} = \underline{B}/B$

$$\underline{\nabla} p = -\underline{\nabla}\left(\frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0}(\underline{B}\cdot\underline{\nabla})\underline{B}$$

$$\underline{\nabla} p = \underline{\nabla}_\perp p$$

$$\underline{\nabla}\left(\frac{B^2}{2\mu_0}\right) = \underbrace{\frac{\partial}{\partial s}\left(\frac{B^2}{2\mu_0}\right)\hat{\underline{b}}}_{\underline{\nabla}_\parallel} + \underline{\nabla}_\perp\left(\frac{B^2}{2\mu_0}\right)$$

$$(\underline{B}\cdot\underline{\nabla})\underline{B} = (B\underbrace{\hat{\underline{b}}\cdot\underline{\nabla}}_{\frac{\partial}{\partial s}})(B\hat{\underline{b}}) = B\frac{\partial}{\partial s}(B\hat{\underline{b}}) = B\frac{\partial B}{\partial s}\hat{\underline{b}} + B^2(\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}}$$

$$= \frac{\partial}{\partial s}\left(\frac{B^2}{2}\right)\hat{\underline{b}} + B^2(\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}}$$

$$\nabla_\perp p = \cancel{-\frac{\partial}{\partial s}\left(\frac{B^2}{2\mu_0}\right)\hat{b}} - \nabla_\perp\left(\frac{B^2}{2\mu_0}\right) + \cancel{\frac{\partial}{\partial s}\left(\frac{B^2}{2\mu_0}\right)\hat{b}} + \frac{B^2}{\mu_0}(\hat{b}\cdot\nabla)\hat{b}$$

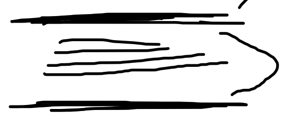
$$\hat{\underline{k}} \overset{def}{=} (\hat{\underline{b}}\cdot\underline{\nabla})\hat{\underline{b}}$$

$$\nabla_\perp\left(p + \frac{B^2}{2\mu_0}\right) - \hat{\underline{k}}\,\frac{B^2}{\mu_0} = 0$$

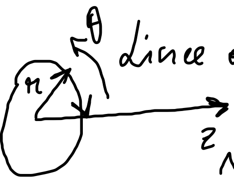
pressione magnetica

se $\hat{\underline{k}} = 0$

$$\nabla_\perp p = -\nabla\frac{B^2}{2\mu_0}$$



$p(r) \quad \downarrow \nabla p \quad \uparrow \nabla\frac{B^2}{2\mu_0}$



Linee di campo curve

$$\hat{\underline{b}} = \hat{\underline{e}}_\theta$$

$$\hat{\underline{k}} = (\hat{\underline{b}}\cdot\underline{\nabla})\,\hat{\underline{e}}_\theta = \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\underline{e}}_\theta = \frac{1}{r}\hat{\underline{e}}_r \neq 0$$

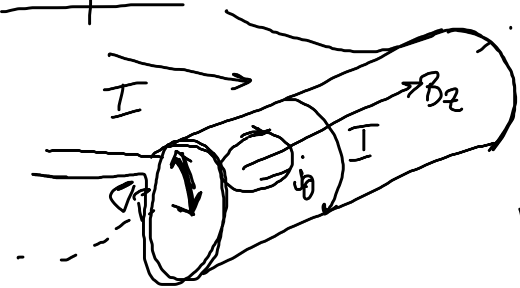
$$\hat{\underline{e}}_r = \cos\theta\,\hat{\underline{i}} + \text{sen}\,\theta\,\hat{\underline{j}}$$

$$\hat{\underline{e}}_\theta = -\text{sen}\,\theta\,\hat{\underline{i}} + \cos\theta\,\hat{\underline{j}}$$

se $\hat{\underline{b}} = \hat{\underline{e}}_r$ $\qquad \hat{\underline{k}} = \frac{\partial}{\partial r}(\hat{\underline{e}}_r) = 0$

se $\hat{\underline{b}} = \hat{\underline{e}}_z$ $\qquad \hat{\underline{k}} = \frac{\partial}{\partial z}(\hat{\underline{e}}_z) = 0$

θ pinch



: Configurazione transiente

$\underline{j}_\theta \quad \underline{B}_z$

Non c'è curvatura delle linee di campo $\Rightarrow \underline{\kappa} = \underline{0}$

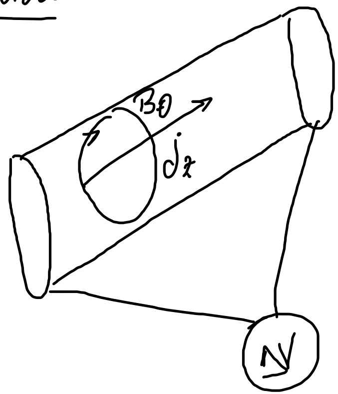
$$\underline{\nabla}_\perp \left(p + \frac{B^2}{2\mu_0} \right) = 0$$

$$p + \frac{B^2}{2\mu_0} = \text{const} = \frac{B_0^2}{2\mu_0}$$

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{j} \Rightarrow -\frac{\partial B_z}{\partial r} = \mu_0 j_\theta \Rightarrow j_\theta = -\frac{\partial B_\theta}{\partial r}$$

$$\beta = \frac{\text{pressione fluido}}{\text{pressione magnetica}} = \frac{p \leq B_0^2/2\mu_0}{B_0^2/2\mu_0} \leq 1$$

Z pinch



$$\nabla_\perp\left(p+\frac{B^2}{2\mu_0}\right) - \hat{\underline{k}}\,\frac{B^2}{2\mu_0} = 0$$

$$\nabla_\perp = \hat{e}_r\frac{\partial}{\partial r} + \cancel{\hat{e}_z\frac{\partial}{\partial z}}$$

$$\hat{\underline{k}} = \left(\underset{\hat{e}_\theta}{\hat{\underline{b}}}\cdot\nabla\right)\hat{\underline{b}} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(\hat{e}_\theta\right) = -\hat{e}_r/r$$

$$\hat{e}_r\left[\left(\frac{\partial p}{\partial r}+\frac{1}{2\mu_0}\frac{\partial}{\partial r}B_\theta^2(r)\right)+\frac{B_\theta^2}{\mu_0 r}\right]=0$$

$$\nabla\times \underline{B}=\mu_0 \underline{j} \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(rB_\theta\right)=\mu_0 j \qquad rB_\theta(r)=\mu_0\int_0^r dr'\, j(r')\, r'$$

$$\frac{\partial}{\partial r}\left(rB_\theta\right)=\mu_0 j(r)\, r$$

Scelta: $j(r')=j_0$

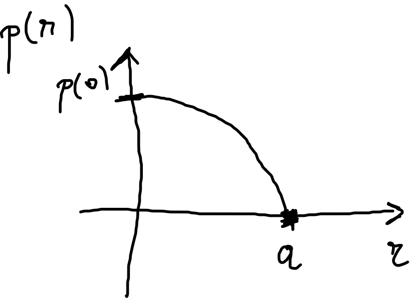
$$rB_\theta(r)=\mu_0 j_0\int_0^r dr' r' = \mu_0 j_0 r^2/2 \Rightarrow B_\theta(r)=\frac{\mu_0 j_0 r}{2}$$

$$\frac{\partial p}{\partial r} + \frac{\partial}{\partial r}\frac{\mu_0^{\cancel{2}}}{4}\frac{j_0^2 r^2}{2\cancel{\mu_0}} + \frac{1}{\cancel{\mu_0} r}\frac{\mu_0^{\cancel{2}} j_0^2 r^{\cancel{2}}}{4} = 0$$

$$\frac{\partial p}{\partial r} + \frac{\mu_0}{\cancel{8}_{4}} j_0^2 \cancel{2}^{1} r + \frac{\mu_0}{4} j_0^2 r = 0$$

$$\frac{\partial p}{\partial r} \overset{4}{=} -\frac{\mu_0}{2} j_0^2 r$$

$$p(r) - \underbrace{p(a)}_{=0} = -\frac{\mu_0}{2} j_0^2 \int_a^r dr\, r = \frac{\mu_0}{4} j_0^2 (a^2 - r^2)$$



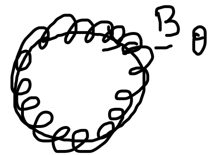
$$p \propto j^2$$

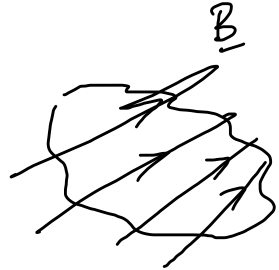
$$p(0) = \frac{\mu_0 j_0^2 a^2}{4}$$

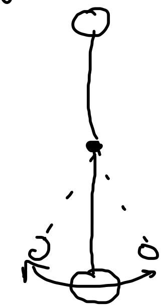
$$\beta = \frac{p(0)}{\left(\frac{\mu_0 j_0 a}{2}\right)^2 \frac{1}{2\mu_0}} = \frac{\cancel{\mu_0 j_0^2 a^2}}{\cancel{4}} \cdot \frac{2\cancel{\mu_0}\cdot\cancel{4}}{\cancel{\mu_0^2 j_0^2 a^2}} = 2$$

Proprietà dello z pinch:

$$\begin{cases} \beta = 2 \\ \text{stazionario} \\ \text{senza perdite agli estremi} \end{cases}$$







$$\underline{E} + \underline{u} \times \underline{B} = 0$$

Instabilità ideali

$$\underline{E} + \underline{u} \times \underline{B} = \eta \underline{j}$$

Instabilità <u>non</u> ideali