$$\omega^2 = \frac{k^2(c_s^2+v_A^2)}{2}\left[1 \pm \sqrt{1-\alpha^2}\right]$$

$$\alpha^2 = \frac{4k_{\parallel}^2}{k^2}\frac{v_A^2 c_s^2}{(v_A^2+c_s^2)^2} = \frac{4k_{\parallel}^2}{k^2}\frac{c_s^2/v_A^2}{(1+c_s^2/v_A^2)^2}$$

$$\frac{4v_A^2c_s^2}{(v_A^2+c_s^2)^2} \le 1; \quad \underbrace{4v_A^2c_s^2}\le v_A^4+c_s^4+2v_A^2c_s^2$$

$$\omega^2>0$$

$$v_A^4+c_s^4-2v_A^2c_s^2\ge 0$$

$$(v_A^2-c_s^2)^2\ge 0$$

Segno ④ onda magnetosonica (o di Alfven compressionale) $\quad c_s^2=\gamma\frac{p}{\rho} \quad v_A^2=\frac{\cancel{2}B^2}{\cancel{2}\mu_0\rho}$

$$\frac{c_s^2}{v_A^2} = \frac{p}{B^2/\mu_0} = \beta \ll 1$$

$$\omega^2 \approx k^2 v_A^2$$

$$\begin{bmatrix} \omega^2-k_{\parallel}^2v_A^2 & 0 & 0 \\ 0 & \omega^2-k^2v_A^2-c_s^2k_{\perp}^2 & -k_{\parallel}k_{\perp}c_s^2 \\ 0 & -k_{\parallel}k_{\perp}c_s^2 & \omega^2-c_s^2k_{\parallel}^2 \end{bmatrix}\begin{pmatrix}\xi_x\\ \xi_y\\ \xi_z\end{pmatrix}=0 \Rightarrow \xi_x=0$$

$$\begin{bmatrix} -c_s^2 k_\perp^2 & -k_\parallel k_\perp c_s^2 \\ -c_s^2 k_\parallel k_\perp & k^2 v_A^2 - c_s^2 k_\parallel^2 \end{bmatrix} \begin{pmatrix}\xi_y \\ \xi_z\end{pmatrix} = 0$$

$$\begin{cases} -\cancel{c_s^2}\, k_\perp^{\cancel{2}}\, \xi_y - k_\parallel \cancel{k_\perp} \cancel{c_s^2}\, \xi_z = 0 \\ -c_s^2 k_\parallel k_\perp \xi_y + \big(k^2 v_A^2 - \underbrace{c_s^2 k_\parallel^2}_{\simeq 0}\big)\xi_z = 0 \end{cases}$$

$$\xi_z / \xi_y = -k_\perp / k_\parallel \qquad \frac{k_\perp}{k_\parallel} = \frac{-\xi_z}{\xi_y}$$

$$-c_s^2 k_\parallel k_\perp \xi_y + k^2 v_A^2 \xi_z = 0$$

$$\xi_z / \xi_y = \underbrace{\frac{c_s^2}{v_A^2}}_{\ll 1} \left(\frac{k_\parallel k_\perp}{k^2}\right)^{\le 1} \ll 1$$

$$\underline{B}_1 = i\,\underline{k} \times (\underline{\xi} \times \underline{B}_0) \simeq i\,(k_\parallel \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y) \times \xi_y \hat{\underline{e}}_y \times B_0 \hat{\underline{e}}_z$$

$$\simeq i B_0 \xi_y (k_\parallel \hat{\underline{e}}_z + k_\perp \hat{\underline{e}}_y) \times \hat{\underline{e}}_x = i B_0 \xi_y (k_\parallel \hat{\underline{e}}_y - k_\perp \hat{\underline{e}}_z)$$

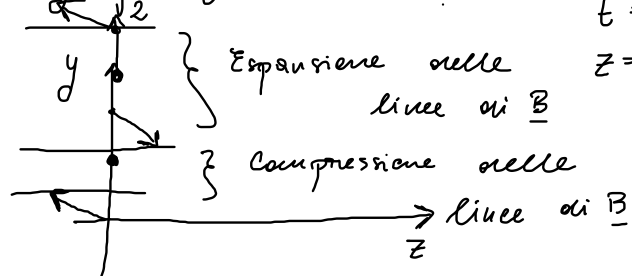
$\underline{B}_1$ è sia lungo $\underline{B}_0$, sia $\perp$ a $\underline{B}_0$

$\rho_1 = -i\rho_0(\underline{k}\cdot\underline{\xi}) = -i\rho_0(k_\parallel \xi_z + k_\perp \xi_y) = 0$ $\quad \forall$ o sep. carica

Suppongo $k_\parallel \sim k_\perp \sim k/\sqrt{2}$

$$\underline{B}_1 = i\xi_y B_0 \frac{k}{\sqrt{2}}(\hat{e}_y - \hat{e}_z) \Rightarrow \frac{B_{1z}}{B_{1y}} = -1$$

$$\underline{B}_1 = i\xi_y B_0 \frac{k}{\sqrt{2}}(\hat{e}_y - \hat{e}_z)\, e^{i\left(\frac{k}{\sqrt{2}}z + \frac{k}{\sqrt{2}}y - k v_A t\right)}$$



$t = 0$ $\quad \underline{B}_1 = i\xi_y B_0 \frac{k}{\sqrt{2}}(\hat{e}_y - \hat{e}_z)\, e^{i\frac{k}{\sqrt{2}}y}$

$z = 0$

<u>Onda sonora</u> $\quad$ segno $\ominus$ $\quad \beta \ll 1$ $\quad \omega^2 \approx k_\parallel^2 c_s^2$ $\quad \underline{\xi}_x = 0$

Sottomatrice 2×2 con $\omega^2 \approx k_\parallel^2 c_s^2$

$$\begin{bmatrix} k_\parallel^2 c_s^2 - k^2 v_A^2 - k_\perp^2 c_s^2 & -k_\parallel k_\perp c_s^2 \\ -c_s^2 k_\parallel k_\perp & \cancel{k_\parallel^2 c_s^2} - \cancel{k_\parallel^2} c_s^2 \end{bmatrix} \begin{pmatrix} \xi_y \\ \xi_z \end{pmatrix} = 0$$

$\simeq 0$ ma non esattamente

$$\begin{cases} \simeq -k^2 v_A^2 \xi_y - k_\parallel k_\perp c_s^2 \xi_z = 0 \\ -c_s^2 k_\parallel k_\perp \xi_y + \varepsilon \cdot \xi_z = 0 \end{cases}$$

$$\left| \xi_y / \xi_z \right| = \left| -\frac{k_\parallel k_\perp c_s^2}{k^2 v_A^2} \right| \ll 1 \rightarrow |\xi_z| \gg |\xi_y|$$

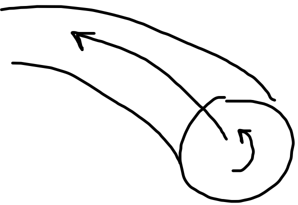
$$\underline{B}_1 = i \underline{k} \times (\underline{\xi}_1 \times \underline{B}_0) \approx i \underline{k} \times (\xi_z \hat{\underline{e}}_z \times B_0 \hat{\underline{e}}_z) \approx 0$$

$\underline{B}$

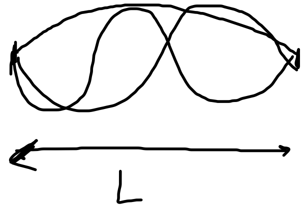
Onda sonora "di un gas"

$$\rho_1 = -i \rho_0 (\underline{k} \cdot \underline{\xi}) \simeq -i \rho_0 k_\parallel \xi_z \neq 0$$

$$p_1 = \gamma p_0 \rho_1 / \rho_0 \simeq -\gamma p_0 k_\parallel \xi_z \neq 0$$

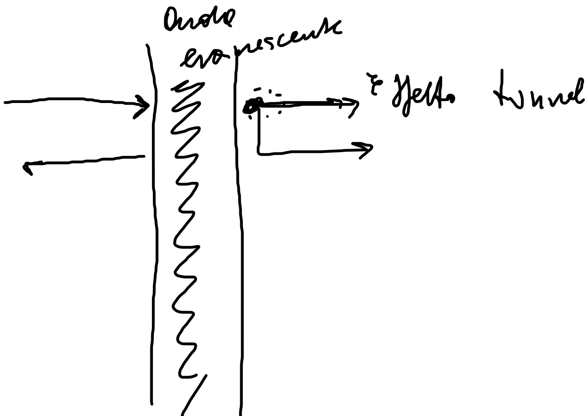


m n

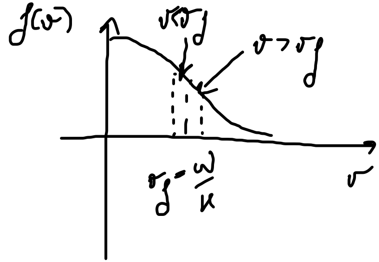


$\lambda = n \frac{L}{2}$

~~$\lambda = \sqrt{2} \frac{L}{2}$~~



Smorzamento di Landau



$v_g = \omega / k$

Se N part con $v < v_g$ > N part. con $v > v_g$

Se $\left.\frac{\partial f}{\partial v}\right|_{v=\frac{\omega}{k}} < 0$, l'onda cede energia al plasma

instabilità $\Leftarrow$ Se $\left.\frac{\partial f}{\partial v}\right|_{v=\frac{\omega}{k}} > 0$, l'onda è amplificata