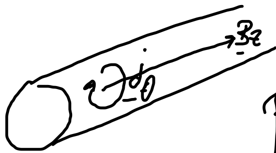
$$\nabla p = \underline{j} \times \underline{B}$$

$$\Rightarrow \nabla_\perp \left(p + \frac{B^2}{2\mu_0} \right) - \frac{B^2}{\mu_0} \hat{\underline{k}} = 0 \qquad \hat{\underline{k}} = (\hat{\underline{b}} \cdot \nabla) \hat{\underline{b}}$$

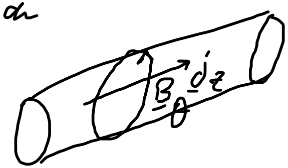
θ - pinch



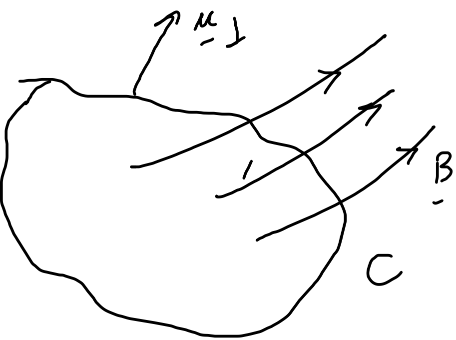
$\beta \le 1$

$$\beta = \frac{p}{B^2/2\mu_0}$$

z - pinch



$\beta = 2$



$$\phi = \int \underline{B} \cdot d\underline{S}$$

$S(t)$: sup. racchiusa da C

$$\frac{d\phi}{dt} = ?$$

$$\phi(t+dt) = \int_{S(t+dt)} \underline{B}(t+dt) \cdot d\underline{S}$$

$$\Phi(t+dt) = \int_{S(t+dt)} \underline{B}(t+dt)\cdot d\underline{S}$$

Esp. di Taylor

$$\simeq \int_{S(t+dt)} \left(\underline{B}(t) + \frac{\partial \underline{B}}{\partial t}\, dt\right)\cdot d\underline{S} =$$

$$S(t+dt) = S(t) + \Delta S$$

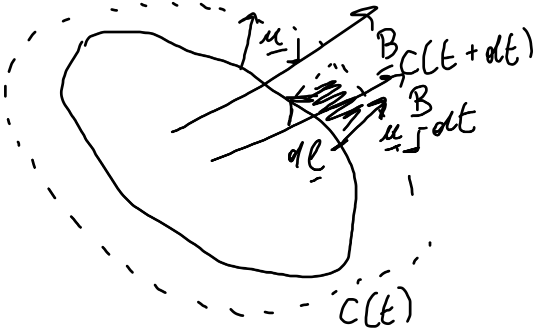
$$= \int_{S} \left(\underline{B}(t) + \frac{\partial \underline{B}}{\partial t} dt\right)\cdot d\underline{S} + \int_{\Delta S} \left[\underline{B}(t) + \cancel{\frac{\partial \underline{B}}{\partial t} dt}\right]\cdot d\underline{S}$$

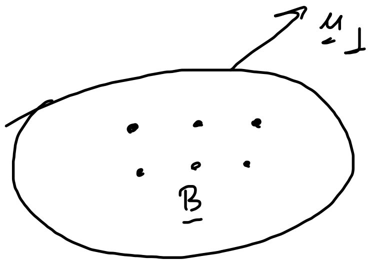
trascuro

$$= \phi(t) + dt \int_{S} \frac{\partial \underline{B}}{\partial t}\cdot d\underline{S} + \int_{\Delta S} \underline{B}(t)\cdot d\underline{S}$$

$$\frac{d\phi}{dt} = \frac{1}{dt}\left(\phi(t+dt) - \phi(t)\right) =$$

$$= \int_{S} \frac{\partial \underline{B}}{\partial t}\cdot d\underline{S} + \int_{\Delta S} \underline{B}(t)\cdot \frac{d\underline{S}}{dt}$$





$$d(\Delta \underline{S}) = \underline{u}_\perp \, dt \times d\underline{l}$$

$$\frac{\int_C \underline{B}(t)\cdot(\underline{u}_\perp \times d\underline{l})\,\cancel{dt}}{\cancel{dt}} =$$

$$= \int_C \underline{B}(t)\cdot(\underline{u}_\perp \times d\underline{l})$$

$$\frac{d\Phi}{dt} = \int_S \frac{\partial \underline{B}}{\partial t}\cdot d\underline{S} + \int_C \underline{B}(t)\cdot(\underline{u}_\perp \times d\underline{l})$$

Legge di Faraday

$$\partial \underline{B}/\partial t = -\underline{\nabla} \times \underline{E}$$

Legge di Ohm
MHD ideale

$$\underline{E} + \underline{u}_\perp \times \underline{B} = 0 \Rightarrow \partial \underline{B}/\partial t = \underline{\nabla} \times (\underline{u}_\perp \times \underline{B})$$

$$\frac{d\Phi}{dt} = \int_S \nabla \times (\mathbf{u}_\perp \times \mathbf{B}) \cdot d\mathbf{S} + \int_C \mathbf{B} \cdot (\mathbf{u}_\perp \times d\mathbf{l})$$

$$= \int_C (\mathbf{u}_\perp \times \mathbf{B}) \cdot d\mathbf{l} + \int_C \mathbf{B} \cdot (\mathbf{u}_\perp \times d\mathbf{l})$$

m Stokes

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$$

$$(\mathbf{u}_\perp \times \mathbf{B}) \cdot d\mathbf{l} = \mathbf{B} \cdot (d\mathbf{l} \times \mathbf{u}_\perp) = -\mathbf{B} \cdot (\mathbf{u}_\perp \times d\mathbf{l})$$

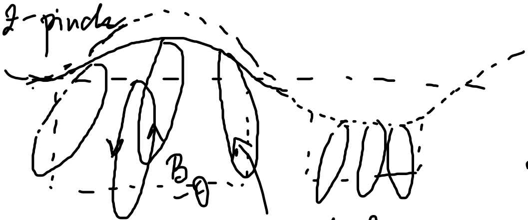
$$\frac{d\Phi}{dt} = \cancel{-\int_C \mathbf{B} \cdot (\mathbf{u}_\perp \times d\mathbf{l})} + \int_C \mathbf{B} \cdot \cancel{(\mathbf{u}_\perp \times d\mathbf{l})} = 0$$

Perturbiamo θ-pinch e z-pinch

schiaccio le linee di $\underline{B}$ ⇒ aumenta la pressione magnetica

si schiacciano le linee di $\underline{B}$ → aumenta la pressione magnetica

θ-pinch stabile rispetto alle perturbazioni ideali



Rigonfiamento vs esterno

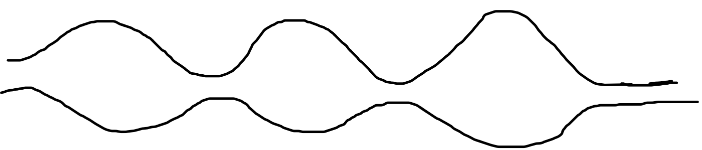
le linee di campo si allargano: meno pressione magnetica e curvatura

Rigonfiamento vs interno: le linee di $\underline{B}$ si addensano

più pressione magnetica

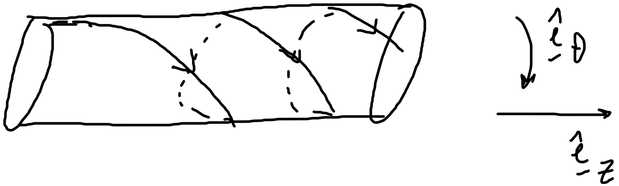
più curvatura

⇒ plasma più strizzato



Z pinch è instabile alle pert. ideali

Screw pinch: somma di uno z-pinch e θ-pinch $\quad \hat{\underline{k}} = (\hat{\underline{b}} \cdot \underline{\nabla}) \hat{\underline{b}}$



$$\underline{B} = B_\theta(r)\, \hat{\underline{e}}_\theta + B_z(r)\, \hat{\underline{e}}_z$$

$$\hat{\underline{b}} = \underline{B} / \|\underline{B}\| = \frac{1}{\sqrt{B_\theta^2 + B_z^2}} \left(B_\theta \hat{\underline{e}}_\theta + B_z \hat{\underline{e}}_z \right)$$

$$\underline{\nabla}_\perp \left(p + \frac{B^2}{2\mu_0} \right) - \frac{B^2}{\mu_0} \hat{\underline{k}} = 0$$

$$\hat{\underline{b}} \cdot \underline{\nabla} = \frac{1}{B} \left(\frac{B_\theta}{r} \frac{\partial}{\partial \theta} + B_z \frac{\partial}{\partial z} \right)$$

$$\hat{\underline{k}} = \frac{1}{B} \left(\frac{B_\theta}{r} \frac{\partial}{\partial \theta} + B_z \frac{\partial}{\partial z} \right) \left(\frac{1}{B} B_\theta \hat{\underline{e}}_\theta + B_z \hat{\underline{e}}_z \right) =$$

$$= \frac{1}{B^2} \frac{B_\theta^2}{r} \frac{\partial \hat{\underline{e}}_\theta}{\partial \theta} = - \frac{B_\theta^2}{B^2 r} \hat{\underline{e}}_r$$

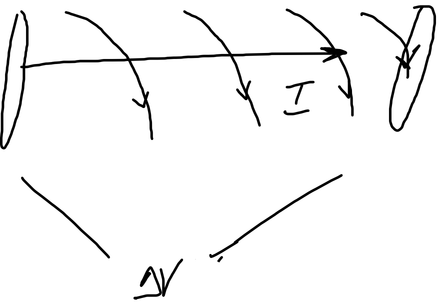
Componente radiale del $\underline{\nabla}$

$$\frac{d}{dr} \left(p(r) + \frac{1}{2\mu_0} B_\theta^2(r) + B_z^2(r) \right) - \frac{1}{\mu_0} \left(\cancel{B_\theta^2(r) + B_z^2(r)} \right) \cdot \left(- \frac{B_\theta^2}{\cancel{B^2} r} \right) = 0$$

$$\frac{d}{dr} \left(p(r) + \frac{1}{2\mu_0} (B_\theta^2 + B_z^2) \right) + \frac{B_\theta^2}{r \mu_0} = 0$$

B_z determinato dalle bobine esterne

Corrente nel plasma è controllato dagli elettrodi



$\Rightarrow B_\theta$

Vantaggi:

Sistema stabile (se B_z è sufficientemente alto)

" stazionario

Svantaggio:

perdita di particelle ai bordi

Come calcoliamo β?

Integro l'equilibrio sulla sezione (e divido per a^2)

$$\frac{1}{a^2}\int_0^a dr\, r^2 \left[\frac{d}{dr}\left(p(r) + \frac{B_\theta^2}{2\mu_0} + \frac{B_z^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r}\right] = 0$$

Considero

$$\frac{1}{a^2}\int_0^a dr\, \underset{f}{r^2}\, \underset{dg/dr}{\frac{dp}{dr}} \underset{\text{per parti}}{=} \frac{1}{a^2}\left[\underbrace{\cancel{r^2 p\Big|_0^a}}_{=\,0} - 2\int_0^a dr\, r\, p(r)\right]$$

perché $p(a) = 0$

$$= -\frac{2}{a^2}\int_0^a dr\, r\, p(r) = -\langle p \rangle$$

$$\langle p \rangle = \frac{1}{\cancel{\pi} a^2}\int_0^a p(r)\cdot \overbrace{2\cancel{\pi} r\, dr}^{\text{area di corona circolare}} = \frac{2}{a^2}\int_0^a p(r)\, r\, dr$$

pressione media sulla sezione

p è cost.

$$\frac{1}{a^2}\int_0^a dr\, r^2 \left[\frac{d}{dr}\left(\frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r}\right] =$$

$$= \frac{1}{a^2}\left[r^2 \frac{B_\theta^2}{2\mu_0}\Big|_0^a \cancel{- \int_0^a dr\, 2r \frac{B_\theta^2}{2\mu_0}} + \cancel{\int_0^r dr\, r\frac{B_\theta^2}{\mu_0}}\right]$$

$$= \frac{B_\theta^2(a)}{2\mu_0}$$

Dal $\underline{\text{th}}$ Ampere: $2\pi a B_\theta(a) = \mu_0 I \Rightarrow B_\theta(a) = \frac{\mu_0 I}{2\pi a}$

applicato all'intera sezione $\oint_C \underline{B}\cdot d\underline{l} = \mu_0 I$

C : circonferenza di raggio a

$$\langle p\rangle = \frac{1}{2\mu_0}\left(B_z^2(a) - \langle B_z^2\rangle\right) + \frac{B_\theta^2(a)}{2\mu_0}$$

$$\beta = \frac{2\mu_0\langle p\rangle}{B_z^2(a)+B_\theta^2(a)} = 1 - \frac{\langle B_z^2\rangle}{B_z^2(a)+B_\theta^2(a)} \le 1$$