Hypothesis Test

Exercises

EXERCISE 1

The blood uric acid value is evaluated in a random sample of 40 subjects drawn from a population. The sample mean is $\bar{x}$ = 5.55mg/dl. If the standard deviation is s = 1.1, what is the 95% confidence interval of the mean?

SOLUTION 1

To calculate the 95% confidence interval for the mean, the following formula should be used:

$$\bar{x} \pm Z\left(\frac{s}{\sqrt{n}}\right)$$

where $\bar{x}$ is the sample mean, s is the sample standard deviation, n is the sample size, and Z is the Z-score corresponding to the established confidence interval (for 95% C.I. = 1.96)

So:

$$95\%\ C.I. = 5.55\ \pm 1.96\left(\frac{1.1}{\sqrt{40}}\right) = 5.55\ \pm 1.96\ \frac{1.1}{6.324555} = 5.55\ \pm 0.348$$
$$= 5.55 - 0.348; 5.55 + 0.348\ = 5.202; 5.898$$

Therefore, the 95% confidence interval of the mean blood uric acid level is approximately (5.202, 5.898) mg/dl.

EXERCISE 2

Blood glucose level in women in menopause follows a Gaussian distribution.
A sample of 100 women in menopause were enrolled in the North of Italy. They showed a mean blood glucose $\bar{x}$ = 86.3 mg/dL, with a standard deviation s = 28.2 mg/dL.
Another sample of 100 women were enrolled in the South of Italy. Their mean blood glucose was $\bar{x}$ = 96.7 mg/dL and standard deviation s = 23.5 mg/dL.
Verify if women in the South had blood glucose levels different from women in the North (α = 5%) by:

a. using a two-sample t-test
b. indicating the 95% confidence interval for the mean difference in blood glucose levels between women based in the North and women based in the South of Italy

SOLUTION 2

a.
The system of hypotheses that must be tested is the following:

$$\begin{cases} H_0: \mu_{North} - \mu_{South} = 0 \\ H_1: \mu_{Nort} - \mu_{South} \neq 0 \end{cases}$$

The null hypothesis (H0) is that there is no difference in mean blood glucose levels between the two groups, and the alternative hypothesis (H1) is that there is a difference.
A two-sample t-test for the difference in means should be performed.
The formula for the two-sample t-test is:

$$t = \frac{(\bar{x}_{South} - \bar{x}_{Nort}\ ) - (\mu_{South} - \mu_{North})}{\sqrt{\left(\frac{s^2_{Sout}}{n_{South}}\right) + \left(\frac{s^2_{North}}{n_{North}}\right)}}$$

So,

$$t = \frac{(96.7 - 86.3) - 0}{\sqrt{\left(\frac{552.25}{100}\right) + \left(\frac{795.24}{100}\right)}} = \frac{10.4}{\sqrt{5.5225 + 7.9524}} = \frac{10.4}{\sqrt{13.4749}} = \frac{10.4}{3.6704} = 2.83$$

To estimate the degrees of freedom we use the rule:
df = the smaller between $n_1 - 1$ and $n_2 - 1$
So, we have 100-1 = 99 degrees of freedom.

Using the **Table A-3** of the t Distribution, we can see that for $\sim$ 99 df the critical t value for a two-tails t-test is ± 1.98. As our t test statistic 2.83> 1.98, it falls within the critical region, so **we can reject the null hypothesis**.
The p-value associated with a t-value of 2.83 with 99 degrees of freedom is 0.0056.

b.
To calculate the 95% confidence interval for the mean difference in blood glucose levels between women based in the North and women based in the South of Italy, the margin of error E should be calculated, using $t_{\alpha/2}$=1.984 (corresponding to 100 degrees of freedom $\approx$ 99 cfr. Table A-3):

$$E = t_{\alpha/2}\sqrt{\left(\frac{s^2_{South}}{n_{South}}\right) + \left(\frac{s^2_{North}}{n_{North}}\right)}$$

So,

$$E = 1.984 \cdot 3.6704 = 7.28207$$

Now we can construct the 95% confidence interval using E= 7.28207, $\bar{x}_{South} = 96.7$ and $\bar{x}_{Nort} \ = 86.3$

$$(\bar{x}_{South} - \bar{x}_{North}) - E < (\mu_{South} - \mu_{Nort}\ ) < (\bar{x}_{South} - \bar{x}_{Nort}\ ) + E$$

So,

$$10.4 - 7.28 < (\mu_{South} - \mu_{Nort}\ ) < 10.4 + 7.28$$

$$\mathbf{3.12 < (\mu_{South} - \mu_{North}) < 17.68}$$

<u>EXERCISE 3</u>

The following table shows the mean and the standard deviation of weight loss (g) by sweating during an insulin-induced hypoglycemic crisis in a sample of 12 treated patients with placebo and 11 patients treated with propanol.

GROUP	n	$\bar{x}$ (g)	s (g)
Placebo	12	120	10
Propanol	11	70	8

a. Define the hypothesis system to test the hypothesis that the mean weight loss does not differ between the two groups (α = 0.05) vs an appropriate one-sided alternative hypothesis assuming a Gaussian distribution of the weight loss
b. Calculate the confidence interval of the difference between the two means.

SOLUTION 3

a.

The system of hypotheses that must be tested is the following:

$$\begin{cases} H_0: \mu_{Propanol} - \mu_{Placebo} = 0 \\ H_1: \mu_{Propanol} < \mu_{Placebo} \end{cases}$$

The null hypothesis (H0) is that there is no difference in mean weight loss between the two groups, and the alternative hypothesis (H1) is that the mean weight loss in the propanol group is less than the one in the placebo group.

A two-sample t-test for the difference in means should be performed.

The formula for the two-sample t-test is:

$$t = \frac{(\bar{x}_{Propanol} - \bar{x}_{Placebo}) - (\mu_{Propanol} - \mu_{Placebo})}{\sqrt{\left(\frac{s^2_{Propanol}}{n_{Propanol}}\right) + \left(\frac{s^2_{Placebo}}{n_{Placebo}}\right)}}$$

So,

$$t = \frac{(70 - 120) - 0}{\sqrt{\left(\frac{64}{11}\right) + \left(\frac{100}{12}\right)}} = \frac{-50}{\sqrt{5.82 + 8.33}} = \frac{-50}{3.76} = -13.298$$

To estimate the degrees of freedom we use the rule:

df = the smaller between $n_1 - 1$ and $n_2 - 1$

So, we have 11-1 = 10 degrees of freedom.

Using the **Table A-3** of the t Distribution, we can see that for 10 df the critical t value for a one-tail t-test is -1.812. As our t test statistic -13.298< 1.812, it falls within the critical region, so **we can reject the null hypothesis**.

The p-value associated with a t-value of -13.298 with 10 degrees of freedom is < 0.00001.

b.

To calculate the 95% confidence interval for the mean difference in weight loss between placebo and propanol group, the margin of error E should be calculated, using t_α=1.812 (corresponding to 10 degrees of freedom, cfr. Table A-3):

$$E = t_\alpha \sqrt{\left(\frac{s^2_{Propanol}}{n_{Propanol}}\right) + \left(\frac{s^2_{Placebo}}{n_{Placebo}}\right)}$$

So,

$$E = 1.812 \cdot 3.76 = 6.813$$

Now we can construct the 95% confidence interval using E= 6.813, $\bar{x}_{Propanol} = 70$ and $\bar{x}_{Placebo} = 120$

$$(\bar{x}_{Propanol} - \bar{x}_{Placebo}) - E < (\mu_{Propanol} - \mu_{Placebo}) < (\bar{x}_{Propanol} - \bar{x}_{Placebo}) + E$$

So,

$$-50 - 6.813 < (\mu_{South} - \mu_{North}) < -50 + 6.813$$

$$-56.813 < (\mu_{South} - \mu_{North}) < -43.187$$

EXERCISE 4

25 children whose parents have type II diabetes and 25 whose parents do not have diabetes were sampled. The former had a mean fasting blood sugar level of 86.1 mg/dl, while the others had a mean fasting blood sugar level of 82.2 mg/dl. The standard deviations of the two samples are 2.09 mg/dl and 2.49 mg/dl, respectively.
Check with one tail t-test whether the parents' illness modifies the average blood sugar level of the children (α = 0.05).

SOLUTION 4

The system of hypotheses that must be tested is the following:

$$\begin{cases} H_0: \mu_{T2DM} - \mu_{Non-T2DM} = 0 \\ H_1: \mu_{T2DM} > \mu_{Non-T2DM} \end{cases}$$

The null hypothesis (H0) is that there is no difference in mean fasting blood sugar level between the two groups, and the alternative hypothesis (H1) is that the mean fasting blood sugar level in children whose parents have T2DM is higher than in the other group.
A two-sample t-test for the difference in means should be performed.
The formula for the two-sample t-test is:

$$t = \frac{(\bar{x}_{T2DM} - \bar{x}_{Non-T2DM}) - (\mu_{T2DM} - \mu_{Non-T2DM})}{\sqrt{\left(\frac{s^2_{T2DM}}{n_{T2DM}}\right) + \left(\frac{s^2_{Non-T2DM}}{n_{Non-T2DM}}\right)}}$$

So,

$$t = \frac{(86.1 - 82.2) - 0}{\sqrt{\left(\frac{4.37}{25}\right) + \left(\frac{6.200}{25}\right)}} = \frac{3.9}{\sqrt{0.175 + 0.248}} = \frac{3.9}{0.650} = 6$$

To estimate the degrees of freedom we use the rule:
df = the smaller between $n_1 - 1$ and $n_2 - 1$
So, we have 25-1 = 24 degrees of freedom.

Using the **Table A-3** of the t Distribution, we can see that for 24 df the critical t value for a one-tail t-test is 1.711. As our t test statistic 6>1.711, it falls within the critical region, so **we can reject the null hypothesis**.
The p-value associated with a t-value of 6 with 24 degrees of freedom is < 0.00001.

EXERCISE 5

The hypnotic effect of a new drug, F2, is tested compared to the same effect of an already known drug, F1, on a group of 8 volunteers. First, F1 was administered and then F2, before the following night. The results, evaluated in terms of additional number of hours of sleep compared to an already known average value per subject, are as follows:

Patient ID	F1	F2	δ_i = F2-F1	$(\delta_i - \delta)^2$
1	+0.4	+0.6		
2	+0.3	+0.5		
3	+0.9	+0.7		

4	+0.4	+0.6
5	+1	+0.9
6	+1	+1.1
7	+1	+1.5
8	+1	+2.1

a. Test if the effect of the two drugs is the same (α = 0.05)

b. Calculate the confidence interval for the difference between the two effects in terms of hours of sleep

SOLUTION 5

Patient ID	F1	F2	d_i = F2-F1	$(d_i - d)^2$
1	+0.4	+0.6	+0.2	0.0025
2	+0.3	+0.5	+0.2	0.0025
3	+0.9	+0.7	-0.2	0.2025
4	+0.4	+0.6	+0.2	0.0025
5	+1	+0.9	-0.1	0.1225
6	+1	+1.1	+0.1	0.0225
7	+1	+1.5	+0.5	0.0625
8	+1	+2.1	+1.1	0.7225
			2	**1.14**

a.

The system of hypotheses that must be tested is the following:

$$\begin{cases} H_0: \delta = 0 \\ H_1: \delta \neq 0 \end{cases}$$

$$\bar{d} = \frac{2}{8} = 0.25$$

$$s_d^2 = \frac{1.14}{7} = 0.163 \rightarrow s_d = 0.4$$

$$t = \frac{0.25}{0.4/\sqrt{8}} = 1.77$$

As our t test statistic $t_{7;0.05} = 2.365$ does not fall within the critical region, **we cannot reject the null hypothesis**. So, the effect of the two drugs is the same.

b.

$$95\%\ C.I = 0.25 \pm 2.365 \cdot \frac{0.4}{\sqrt{8}} = -0.08; 0.58$$

The values of the extremes of the confidence interval are consistent with the test result.

EXERCISE 6

The distribution according to sex and age in classes of a sample of people who have had a heart attack results to be the following:

Age	Sex	
	Male	Female
(35, 45]	2	1
(45, 55]	8	3
(55, 65]	14	10
(65, 85]	17	23

Total	41	37

It can be stated that the mean age of males with heart attack is significantly different from that of females (α = 0.05)?

SOLUTION 6

The system of hypotheses that must be tested is the following:

$$\begin{cases} H_0: \mu_M = \mu_F \\ H_1: \mu_M \neq \mu_F \end{cases}$$

$$\bar{x}_M = \frac{40 \cdot 2 + 50 \cdot 8 + 60 \cdot 14 + 75 \cdot 17}{41} = 63.3$$

$$\bar{x}_F = \frac{40 \cdot 1 + 50 \cdot 3 + 60 \cdot 10 + 75 \cdot 23}{37} = 68$$

$$s_M^2 = \frac{(40-63.3)^2 \cdot 2 + (50-63.3)^2 \cdot 8 + (60-63.3)^2 \cdot 14 + (75-63.3)^2 \cdot 17}{40} = \frac{4980.5}{40} = 124.5$$

$$\rightarrow s_M = \sqrt{124.5} = 11.2$$

$$s_F^2 = \frac{(40-68)^2 + (50-68)^2 \cdot 3 + (60-68)^2 \cdot 10 + (75-68)^2 \cdot 23}{36} = \frac{3523}{36} = 97.86 \rightarrow s_F = \sqrt{97.86}$$

$$= 9.89$$

$$t = \frac{(\bar{x}_F - \bar{x}_M) - (\mu_F - \mu_M)}{\sqrt{\left(\frac{s_F^2}{n_F}\right) + \left(\frac{s_M^2}{n_M}\right)}} = \frac{68 - 63.3 - 0}{\sqrt{\frac{97.86}{37} + \frac{124.5}{41}}} = \frac{4.7}{\sqrt{2.64 + 3.04}} = \frac{4.7}{\sqrt{5.68}} = \frac{4.7}{2.38} = 1.97$$

As our t test statistic $t_{36;0.05} = 2.028$ does not fall within the critical region, **we cannot reject the null hypothesis**. So, it cannot be stated that the mean age of males with heart attack is significantly different from that of females (α = 0.05).

EXERCISE 7

In a study on the incidence of migraine in individuals engaged in sports activities, a sample of 150 boys and 200 girls, aged between 16 and 20 years, is examined. It is revealed that 30 boys and 48 girls suffer from frequent migraines. Based on these data, is it possible to conclude, at a significance level of α = 0.05, that there is no significant difference between the boys and girls on the proportions of athletes affected by migraines?

Calculate the 95% confidence interval of the difference between the proportion of boys and girls affected by migraines.

SOUTION 7

The system of hypotheses that must be tested is the following:

$$\begin{cases} H_0: \pi_B = \pi_G \\ H_1: \pi_B \neq \pi_G \end{cases}$$

$$p_B = \frac{30}{150} = 0.2$$

$$p_G = \frac{48}{200} = 0.24$$

$$p = \frac{30 + 48}{150 + 200} = \frac{78}{350} = 0.22$$

$$z = \frac{p_G - p_B}{\sqrt{\frac{p(1-p)}{n_G} + \frac{p(1-p)}{n_B}}} = \frac{0.24 - 0.2}{\sqrt{\frac{0.22(1-0.22)}{200} + \frac{0.22(1-0.22)}{150}}} = \frac{0.04}{\sqrt{\frac{0.17}{200} + \frac{0.17}{150}}} = \frac{0.04}{0.045} = 0.89$$

As our z test statistic $z = 0.89$ does not fall within the critical region, **we cannot reject the null hypothesis**. So, it cannot be stated that there is significant difference between boys and girls on the proportions of athletes affected by migraines (α = 0.05).

For calculating the 95% C.I., the formula is:

$$95\%\ C.I. = (p_G - p_B) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p_G(1-p_G)}{n_G} + \frac{p_B(1-p_B)}{n_B}}$$

$$= 0.04 \pm 1.96 \sqrt{\frac{0.24\ (1-0.24)}{200} + \frac{0.20\ (1-0.20)}{150}} = 0.04 \pm 1.96 \cdot \sqrt{0.001979}$$

$$= 0.04 \pm 1.96 \cdot 0.0445 = [-0.04722;\ 0.12722]$$

With 95% of confidence, we can say that the difference between boys and girls $(p_G - p_B)$ on the proportions of athletes affected by migraines will be included between -0.04722 and 0.12722.

EXERCISE 8

In a study to assess the efficacy of two different antibiotic treatment regimens for urinary tract infections in adult patients 200 patients were enrolled. 100 patients were randomly assigned to group A, for receiving the standard antibiotic treatment, or to group B, for receiving a newly developed antibiotic. At the end of the treatment, the number of patients showing clinical improvement (cure) in each group was recorded.
Group A: 75 patients cured out of 100 treated.
Group B: 85 patients cured out of 100 treated.
Examine whether there is a significant difference in the efficacy of the two treatments in curing urinary tract infections:

a. Formulate the null and alternative hypotheses for the comparison of proportions
b. Calculate the cure proportion for each group
c. Perform the appropriate hypothesis test at a significance level of 5%. State the critical value and make a decision on rejecting or not rejecting the null hypothesis
d. Interpret the results of the hypothesis test: what conclusions can be drawn from the study?

SOLUTION 8

The system of hypotheses that must be tested is the following:

$$\begin{cases} H_0: \pi_A = \pi_B \\ H_1: \pi_A \neq \pi_B \end{cases}$$

$$p_A = \frac{75}{100} = 0.75$$

$$p_B = \frac{85}{100} = 0.85$$

$$p = \frac{75 + 85}{100 + 100} = \frac{160}{200} = 0.8$$

$$z = \frac{p_A - p_B}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} = \frac{0.75 - 0.85}{\sqrt{\frac{0.80(1-0.80)}{100} + \frac{0.80(1-0.80)}{100}}} = \frac{0.10}{\sqrt{\frac{0.16}{100} + \frac{0.16}{100}}} = \frac{0.10}{0.0566} = 1.767$$

As our z test statistic $z = 1.767$ does not fall within the critical region, **we cannot reject the null hypothesis**. So, it cannot be stated that there is significant difference between treatment A and B in determining clinical improvement in urinary tract infections.

EXERCISE 9

In a study for assessing whether there are significant differences in the frequency of postoperative complications between two different surgical procedures for laparoscopic cholecystectomy 300 patients were enrolled and randomly assigned to procedure A (traditional technique) or procedure B (innovative technique). The following data were collected:
Procedure A: 25 patients developed complications.
Procedure B: 15 patients developed complications.
Determine if the innovative technique (Procedure B) is associated with a significantly lower frequency of postoperative complications compared to the traditional technique (Procedure A).

a. Formulate the null and alternative hypotheses for the comparison of proportions
b. Calculate the proportion of patients with complications for each surgical procedure
c. Perform the appropriate hypothesis test at a significance level of 5%. State the critical value and make a decision on rejecting or not rejecting the null hypothesis
d. Interpret the results of the hypothesis test: what conclusions can be drawn from the study?

SOLUTION 9

The system of hypotheses that must be tested is the following:

$$\begin{cases} H_0: \pi_A = \pi_B \\ H_1: \pi_A > \pi_B \end{cases}$$

$$p_A = \frac{25}{150} = 0.167$$

$$p_B = \frac{15}{150} = 0.1$$

$$p = \frac{25+15}{150+150} = \frac{40}{300} = 0.133$$

$$z = \frac{p_A - p_B}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} = \frac{0.167 - 0.1}{\sqrt{\frac{0.133(1-0.133)}{150} + \frac{0.133(1-0.133)}{150}}} = \frac{0.067}{\sqrt{\frac{0.1153}{150} + \frac{0.1153}{150}}} = \frac{0.067}{0.0392} = 1.709$$

The critical value for a one-tailed test at a 5% significance level is approximately 1.645.
As our z value $= 1.709$ falls within the critical region, **we can reject the null hypothesis**.
There is enough evidence to suggest that procedure B has a significantly lower proportion of postoperative complications compared to procedure A at the 5% significance level.