$\underline{\xi}$ $\qquad \underline{\underline{M}} \cdot \underline{\xi} = 0$

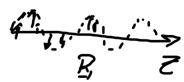
$$\underline{\underline{M}} = \begin{bmatrix} \omega^2 - v_A^2 k_\parallel^2 & 0 & 0 \\ 0 & \omega^2 - k^2 v_A^2 - c_s^2 k_\perp^2 & -k_\parallel k_\perp c_s^2 \\ 0 & -c_s^2 k_\parallel k_\perp & \omega^2 - c_s^2 k_\parallel^2 \end{bmatrix}$$

$$\det \underline{\underline{M}} = 0$$

$$(\omega^2 - v_A^2 k_\parallel^2)\left[(\omega^2 - k^2 v_A^2 - c_s^2 k_\perp^2)(\omega^2 - c_s^2 k_\parallel^2) - k_\parallel^2 k_\perp^2 c_s^4\right] = 0$$

Eq. 3° grado in ω^2

Se $\omega = v_A k_\parallel$: onda Alfven torsionale



$\underline{B}_1 \parallel \hat{\underline{e}}_x \qquad \xi_x \neq 0 \qquad \rho_1 = 0$

$$\left[\qquad \right] = 0 \qquad -\omega^2 c_s^2 k^2$$

$$\omega^4 - c_s^2 k_\parallel^2 \omega^2 - k^2 v_A^2 \omega^2 + c_s^2 v_A^2 k^2 k_\parallel^2 - \omega^2 c_s^2 k_\perp^2 + \cancel{c_s^4 k_\parallel^2 k_\perp^2} - \cancel{c_s^4 k_\parallel^2 k_\perp^2} = 0$$

$$\omega^4 - \omega^2 k^2 (v_A^2 + c_s^2) + c_s^2 v_A^2 k^2 k_\parallel^2 = 0$$

$$\omega^4 - \omega^2 k^2 (c_S^2 + v_A^2) + k^2 k_{\parallel}^2 v_A^2 c_S^2 = 0$$

$a = 1 \quad b = -k^2(c_S^2 + v_A^2)$

$c = k^2 k_{\parallel}^2 v_A^2 c_S^2$

$$\omega^2_{1,2} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a}\left[1 \mp \sqrt{1 - \frac{4ac}{b^2}}\right]$$

$$c_S^2 = \frac{\gamma P}{\rho}$$

$$\alpha^2 = \frac{4ac}{b^2} = \frac{4 \cdot 1 \cdot k_{\parallel}^2 \cancel{k^2} v_A^2 c_S^2}{k^{\cancel{4}\,2}(c_S^2 + v_A^2)^2} = 4 \cdot \frac{k_{\parallel}^2}{k^2} \frac{v_A^2 c_S^2}{(c_S^2 + v_A^2)^2} \leq 1$$

α^2

$$\frac{k_{\parallel}}{k} \leq 1 \qquad v_A^2 = \frac{\cancel{2} \cdot B^2}{\cancel{2}\mu_0 \rho}$$

$$\alpha^2 = 4\frac{k_{\parallel}^2}{k^2} \frac{c_S^2 / v_A^2}{(1 + c_S^2/v_A^2)^2} \ll 1$$

$$\frac{c_S^2}{v_A^2} = \gamma \frac{P}{2\frac{B^2}{2\mu_0}} \propto \beta \ll 1$$

$$\omega^2 = \frac{k^2(c_S^2 + v_A^2)}{2}\left(1 \pm \sqrt{1 - \alpha^2}\right)$$

(+) onda magnetosonica compressionale (onda di Alfven compress.)

(−) onda acustica

Onda Alfven compressionale

$\rightarrow \beta \ll 1$

$$\omega^2 = \frac{k^2(c_s^2+v_A^2)}{2}\left(1 + 1 - \frac{\alpha^2}{2}\right) = k^2(c_s^2+v_A^2) - \frac{\cancel{k^2}}{\cancel{4}}(c_s^2+v_A^2)\frac{\cancel{4}k_{\parallel}^2}{\cancel{k^2}}\frac{v_A^2 c_s^2}{(v_A^2+c_s^2)^2}$$

$$= k^2c_s^2 + k^2v_A^2 - v_A^2\left(1+\frac{c_s^2}{v_A^2}\right)\frac{k_{\parallel}^2\, c_s^2\, \cancel{v_A^2}}{v_A^{\cancel{4}\,2}\left(1+\frac{c_s^2}{v_A^2}\right)^2}$$

$$= k^2c_s^2 + k^2v_A^2 - \frac{k_{\parallel}^2\left(1+\frac{c_s^2}{v_A^2}\right)c_s^2}{\left(1+\frac{c_s^2}{v_A^2}\right)^2}$$

$$\approx k^2v_A^2 + \underbrace{k^2c_s^2 - k_{\parallel}^2c_s^2}_{k_{\perp}^2c_s^2} \approx k^2v_A^2$$

$\omega^2 = k_{\parallel}^2 v_A^2$ onda Alfven torsionale

$\omega^2 = k^2 v_A^2$ 〃 〃 compressionale

Moto del fluido? $\omega = k v_A$ $\longrightarrow z$

$$\begin{bmatrix} k^2 v_A^2 - v_A^2 k_{\parallel}^2 & 0 & 0 \\ 0 & -c_S^2 k_\perp^2 & -k_{\parallel} k_\perp c_S^2 \\ 0 & -k_{\parallel} k_\perp c_S^2 & k^2 v_A^2 - k_{\parallel}^2 c_S^2 \end{bmatrix} \begin{pmatrix} \xi_x \\ \xi_y \\ \xi_z \end{pmatrix} = 0 \qquad \xi_x = 0$$

$$-k_{\parallel} k_\perp c_S^2 \xi_y + \left(k^2 v_A^2 - k_{\parallel}^2 c_S^2\right) \xi_z = 0$$

$$\frac{\xi_z}{\xi_y} = \frac{k_{\parallel} k_\perp c_S^2}{k^2 v_A^2 - k_{\parallel}^2 c_S^2} \sim \frac{k_{\parallel} k_\perp c_S^2}{k^2 v_A^2} \leq \frac{c_S^2}{v_A^2} << 1$$

Moto fluido prevalentemente in direzione $\perp$ a $\underline{B}$

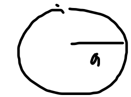
$$\underline{B}_1 = i\, \underline{k} \times (\underline{\xi}_1 \times \underline{B}_0) =$$

$$= i\,(k_{\parallel} \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y) \times \left[(\xi_y \hat{\underline{e}}_y + \xi_z \hat{\underline{e}}_z) \times B_0 \hat{\underline{e}}_z\right] = i\,(k_{\parallel} \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y) \times \xi_y B_0 \hat{\underline{e}}_x$$

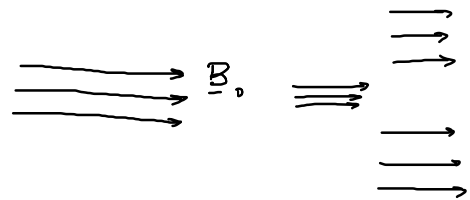
$$= i \xi_y B_0 \left[k_{\parallel} \hat{\underline{e}}_y - k_\perp \hat{\underline{e}}_z\right]$$

$$\frac{B_{1y}}{B_{1z}} = -\frac{k_\parallel}{k_\perp} \gg 1 \qquad k_\parallel \,?\, k_\perp \qquad -\cancel{c_s^2 k_\perp^2}\xi_y - k_\parallel \cancel{k_\perp c_s^2}\,\xi_z = 0$$

$$\frac{\xi_z}{\xi_y} = -\frac{k_\perp}{k_\parallel} \ll 1 \quad ; \quad k_\perp \ll k_\parallel$$



$$\rho_1 = -i\rho_0(\underline{k}\cdot\underline{\xi}) = -i\rho_0(k_\parallel \xi_z + k_\perp \xi_y) \neq 0$$



$\underline{B}_0$

$\lambda \sim a$

$\lambda_\parallel \sim 2\pi R_0$

$R_0 \gg a$

$\Rightarrow \frac{1}{R_0} \ll \frac{1}{a}$

$(k_\parallel \ll k_\perp)$

$\overleftrightarrow{z}$

Scelgo segno $\ominus$

$$\omega^2 = \frac{k^2 v_A^2}{2}\left(1-\sqrt{1-\alpha^2}\right) = \frac{k^2 v_A^2}{2}\,\frac{\alpha^2}{2} \approx \underbrace{k_\parallel^2 c_s^2}_{\equiv}$$

$$\begin{bmatrix} k_\parallel^2 c_s^2 - k^2 v_A^2 - c_s^2 k_\perp^2 & -k_\parallel k_\perp c_s^2 \\ -c_s^2 k_\parallel k_\perp & k_\parallel^2 \cancel{c_s^2 - c_s^2 k_\parallel^2} \end{bmatrix} \begin{pmatrix} \xi_y \\ \xi_z \end{pmatrix} = 0$$

$$-k^2 v_A^2 \xi_y - k_\parallel k_\perp c_s^2 \xi_z = 0 ; \qquad \frac{\xi_y}{\xi_z} = -\frac{k_\parallel k_\perp}{k^2}\frac{c_s^2}{v_A^2} \ll 1$$