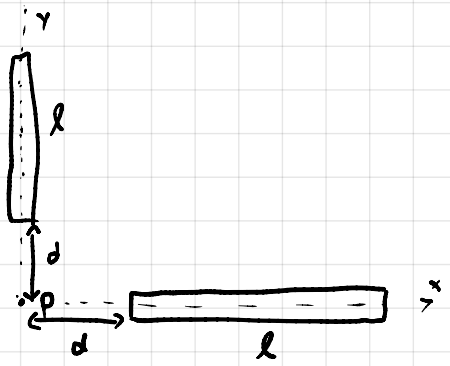
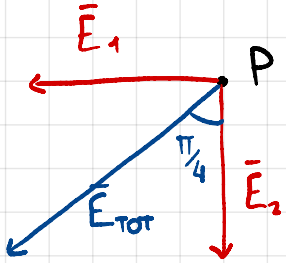
TUTORAGGIO 26/11 ELETTROSTATICA

ESERCIZI PRESI DAL MAZZOLDI PER LA TRACCIA VEDERE LIBRO

1.6)



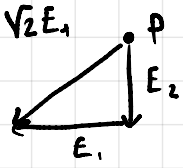


$$\lambda = \frac{q}{l} \qquad dq = \lambda dx \qquad dE_1 = K_0 \frac{dq}{x^2} \qquad E_1 = E_2$$

$$E_1 = K \int_d^{d+l} dx \, \frac{\lambda}{x^2} = K_0 \lambda \left(\frac{1}{x}\right)_d^{d+l}$$

$$= K_0 \lambda \left(\frac{1}{d+l} - \frac{1}{d}\right)$$

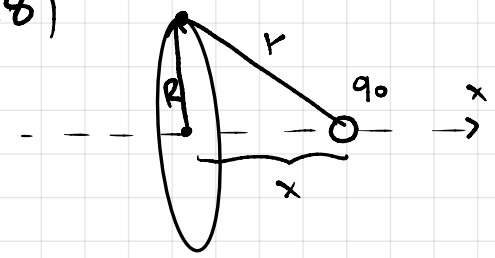
$$= -\frac{K \lambda l}{(d+l) l}$$



$$E = K_0 \frac{q}{(d+l) l}$$

$$E_{TOT} = \sqrt{2 E_1^2} = K_0 \frac{\sqrt{2} q}{l(d+l)} = 106 \ V/m$$

1.8)



$$dV(x) = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda R \, d\theta}{(R^2 + x^2)^{1/2}}$$

$$= \frac{2\pi\lambda R}{4\pi\varepsilon_0} \cdot \frac{1}{(R^2 + x^2)^{1/2}}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{1}{(R^2 + x^2)^{1/2}}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{2}\right) \frac{2x}{(R^2 + x^2)^{3/2}} \qquad x \ll R \longrightarrow E_x \approx \frac{q}{4\pi\varepsilon_0} \frac{x}{R^3}$$

$$F_x = m\ddot{x} \rightsquigarrow q_0 E_x = m\ddot{x} \rightsquigarrow \ddot{x} = \underbrace{\frac{q_0 q}{m(4\pi\varepsilon_0) R^3}}_{\omega^2} \cdot x \rightsquigarrow \ddot{x} = \omega^2 x$$

SOLUZIONE $\rightsquigarrow$ $x(t) = a_1 \cos(\omega t) + a_2 \operatorname{sen}(\omega t)$ C.I $x(0) = x_0$ $V(0) = 0$

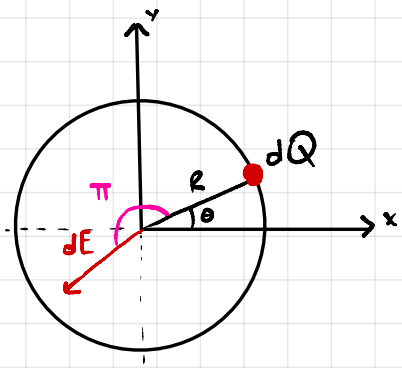
$\rightsquigarrow$ $x(t) = x_0 \cos(\omega t)$ $\omega = \frac{2\pi}{T}$ $\rightsquigarrow T = \frac{2\pi}{\omega}$

$E_c(x=0) \rightsquigarrow x(t^*) = 0 \rightsquigarrow \cos(\omega t^*) = 0$ $\omega t^* = \frac{\pi}{2}$ $t^* = \frac{\pi}{2\omega}$

$V(t) = -x_0 \omega \operatorname{sen}(\omega t)$ $V(t^*) = -x_0 \omega \operatorname{sen}\left(\frac{\pi}{2}\right) = -x_0 \omega$

$E_c(t^*) = \frac{1}{2} m x_0^2 \omega^2$ (OPPURE CON CONS ENERGIA)

2.2)



$\lambda = \lambda_0 S_\theta$ $\lambda = \frac{dQ}{d\ell}$ $dQ = \lambda d\ell$ ($R d\theta$)

$= \lambda_0 R S_\theta d\theta$

$$d\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{R^2} \bar{u}_x \cos(\theta+\pi) + \frac{1}{4\pi\varepsilon_0} \frac{dQ}{R^2} \bar{u}_y \operatorname{sen}(\theta+\pi)$$

$$= -\frac{\lambda_0}{4\pi\varepsilon_0 R} \left[\underbrace{\int_0^{2\pi} d\theta\, S_\theta C_\theta \bar{u}_x}_{0} + \underbrace{\int_0^{2\pi} d\theta\, S_\theta^2 \bar{u}_y}_{\frac{1}{2}\int (1 - C_{2\theta})} \right]$$

$$= -\frac{\lambda_0}{4\pi\varepsilon_0 R} \cdot \underbrace{\frac{1}{2} \int_0^{2\pi} d\theta (1 - C_{2\theta}) \bar{u}_y}$$

$= \frac{1}{2} \left[\theta - \frac{1}{2} S_{2\theta} \right]_0^{2\pi} = \pi$

$$= -\frac{\lambda_0}{4\varepsilon_0 R} \bar{u}_y$$

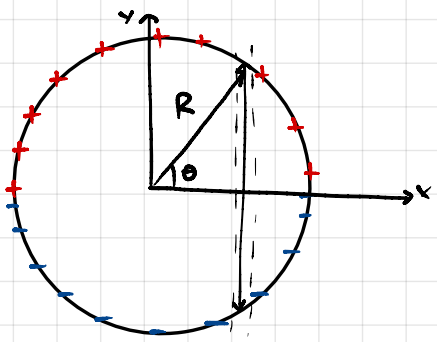
CAMPO A GRANDE x e y (APPROX. DIPOLO)

$d\bar{p} = dq\,\bar{a}$ ($2RS_\theta \bar{u}_y$)

$\lambda \cdot d\ell$

$= \lambda_0 S_\theta R d\theta$

$-2\lambda_0 R^2 S_\theta^2 d\theta \bar{u}_y$



$$x(t)\bar{p} = x_0 \cdot \lambda_0 R^2 \bar{u}_y \int_0^{\pi} S_\theta^2 \, d\theta$$

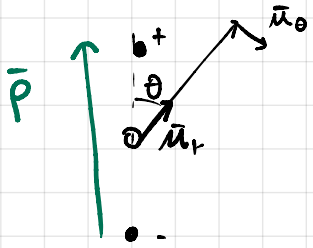
$$x(t) = \frac{V_0}{\omega \lambda_0 R^2 \bar{u}_y} \operatorname{sen}(\omega t) \int_0^{\pi} (1 - C_{2\theta}) \, d\theta$$

DIPOLO EQUIVALENTE

$$\left[\theta - \frac{1}{2} S_{2\theta}\right]_0^{\pi} = \pi$$

$$= \pi \lambda_0 R^2 \bar{u}_y$$

$$= \frac{1}{2} \pi R q \bar{u}_y$$



$$V = \frac{1}{4\pi\varepsilon_0 r^2} \bar{p} \cdot \bar{u}_r = \frac{p C_\theta}{4\pi\varepsilon_0 r^2}$$

$$\begin{cases} E_r = -\dfrac{\partial V}{\partial r} = \dfrac{2p C_\theta}{4\pi\varepsilon_0 r^3} \\ E_\theta = -\dfrac{1}{r}\dfrac{\partial V}{\partial \theta} = \dfrac{p S_\theta}{4\pi\varepsilon_0 r^3} \\ E_\varphi = 0 \end{cases}$$

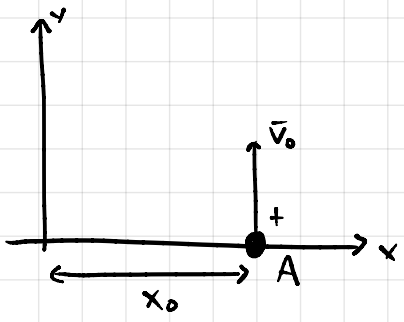
$$\bar{E} = E_r \bar{u}_r + E_\theta \bar{u}_\theta = \frac{p}{4\pi\varepsilon_0 r^3} \left(2 C_\theta \bar{u}_r + S_\theta \bar{u}_\theta\right)$$

GRANDE DISTANZA IN x (e $y = 0$) $\rightsquigarrow \theta = \pi/2 \rightsquigarrow \bar{E} = \dfrac{p}{4\pi\varepsilon_0 r^3} \underbrace{\bar{u}_\theta}_{\hookrightarrow -\bar{u}_y}$ ($r^3 \to x^3$)

GRANDE DISTANZA IN y (e $x = 0$) $\rightsquigarrow \theta = 0 \rightsquigarrow \bar{E} = \dfrac{2p}{4\pi\varepsilon_0 r^3} \underbrace{\bar{u}_r}_{\hookrightarrow \bar{u}_y} = \dfrac{2p}{4\pi\varepsilon_0 y^3} \bar{u}_y$

2.8) $V = V_0(x^2 + y^2)$ $\quad V_0 = 10^7 \ V/m$ $\quad \dfrac{q}{m} = 5 \cdot 10^6 \ C/kg$

x_0 $\qquad \bar{V} = V_0 \bar{u}_y$



$$\bar{F} = q\bar{E} \rightsquigarrow \bar{a} = \frac{q}{m}\bar{E}$$

$$\bar{E} = -\nabla \bar{V} \qquad \frac{\partial V}{\partial x} = 2V_0 x \qquad \frac{\partial V}{\partial y} = 2V_0 y$$

$$\ddot{x} = -\underbrace{\frac{q}{m}}_{\omega^2} 2V_0 x \qquad \ddot{y} = -\underbrace{\frac{q}{m}}_{\omega^2} 2V_0 y$$

$$\begin{cases} x(t) = x_0 \cos \omega t \\ y(t) = \dfrac{V_0}{\omega} \operatorname{sen}(\omega t) \end{cases}$$