$$\underline{k}\cdot\underline{E}_1\left[\underbrace{1-\sum_j \frac{\omega_{p_j}^2}{\omega^2-\frac{\gamma T_{0_j}k^2}{m_j}}}_{=0}\right]=0$$

$$\omega_{p_j}^2=\frac{n_{0_j}q_j^2}{\varepsilon_0 m_j}$$

Se $T_{0_j}=0$ $\quad \omega^2=\omega_{p_e}^2+\omega_{p_i}^2$

$\omega\approx\omega_{p_e}$

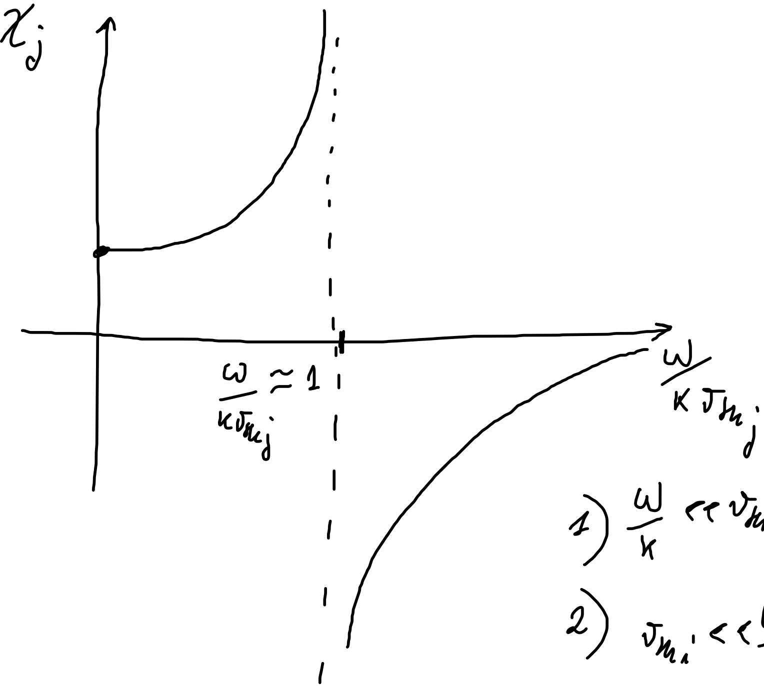
suscettività della specie j-esima χ_j

$\underline{\nabla}\cdot\underline{D}=0$ $\quad \underline{D}=\varepsilon_0\underline{E}+\underline{P}=\varepsilon_0\underline{E}+\varepsilon_0\chi_n\underline{E}\Rightarrow\underline{D}=\varepsilon_0(1+\chi_n)\underline{E}$

vett. polarizzazione

$i\,\underline{k}\cdot\underline{E}\,\varepsilon_0(1+\chi_n)=0$ $\quad \chi=\sum_j\chi_j$ $\quad v_{th_j}^2=\frac{2T_{0j}}{m_j}$

$$1+\chi_n=0\Rightarrow\chi_j=-\frac{\omega_{p_j}^2}{\omega^2-\gamma T_{0j}k^2/m_j}=-\frac{\omega_{p_j}^2}{\omega^2-k^2\frac{\gamma}{2}v_{th_j}^2}=\frac{-\omega_{p_j}^2}{k^2\left[\frac{\omega^2}{k^2}-\frac{\gamma v^2}{2 v_{th_j}}\right.}$$



$\omega^2 - \frac{\gamma}{2}$

$\frac{\omega}{k} \ll v_{th_j}$ $\qquad$ $\frac{\omega}{k} \gg v_{th_j}$

$\chi = \chi_e + \chi_i$ $\qquad$ se $T_e \sim T_i$

$v_{th_e} \gg v_{th_i}$

$v_{th} \propto \frac{1}{\sqrt{m}}$

1) $\frac{\omega}{k} \ll v_{th_e}, v_{th_i}$

2) $v_{th_i} \ll \frac{\omega}{k} \ll v_{th_e}$

3) $\frac{\omega}{k} \gg v_{th_e}, v_{th_i}$

se $\frac{\omega}{k} \sim v_{th_j}$: modello inadeguato

1) $\frac{\omega}{k} \ll v_{th\,i}, v_{th\,e}$

Il plasma segue un'isoterma per ioni ed elettroni $\gamma_e = \gamma_i = 1$

$$1 + \chi_e + \chi_i = 0$$

$$\chi_j = \frac{-\omega_{p_j}^2}{\omega^2 - \frac{k^2 T_{0j}}{m_j}} = \frac{-\omega_{p_j}^2}{k^2\left(\frac{\omega^2}{k^2} - \frac{v_{th_j}^2}{2}\right)} \simeq \frac{2\omega_{p_j}^2}{k^2 v_{th_j}^2}$$

$$1 + \frac{2\omega_{pe}^2}{k^2 v_{the}^2} + \frac{2\omega_{pi}^2}{k^2 v_{thi}^2} = 0$$

$$\frac{v_{th_j}^2}{2\omega_{p_j}^2} = \frac{T_{0j}}{2\cancel{m_j}} \frac{\cancel{m_j}\varepsilon_0}{n_{0j} e^2} = \frac{\lambda_{D_j}^2}{2}$$

$$1 + \frac{1}{k^2\lambda_{De}^2} + \frac{1}{k^2\lambda_{Di}^2} = 0$$

$$1 + \frac{1}{k^2\lambda_D^2} = 0; \qquad \frac{1}{\lambda_D^2} = \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2}$$

$k = \pm i \lambda_D$

$$e^{ikx} \sim e^{-x/\lambda_D}$$

2) $v_{thi} \ll \frac{\omega}{k} \ll v_{the}$

$\gamma_e = 1$

$N=1$

$\gamma_i = \frac{2+N}{N} = 3$

$$\chi_e \sim \frac{\omega_{pe}^2}{\frac{v_{the}^2 k^2}{2}} = \frac{1}{k^2 \lambda_{De}^2}$$

$$\chi_i \sim \frac{-\omega_{pi}^2}{\omega^2\left(1 - \frac{3}{2} \frac{v_{thi}^2 k^2}{\omega^2}\right)} \simeq -\frac{\omega_{pi}^2}{\omega^2}\left(1 + \frac{3}{2}\frac{v_{thi}^2 k^2}{\omega^2}\right)$$

$\frac{v_{thi} k}{\omega} \ll 1$

$$1 + \sum_j \chi_j = 0, \quad 1 + \frac{1}{k^2\lambda_{De}^2} - \frac{\omega_{pi}^2}{\omega^2}\left(1 + \frac{3}{2}\frac{v_{thi}^2 k^2}{\omega^2}\right) = 0$$

$$\omega^2 = \frac{k^2 c_s^2}{1 + k^2\lambda_{De}^2}\left(1 + \frac{3k^2}{\omega^2}\frac{T_{i0}}{m_i}\right) \qquad c_s^2 = \frac{T_e}{m_i}$$

All'ordine più basso $\frac{k}{\omega}\frac{T_{i0}}{m_i} \ll 1$ $\qquad \omega^2 = \frac{k^2 c_s^2}{1 + k^2 \lambda_{De}^2}$

Se $k^2\lambda_{De}^2 \gg 1$ $\qquad \omega^2 = \frac{c_S^2}{\lambda_{De}^2} = \frac{\cancel{T_e}}{m_i}\frac{e^2 n_0}{\varepsilon_0 \cancel{T_e}} = \frac{n_0 e^2}{\varepsilon_0 m_i} = \omega_{pi}^2$

$\Downarrow$

Se $\lambda \ll \lambda_{De}$

$k^2\lambda_{De}^2 \ll 1 \Rightarrow \lambda > \lambda_{De} \Rightarrow \frac{\omega}{k} = c_S$ $\qquad$ Onda ionico-acustica

All'ordine successivo: $\qquad \omega^2 \approx \frac{k^2 c_S^2}{1+k^2\lambda_{De}^2}\left(1 + \frac{3T_{i0}}{m_i}\frac{1+k^2\lambda_{De}^2}{c_S^2}\right)$ $\qquad k\lambda_{De} \ll 1$

$$\omega^2 \approx k^2 c_S^2 + \frac{3T_{i0}}{m_i}k^2 = k^2 c_S^2 + \frac{3}{2}v_{th\,i}^2 k^2$$

$\frac{\omega}{k} \gg v_{th,i} \Rightarrow c_S \gg v_{th\,i}$ $\qquad c_S^2 \gg v_{th\,i}^2$ $\qquad \frac{T_{e0}}{\cancel{m_i}} \gg \frac{T_{i0}}{\cancel{m_i}} \Rightarrow T_{e0} \gg T_{i0}$

3) $\frac{\omega}{k} \gg v_{th\,i}\,,\, v_{th\,e}$ $\qquad \gamma_i = \gamma_e = 3$

$$\chi_i = -\frac{\omega_{pi}^2}{\omega^2}\left(1 + \frac{3}{2}\frac{k^2 v_{th\,i}^2}{\omega^2}\right)$$

$$\chi_e = -\frac{\omega_{pe}^2}{\omega^2}\left(1 + \frac{3}{2}\frac{k^2 v_{th\,e}^2}{\omega^2}\right) \qquad 1 + \chi_e + \chi_i = 0$$

$$1 - \frac{\omega_{pe}^2}{\omega^2}\left(\quad\right) - \frac{\omega_{pi}^2}{\omega^2}\left(\quad\right) = 0 \qquad \frac{k v_{th}}{\omega} \ll 1$$

Ordine più basso: $\omega^2 = \omega_{pe}^2 + \omega_{pi}^2 \simeq \omega_{pe}^2$

Oscillazione ad $\omega = \omega_{pe}$

$$\omega^2 \simeq \omega_{pe}^2 + \frac{3T_{e0}}{m_e}k^2$$ Onde di Bohm-Gross

Onde e.m.

$\underline{k} \perp \underline{E}_1$ Se $\underline{k}\cdot\underline{E}_1 = 0$: $\underline{\nabla}\cdot\underline{E}_1 = \cancel{\rho}$

$0 = i\underline{k}\cdot\underline{E}_1 = \cancel{\rho}$

conducibilità

$$\underline{\nabla}\times\underline{E}_1 = -\frac{\partial \underline{B}_1}{\partial t} \qquad \underline{\nabla}\times\underline{B}_1 = \cancel{\mu_0}\,\underline{j}_1 + \cancel{\varepsilon_0\mu_0}\frac{\partial \underline{E}_1}{\partial t}$$

$$\downarrow$$

$$\underline{j}_1 = \sigma\underline{E}_1$$

$$i\underline{k}\times\underline{E}_1 = +i\omega\underline{B}_1 \Rightarrow \underline{B}_1 = \frac{\underline{k}\times\underline{E}_1}{\omega}$$

$$\Downarrow$$

$$i\underline{k}\times\underline{B}_1 = \cancel{\mu_0}\,\underline{j}_1 - i\omega\cancel{\varepsilon_0\mu_0}\,\underline{E}_1$$

$$i\,\underline{k}\times\underbrace{(\underline{k}\times\underline{E}_1)}_{\omega} = \cancel{\mu_0}\,\sigma\,\underline{E}_1 - i\omega\cancel{\varepsilon_0\mu_0}\,\underline{E}_1$$

$$\frac{i}{\omega}\left[\cancel{\underline{k}(\underline{k}\cdot\underline{E}_1)} - k^2\underline{E}_1\right] = \underline{E}_1\left(\cancel{\mu_0}\,\sigma - \frac{i\omega}{c^2}\right)$$

$\parallel$

0 onde trasversali

$$\underline{E}_1\left(k^2 - \frac{\omega^2}{c^2} - i\sigma\cancel{\mu_0}\,\omega\right) = 0$$

$\underline{E}_1 = 0$

$\underline{E}_1 \neq 0$: $\underline{\omega^2} = k^2\underline{c^2} + \underline{i\sigma\cancel{\mu_0}\,\omega\,c^2}$

Se $\sigma = 0$: $\frac{\omega}{k} = c$

$\sigma = ?$

$$\underline{j}_1 = \sum_s \underline{j}_{1s} = \sum_s q_s n_{0s} \underline{u}_{1s}$$

Eq. bilancio forze

$$m_s n_{0s} \frac{\partial \underline{u}_{1s}}{\partial t} = n_{0s} q_{0s} \underline{E}_1 - \underline{\nabla} p_{1s}$$

$\underline{\nabla} p_1$ non contribuisce: $\underline{\nabla} p_1 = \gamma T_0 \underline{\nabla} n_1 = \gamma T_0 i \underline{k} n_1 \propto \underline{k}$

(legge di stato)

Ci aspettiamo $\underline{j}_1 \perp \underline{k} \Rightarrow \underline{\nabla} p_1$ non influenza

perché, per ipotesi, $\underline{E}_1 \perp \underline{k}$

$\underline{j}_1 \Rightarrow \underline{j}_1 \perp \underline{k}$

$\Rightarrow \underline{k} \cdot \underline{j}_1 = 0$

eq. continuità

$$\frac{\partial n_{1s}}{\partial t} + n_{0s} \underline{\nabla} \cdot \underline{u}_{1s} = 0$$

$$\sum_s q_s \times \left(-i\omega n_{1s} + n_{0s} i \underline{k} \cdot \underline{u}_{1s}\right) = 0$$

$$-i\omega \underbrace{\sum_s q_s n_{1s}}_{\rho_1 = 0} + i \underline{k} \cdot \underbrace{\sum_s n_{0s} q_s \underline{u}_{1s}}_{\underline{j}_1} = 0$$

$$m_s n_{0s} \frac{\partial \underline{u}_{1s}}{\partial t} = q_s n_{0s} \underline{E}_1$$

$$-i\omega \underline{u}_{1s} m_s \cancel{n_{0s}} = q_s \cancel{n_{0s}} \underline{E}_1 ; \qquad \underline{u}_{1s} = i \frac{q_s \underline{E}_1}{m_s \omega}$$

$$\underline{j}_1 = \sum_s q_s n_{0s} \underline{u}_{1s} = \frac{i}{\omega} \sum_s \frac{q_s^2 n_{0s}}{m_s} \underline{E}_1 \Rightarrow \sigma = \frac{i}{\omega} \sum_s \frac{q_s^2 n_{0s}}{m_s}$$

$$\omega^2 = k^2 c^2 - i \sigma \omega \cancel{\mu_0} c^2 = k^2 c^2 + \cancel{\varepsilon_0 \mu_0} \sum_s \underbrace{\frac{n_{0s} q_s^2}{m_s \varepsilon_0}}_{\omega_{ps}^2} c^2 = k^2 c^2 + \sum_s \omega_{ps}^2 \approx c^2 k^2 + \omega_{pe}^2$$

$$\omega^2 = c^2 k^2 + \omega_{pe}^2$$

onde e.m. nel plasma

Se $n_{e0} \to 0$ $\quad \omega^2 = c^2 k^2$

onde e.m. nel vuoto

$$k^2 = \frac{\omega^2 - \omega_{pe}^2}{c^2}$$

$k^2 > 0$ se $\omega > \omega_{pe}$ onda può propagare: $k \in \mathbb{R}$

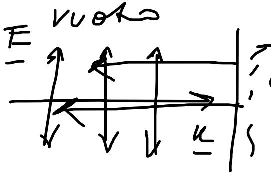
$k^2 < 0$ se $\omega < \omega_{pe}$ k è immaginario: onda evanescente

$\underline{E} \propto e^{ikx} = e^{-\alpha x}$

se $k = i\alpha$

Cosa succede all'onda se $\omega < \omega_{pe}$?

$\omega < \omega_{pe}$



$v_f = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \omega_{pe}^2/\omega^2}}$ se $\omega \to \omega_{pe}$ $v_f \to +\infty$

$v_g = \frac{d\omega}{dk} = \frac{1}{dk/d\omega} = \frac{c}{\omega}\sqrt{\omega^2 - \omega_{pe}^2} \to 0$ $\omega \to \omega_{pe}$

$\underline{S} = \frac{1}{\mu_0} \underline{E} \times \underline{B}$

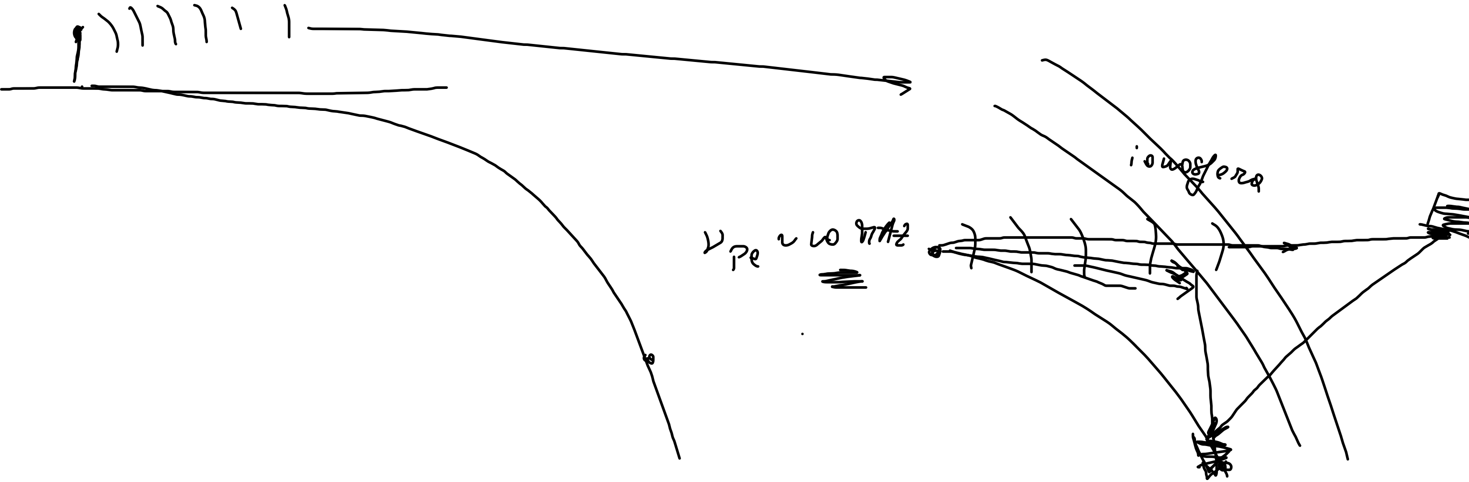
$= \ldots = \frac{E_0^2}{\omega \mu_0} \hat{\underline{k}}$

$k(\omega) = \frac{\sqrt{\omega^2 - \omega_{pe}^2}}{c}$

$\frac{dk}{d\omega} = \frac{1}{c} \frac{1}{\cancel{2}} \frac{\cancel{2}\omega}{\sqrt{\omega^2 - \omega_{pe}^2}} = \frac{\omega}{c\sqrt{\omega^2 - \omega_{pe}^2}}$

$\underline{S} = \frac{|\underline{E}|^2}{\omega \mu_0} \mathrm{Re}(\hat{\underline{k}})$

deve essere reale


ionosfera
$\nu_{pe} \sim 10\ MHz$