Equilibrio $\quad n_0, \; p_0, \; T_0, \; \underline{B} = \underline{B}_0$
$\underline{u} = 0$

$\underline{\xi}$ spostamento dell'elemento fluido

eq. Maxwell per MHD
eq. del plasma in MHD $\Bigg\}$ linearizzate ed espresse in funzione di $\underline{\xi}$

eq. del bilancio delle forze in funzione di $\underline{\xi}$

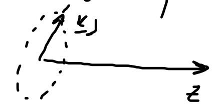
$$\omega^2 \rho_0 \underline{\xi} = \frac{B^2}{\mu_0}\left[\underline{k}\times\left(\underline{k}\times\left(\underline{\xi}\times\hat{\underline{e}}_z\right)\right)\right]\times\hat{\underline{e}}_z + \gamma p_0 \underline{k}(\underline{k}\cdot\underline{\xi})$$

$$\xrightarrow[\underline{B}]{} \hat{\underline{e}}_z \quad z$$

$$\underline{\underline{M}}\cdot\underline{\xi} = 0 \quad \nearrow \; \underline{\xi} = \underline{0}$$

$$\searrow \; \det \underline{\underline{M}} = 0 \Rightarrow \text{relazione di dispersione}$$

$$\underline{k} = k_{\parallel}\hat{\underline{e}}_z + \underline{k}_{\perp}$$



$$\underline{k}_{\perp} = k_{\perp}\hat{\underline{e}}_y$$

$$\underset{\underline{A}}{\underline{k}}\times\Big(\underset{\underline{B}}{\underline{\xi}}\times\underset{\underline{C}}{\hat{\underline{e}}_z}\Big) = k_{\parallel}\underline{\xi} - (\underline{k}\cdot\underline{\xi})\hat{\underline{e}}_z$$

$$\underline{A}\times(\underline{B}\times\underline{C}) = (\underline{A}\cdot\underline{C})\underline{B} - (\underline{A}\cdot\underline{B})\underline{C}$$

$$\underbrace{k_{\parallel}\xi_z + k_{\perp}\xi_y}$$

$$\underline{k}\times\left(\underline{k}\times\left(\underline{\xi}\times\hat{\underline{e}}_z\right)\right) = \underline{k}\times\left(k_{\parallel}\underline{\xi} - (\underline{k}\cdot\underline{\xi})\hat{\underline{e}}_z\right) = k_{\parallel}\left(\underline{k}\times\underline{\xi}\right) - (\underline{k}\cdot\underline{\xi})\underbrace{\left(\underline{k}\times\hat{\underline{e}}_z\right)}_{k_{\perp}\hat{\underline{e}}_x}$$

$$\underline{k} = k_{\parallel}\hat{\underline{e}}_z + k_{\perp}\hat{\underline{e}}_y$$

$$\underline{\xi} = \xi_x\hat{\underline{e}}_x + \xi_y\hat{\underline{e}}_y + \xi_z\hat{\underline{e}}_z$$

Valuto $\underline{k} \times \underline{\xi}$

$$(k_\parallel \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y) \times (\xi_x \hat{\underline{e}}_x + \xi_y \hat{\underline{e}}_y + \xi_z \hat{\underline{e}}_z) = k_\parallel \xi_x \hat{\underline{e}}_y - k_\parallel \xi_y \hat{\underline{e}}_x - k_\perp \xi_x \hat{\underline{e}}_z + k_\perp \xi_z \hat{\underline{e}}_x$$

$$(\underline{k} \cdot \underline{\xi})(\underline{k} \times \hat{\underline{e}}_z) = (k_\parallel \xi_z + k_\perp \xi_y) + k_\perp \hat{\underline{e}}_x = (k_\parallel k_\perp \xi_z + k_\perp^2 \xi_y)\hat{\underline{e}}_x$$

Valutiamo

$$\left[k_\parallel (\underline{k} \times \underline{\xi}) - (\underline{k} \cdot \underline{\xi})(\underline{k} \times \hat{\underline{e}}_z)\right] \times \hat{\underline{e}}_z =$$

$$= \left[k_\parallel^2 \xi_x \hat{\underline{e}}_y - k_\parallel^2 \xi_y \hat{\underline{e}}_x - k_\parallel k_\perp \xi_x \hat{\underline{e}}_z + \cancel{k_\parallel k_\perp \xi_z \hat{\underline{e}}_x} - \cancel{k_\parallel k_\perp \xi_z \hat{\underline{e}}_x} - k_\perp^2 \xi_y \hat{\underline{e}}_x\right] \times \hat{\underline{e}}_z =$$

$$- k^2 \xi_y \hat{\underline{e}}_x$$

$$= \left[k_\parallel^2 \xi_x \hat{\underline{e}}_y - k^2 \xi_y \hat{\underline{e}}_x - k_\parallel k_\perp \xi_x \hat{\underline{e}}_z\right] \times \hat{\underline{e}}_z$$

$$= k_\parallel^2 \xi_x \hat{\underline{e}}_x + k^2 \xi_y \hat{\underline{e}}_y$$

Valuto $\quad \underline{k}(\underline{k}\cdot\underline{\xi}) = (k_\parallel \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y)(k_\parallel \xi_z + k_\perp \xi_y)$

$$= k_\parallel^2 \xi_z \hat{\underline{e}}_z + k_\parallel k_\perp \xi_y \hat{\underline{e}}_z + k_\perp k_\parallel \xi_z \hat{\underline{e}}_y + k_\perp^2 \xi_y \hat{\underline{e}}_y$$

$$\omega^2 \left(\xi_x \hat{\underline{e}}_x + \xi_y \hat{\underline{e}}_y + \xi_z \hat{\underline{e}}_z\right) = \frac{B^2}{\mu_0 \rho_0}\left[k_\parallel^2 \xi_x \hat{\underline{e}}_x + k^2 \xi_y \hat{\underline{e}}_y\right] +$$

$$+ \frac{\gamma p_0}{\rho_0}\left[\hat{\underline{e}}_y\left(k_\perp k_\parallel \xi_z + k_\perp^2 \xi_y\right) + \hat{\underline{e}}_z\left(k_\parallel^2 \xi_z + k_\parallel k_\perp \xi_y\right)\right]$$

$\left[\frac{B^2}{\mu_0}\right]$ = pressione

$\frac{B^2}{\mu_0 \rho_0} = v_A^2$

$\frac{\gamma p_0}{\rho_0} = c_S^2$

$\hat{\underline{e}}_x$: $\quad \omega^2 \xi_x = v_A^2 k_\parallel^2 \xi_x \; ; \quad (\omega^2 - v_A^2 k_\parallel^2)\xi_x = 0$

$\hat{\underline{e}}_y$: $\quad \omega^2 \xi_y = v_A^2 k^2 \xi_y + c_S^2 k_\parallel k_\perp \xi_z + c_s^2 k_\perp^2 \xi_y \; ; \quad \left[\omega^2 - k^2 v_A^2 - k_\perp^2 c_S^2\right]\xi_y - c_S^2 k_\parallel k_\perp \xi_z = 0$

$\hat{\underline{e}}_z$: $\quad \omega^2 \xi_z = c_S^2 k_\parallel^2 \xi_z + c_S^2 k_\parallel k_\perp \xi_y \; ; \quad (\omega^2 - k_\parallel^2 c_S^2)\xi_z - c_S^2 k_\parallel k_\perp \xi_y = 0$

$$\underline{\underline{M}} \cdot \underline{\xi} = 0$$

$$\underline{\underline{M}} = \begin{bmatrix} \omega^2 - k_\parallel^2 v_A^2 & 0 & 0 \\ 0 & \omega^2 - k^2 v_A^2 - c_s^2 k_\perp^2 & -k_\parallel k_\perp c_s^2 \\ 0 & -c_s^2 k_\parallel k_\perp & \omega^2 - c_s^2 k_\parallel^2 \end{bmatrix} \qquad \det \underline{\underline{M}} = 0$$

$$\left(\omega^2 - k_\parallel^2 v_A^2\right)\left[\left(\omega^2 - k^2 v_A^2 - c_s^2 k_\perp^2\right)\left(\omega^2 - c_s^2 k_\parallel^2\right) - c_s^4 k_\parallel^2 k_\perp^2\right] = 0$$

eq. di 3 grado per ω^2 : 3 onde diverse

Onda di Alfven torsionale
(shear Alfven wave) $\qquad \omega = \pm k_\parallel v_A \qquad v_A^2 = \dfrac{B_0^2}{\mu_0 \rho_0} = 2\,\dfrac{B_0^2/2\mu_0}{\rho_0} = \gamma \dfrac{P_m}{\rho_0}$

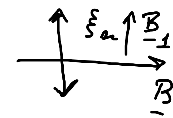
$$\gamma = 2 = \frac{N+2}{N} \Rightarrow N = 2 \qquad c_s^2 = \frac{\gamma P}{\rho_0}$$

Verifico che

$$\det \begin{bmatrix} \overbrace{-k^2 v_A^2}^{k_{\parallel}^2 v_A^2 \;\to\; -k_\perp^2 v_A^2} - c_s^2 k_\perp^2 & -k_{\parallel} k_\perp c_s^2 \\ -c_s^2 k_{\parallel} k_\perp & k_{\parallel}^2 v_A^2 - c_s^2 k_{\parallel}^2 \end{bmatrix} \neq 0$$

$$-k_\perp^2 (v_A^2 + c_s^2) k_{\parallel}^2 (v_A^2 - c_s^2) - k_{\parallel}^2 k_\perp^2 c_s^4 \qquad v_A \neq c_s$$

$$= -k_{\parallel}^2 k_\perp^2 \left[v_A^4 - \cancel{c_s^4} + \cancel{c_s^4} \right] \neq 0 \implies \xi_y = \xi_z = 0$$



$\underline{B}_1$?

$$\underline{B}_1 = i \underline{k} \times (\underline{\xi} \times \underbrace{\underline{B}_0}_{B_0 \hat{\underline{e}}_z}) =$$

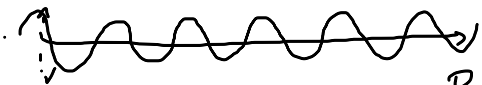
$$= i \left[k_{\parallel} \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y \right] \times \left(-\xi_x \hat{\underline{e}}_y + \xi_y \hat{\underline{e}}_x \right) B_0$$

$$= i B_0 \left[k_{\parallel} \xi_x \hat{\underline{e}}_x + \overbrace{\cancel{k_{\parallel} \xi_y \underline{e}_y}}^{=0} + \overbrace{\cancel{k_\perp \xi_y \hat{\underline{e}}_z}}^{=0} \right] = i B_0 k_{\parallel} \xi_x \hat{\underline{e}}_x$$

$\rho_1 = ?$

$$\rho_1 = -i\rho_0(\underline{k}\cdot\underline{\xi}) = 0 \Rightarrow \rho_1 = 0$$

$$\underline{B} = B_0\,\hat{\underline{e}}_z + B_1\,\hat{\underline{e}}_x$$



$\underline{B}_0$.