$\omega = \omega(k)$

1) Due fluidi $\underline{B} = \underline{0}$

2) MHD con $\underline{B} \neq \underline{0}$

Plasma uniforme
n T
ed è in equilibrio con $\underline{u} = \underline{0}$
Infinito (no effetti di geometria)

Onda: piccola perturbazione rispetto all'equilibrio

$$Q(\underline{x}, t) = Q_0(\cancel{\underline{x}}, \cancel{t}) + Q_1(\underline{x}, t) \qquad Q_1 \ll Q_0$$

Linearizzare le equazioni rilevanti

Considerate onde monocromatiche

$$Q_1(\underline{x}, t) \propto e^{i(\underline{k} \cdot \underline{x} - \omega t)}$$

$$\underline{\nabla} Q_1 \longrightarrow i \underline{k} Q_1$$

$$\underline{\nabla} \times \underline{Q}_1 \longrightarrow i \underline{k} \times \underline{Q}_1$$

$$\underline{\nabla} \cdot \underline{Q}_1 \longrightarrow i \underline{k} \cdot \underline{Q}_1$$

Equazioni rilevanti → Eq. Maxwell
→ modello di plasma

Eq. Maxwell

$$\underline{\nabla}\cdot\underline{E} = \rho/\varepsilon_0 \qquad \underline{\nabla}\cdot\underline{B} = 0$$

$$\underline{\nabla}\times\underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad \underline{\nabla}\times\underline{B} = \cancel{\mu_0 \underline{j}} + \cancel{\varepsilon_0\mu_0\, \partial \underline{E}/\partial t} \qquad \underline{E} = \cancel{\underline{E}_0} + \underline{E}_1, \quad \underline{E}_0 = 0$$

$$\underline{\nabla}\cdot\underline{E}_1 = \rho_1/\varepsilon_0 \qquad \underline{\nabla}\cdot\underline{B}_1 = 0 \qquad \underline{B} = \underline{B}_0 + \underline{B}_1 \qquad \rho = \cancel{\rho_0} + \rho_1$$

$$\underline{j} = \underline{j}_0 + \underline{j}_1$$

In MHD: $\underline{\nabla} p = \underline{j}\times\underline{B}$

$$\underline{\nabla}\times\underline{E}_1 = -\frac{\partial \underline{B}_1}{\partial t} \qquad \underline{\nabla}\times\underline{B}_1 = \cancel{\mu_0 \underline{j}_1} + \cancel{\varepsilon_0\mu_0 \frac{\partial \underline{E}_1}{\partial t}}$$

0 in MHD

Plasma unif.: $\underline{\nabla} p = 0$

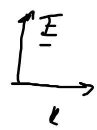
$\Rightarrow \underline{j}_0 = \underline{0}$

modi normali

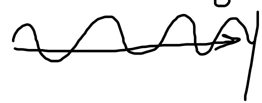
$$i\,\underline{k}\cdot\underline{E}_1 = \rho_1/\varepsilon_0$$

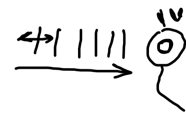
$\rho_1 \neq 0$ se $\underline{k}$ ha una comp // ad $\underline{E}_1$: $\underline{k}\cdot\underline{E}_1 \neq 0$

Ipotesi: $\underline{k}\cdot\underline{E}_1 \neq 0$



Onde longitudinali





Equazione continuità specie j-esima

$$\frac{\partial n_j}{\partial t} + \underline{\nabla}\cdot(n_j \underline{u}_j) = 0$$

$$n_j = n_{0j} + n_{1j} \qquad \underline{u}_j = \cancel{\underline{u}_{0j}} + \underline{u}_{1j}$$

$$\propto e^{i(\underline{k}\cdot\underline{r} - \omega t)}$$

$$\frac{\partial}{\partial t}(n_{0j} + n_{1j}) + \underline{\nabla}\cdot\Big((n_{0j} + \underbrace{n_{1j})\,\underline{u}_{1j}}_{\text{ordine} > 1}\Big) = 0$$

$$\frac{\partial n_{1j}}{\partial t} + n_{0j}\,\underline{\nabla}\cdot\underline{u}_{1j} = 0$$

Conserv. del momento lineare j-esima

$$m_j n_j \left[\frac{\partial}{\partial t}\underline{u}_j + \underbrace{(\underline{u}_j\cdot\underline{\nabla})\,\underline{u}_j}_{\text{ordine } 2}\right] = q\, n_j(\underline{E}_j + \overbrace{\underline{u}_j\times\underline{B}}^{\text{ordine } 2}) - \underline{\nabla} p_j$$

$$m_j (\underbrace{n_{0j} + n_{1j}}_{\text{ordine } 2})\left[\frac{\partial \underline{u}_{1j}}{\partial t} + \cancel{(\underline{u}_{1j}\cdot\underline{\nabla})\,\underline{u}_{1j}}\right] = q\,(\underbrace{n_{0j} + n_{1j})(\underline{E}_1}_{\text{ordine } 2} + \cancel{\underline{u}_{1j}\times\underline{B}_1}) - \underline{\nabla}(\cancel{p_{0j}} + p_{1j})$$

$$m_j n_{0j} \frac{\partial}{\partial t} \underline{u}_{1j} = n_{0j} q \underline{E}_1 - \underline{\nabla} p_{1j}$$

$$\frac{\underline{\nabla} p_j}{p_j} = \gamma \frac{\underline{\nabla} n_j}{n_j} \qquad \frac{\underline{\nabla}(\cancel{p_{0j}} + p_{1j})}{p_{0j} + \cancel{p_{1j}}} = \gamma \frac{\underline{\nabla}(\cancel{n_{0j}} + n_{1j})}{n_{0j} + \cancel{n_{1j}}}$$

$$\frac{\underline{\nabla} p_{1j}}{p_{0j}} = \gamma \frac{\underline{\nabla} n_{1j}}{n_{0j}}$$

Equazioni → ~~eq.~~ Poisson, = cont, = cons. momenti, = stato } 4 equazioni

Incognite: $n_1, p_1, \underline{u}_1, \underline{E}_1$

Inseriamo i modi normali: Eq. cont. linearizzata

$$-\cancel{i}\omega n_{1j} + n_{0j} \cancel{i} \underline{k} \cdot \underline{u}_{1j} = 0 \Rightarrow n_{1j} = \frac{n_{0j}\, i \underline{k} \cdot \underline{u}_{1j}}{\omega}$$

Cons. momento linearizzata

$$\underline{k} \cdot \quad -m_j n_{0j} i \omega \underline{u}_{1j} = n_{0j} q \underline{E}_1 - \underline{\nabla} p_{1j} = n_{0j} q \underline{E}_1 - \gamma \frac{p_{0j}}{n_{0j}} i \underline{k} n_{1j} \quad \cdot \underline{k}$$

↑ eq. stato

$$-i\omega m_j n_{0j}\, \underline{k}\cdot\underline{u}_{1j} = n_{0j}\, q_j\, \underline{k}\cdot\underline{E}_1 - \frac{\gamma p_{0j}}{n_{0j}} k^2 n_{1j}\, i$$

$\hookrightarrow$ eq. continuità

$$-i\omega^2 m_j n_{1j} = n_{0j}\, q_j\, \underline{k}\cdot\underline{E}_1 - \frac{\gamma p_{0j}}{n_{0j}} k^2 n_{1j}\, i$$

$$i n_{1j}\left(\frac{k^2\gamma p_{0j}}{n_{0j}} - \omega^2 m_j\right) = n_{0j}\, q_j\, \underline{k}\cdot\underline{E}_1 \; ; \quad n_{1j} = \frac{n_{0j}}{i} q_j\, \underline{k}\cdot\underline{E}_1 \frac{1}{\frac{\gamma k^2 p_{0j}}{n_{0j}} - \omega^2 m_j}$$

$p_{0j} = n_{0j} T_{0j}$

eq. $$i\,\underline{k}\cdot\underline{E}_1 = \rho_1/\varepsilon_0 \qquad \rho_1 = \sum_j \rho_{1j} = \sum_j q_j\, n_{1j}$$

$$\cancel{i}\,\underline{k}\cdot\underline{E}_1 = \frac{\rho_1}{\varepsilon_0} = \sum_j -\frac{\cancel{i}\, n_{0j}\, q_j^2\, \cancel{\underline{k}\cdot\underline{E}_1}}{\varepsilon_0 m_j} \frac{1}{\frac{\gamma k^2 T_{0j}}{m_j} - \omega^2}$$

ω_{pj}^2

$$= \frac{n_{0j}}{i} \frac{q_j\, \underline{k}\cdot\underline{E}_1}{m_j} \frac{1}{\frac{\gamma k^2 T_{0j}}{m_j} - \omega^2}$$

$$i\underline{k}\cdot\underline{E}_1 \left(1 - \sum_j \frac{\omega_{p_j}^2}{\omega^2 - \gamma \frac{k^2 T_{0j}}{m_j}}\right) = 0 \qquad \omega_{p_j}^2 \overset{\text{ory}}{=} \frac{n_{0j}\, q_j^2}{m_j\, \varepsilon_0}$$

Due soluzioni $\quad \underline{k}\cdot\underline{E}_1 = 0 \Rightarrow$ escluso

$$1 - \sum_j \frac{\omega_{p_j}^2}{\omega^2 - \gamma \frac{k^2 T_{0j}}{m_j}} = 0 \Rightarrow \omega = \omega(k)$$

Se $T_{0j} = 0$ $\quad 1 - \sum_j \frac{\omega_{p_j}^2}{\omega^2} = 0 \,; \quad \omega^2 = \sum_j \omega_{p_j}^2$

Plasma di idrogeno $\to$ protoni, elettroni

$$\omega^2 = \omega_{p_e}^2 + \omega_{p_i}^2 \simeq \omega_{p_e}^2$$