• *Ripasso*

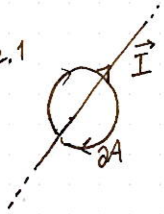
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\hat{\imath}}{\imath^2}\,d\tau \xrightarrow{\nabla\times} \nabla\times\vec{B} = \mu_0\vec{J} \xrightarrow{\int dA} \int_A \nabla\times\vec{B}\cdot d\vec{A} = \mu_0\int_A \vec{J}\cdot d\vec{A} \quad \text{Stokes}$$

Biot-Savart

$$\oint_{\partial A}\vec{B}\cdot d\vec{l} = \mu_0\int_A \vec{J}\cdot d\vec{A} = \mu_0 I_{enc}$$

Ampere

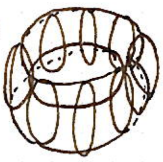
Es.1



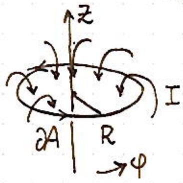
$$2\pi s\, B(s) = \mu_0 I$$
$$\rightarrow \vec{B}(s) = \frac{\mu_0 I}{2\pi s}\hat{\varphi}$$

filo infinito

Es.2

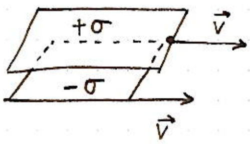


toro



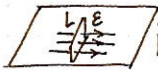
$$2\pi R\, B(R) = \mu_0 N I$$
$$\rightarrow \vec{B}(R) = \frac{\mu_0 N I}{2\pi R}\hat{\varphi}$$

6• *Condensatore in moto* [ex. 16 magnetostatica]



2. $\vec{f}_B, \vec{f}_E$?
1. $\vec{E}, \vec{B}$?
3. v: $\vec{f}_B = -\vec{f}_E$?

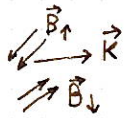
Ampere



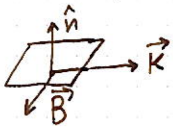
$\vec{K} = \sigma\vec{v}$ $\qquad B_\uparrow L - B_\downarrow L = \mu_0 K L$

$\varepsilon \to 0$

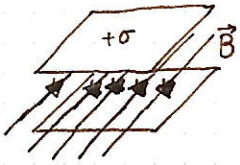
Biot-Savart



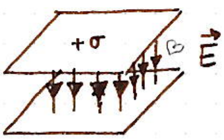
$$2\vec{B}_\uparrow = \mu_0 K$$
$$\boxed{\vec{B} = \frac{1}{2}\mu_0\vec{K}\times\hat{n}}$$



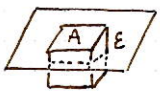
1.
$$\vec{B} = \begin{cases} 0 & \text{outside} \\ \mu_0\vec{K}\times\hat{n} & \text{inside}\end{cases}$$



$$\vec{E} = \begin{cases} 0 & \text{outside} \\ \frac{\sigma}{\varepsilon_0}\hat{n} & \text{inside}\end{cases}$$

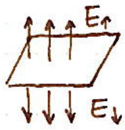


Gauss



$$\oint\vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} \qquad E_\uparrow A - E_\downarrow A = \frac{\sigma A}{\varepsilon_0}$$

Coulomb



$$\boxed{\vec{E} = \frac{1}{2}\frac{\sigma}{\varepsilon_0}\hat{n}}$$

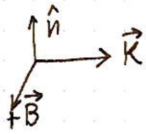
2. $\vec{F}_B = \int I\, d\vec{l}\times\vec{B} = \int dl\, \vec{I}\times\vec{B}$

$$= \int dA\, \underbrace{\vec{K}\times\vec{B}}_{f_B}$$

$$\vec{F}_E = \int dq\,\vec{E} = \int dA\, \underbrace{\sigma\vec{E}}_{f_E}$$

⚠ $\vec{E}, \vec{B}$ prodotto dall'altro piano, non il totale!

$$\begin{cases} \vec{f}_B = \vec{K}\times\vec{B} = \vec{K}\times\left(\frac{1}{2}\mu_0\vec{K}\times\hat{n}\right) = -\frac{1}{2}\mu_0 K^2\hat{n} \\ \vec{f}_E = \sigma\vec{E} = \frac{1}{2}\frac{\sigma^2}{\varepsilon_0}\hat{n}\end{cases}$$



3. $\vec{F}_E = -\vec{F}_B \iff \cancel{\tfrac{1}{2}}\mu_0 K^2 = \cancel{\tfrac{1}{2}}\frac{\sigma^2}{\varepsilon_0} \qquad \mu_0\cancel{\sigma^2} v^2 = \frac{\cancel{\sigma^2}}{\varepsilon_0} \qquad v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$!

• *Potenziale vettore*

$\nabla\cdot\vec{B} = 0 \Rightarrow$ (Helmholtz) esiste $\vec{A}$: $\nabla\times\vec{A} = \vec{B}$

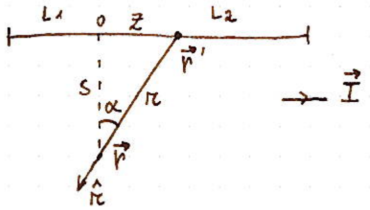
$$\begin{array}{ccc} \vec{A} & \longrightarrow & A + \nabla f \\ \downarrow \nabla\times & & \downarrow \nabla\times \\ B & \xrightarrow{=} & B \end{array} \quad , \quad f:\ \nabla\cdot A = 0$$

Legge di Ampere: $\nabla\times\vec{B} = \nabla\times(\nabla\times\vec{A}) = \mu_0\vec{J}$

$$\underbrace{\nabla(\nabla\cdot A)}_{0} - \nabla^2 A = \mu_0\vec{J}$$

$$-\nabla^2\vec{A} = \mu_0\vec{J} \longrightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(r')}{\imath(\vec{r},\vec{r}')}\,d\tau(r') \quad \text{"Biot-Savart per A"}$$

7. Potenziale vettore del filo di corrente [es. 23 magnetostatica]



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{-L_1}^{+L_2} \frac{I\,dz}{\mathcal{r}} \qquad \begin{cases} \mathcal{r} = \sqrt{z^2+s^2} \\ z = s\tan\alpha \end{cases} \rightarrow \begin{cases} dz = s\left(\frac{1}{\cos^2\alpha}\right)d\alpha \\ \mathcal{r} = s\frac{1}{\cos\alpha} \end{cases}$$

a) $$\vec{A}(\vec{r}) = \frac{\mu_0\vec{I}}{4\pi}\int_{-\alpha_1}^{+\alpha_2} d\alpha\left(\frac{1}{\cos^2\alpha}\right)\cos\alpha \qquad \text{dove} \quad \alpha_i = \operatorname{atan}\left(\frac{L_i}{s}\right)$$

$$= \frac{\mu_0\vec{I}}{4\pi}\int_{-\alpha_1}^{+\alpha_2}\frac{d\alpha\cos\alpha}{1-\sin^2\alpha} \qquad \begin{cases} u = \sin\alpha \\ du = d\alpha\cos\alpha \end{cases}$$

$$= \frac{\mu_0\vec{I}}{4\pi}\int_{-\sin\alpha_1}^{+\sin\alpha_2}\frac{du}{1-u^2}$$

$$\frac{1}{1-u^2} = \frac{A}{1+u} + \frac{B}{1-u} = \frac{A(1-u)+B(1+u)}{(1+u)(1-u)} = \frac{1(A+B)+u(B-A)}{1-u^2}$$

$$A, B = \frac{1}{2} \Rightarrow \frac{1}{1-u^2} = \frac{1}{2}\left(\frac{1}{1+u}+\frac{1}{1-u}\right)$$

$$= \frac{\mu_0\vec{I}}{4\pi}\,\frac{1}{2}\Big[\log(1+u) - \log(1-u)\Big]_{-\sin\alpha_1}^{+\sin\alpha_2}$$

$$= \frac{\mu_0\vec{I}}{8\pi}\log\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right)\Bigg|_{-\alpha_1}^{+\alpha_2}$$

$$= \frac{\mu_0\vec{I}}{8\pi}\log\left(\frac{L_1+\sqrt{s^2+L_1^2}}{L_1-\sqrt{s^2+L_1^2}}\cdot\frac{L_2+\sqrt{s^2+L_2^2}}{L_2-\sqrt{s^2+L_2^2}}\right)$$

$$z = \mathcal{r}\sin\alpha \Rightarrow \sin\alpha = \frac{z}{\sqrt{s^2+z^2}}$$

$$\sin\alpha_i = \frac{L_i}{\sqrt{s^2+L_i^2}}$$

$$= \frac{\mu_0\vec{I}}{4\pi}\,\frac{1}{2}\log\left[\left(\frac{L_2+\sqrt{s^2+L_2^2}}{\sqrt{s^2+L_1^2}-L_1}\right)^2\right]$$

$$\times\left(L_1-\sqrt{s^2+L_1^2}\right)\left(L_2+\sqrt{s^2+L_2^2}\right)$$
$$\times\left(L_1-\sqrt{s^2+L_1^2}\right)\left(L_2+\sqrt{s^2+L_2^2}\right)$$

$$= \frac{\mu_0\vec{I}}{4\pi}\log\left(\frac{L_2+\sqrt{s^2+L_2^2}}{-L_1+\sqrt{s^2+L_1^2}}\right)$$

b) $$\vec{B} = \nabla\times\vec{A} = -\frac{\partial A_z}{\partial s}\hat{\varphi}$$

$$\left[\nabla\times A = \left(\underbrace{\frac{1}{s}\frac{\partial A_z}{\partial\varphi}}_{\to 0} - \overbrace{\frac{\partial A_\varphi}{\partial z}}^{\to 0}\right)\hat{s} + \left(\overbrace{\frac{\partial A_s}{\partial z}}^{\to 0} - \underline{\frac{\partial A_z}{\partial s}}\right)\hat{\varphi} + \frac{1}{s}\left(\frac{\partial}{\partial s}(s\overbrace{A_\varphi}^{\to 0}) - \overbrace{\frac{\partial A_s}{\partial\varphi}}^{\to 0}\right)\hat{z}\right]$$

$$= \dots = \frac{\mu_0 I}{4\pi s}(\sin\alpha_1 + \sin\alpha_2)\hat{\varphi}$$

c) per $L_1, L_2 \to +\infty$ $\left(\alpha_1, \alpha_2 \to \frac{\pi}{2}\right)$ $\quad \vec{B} \to \frac{\mu_0 I}{2\pi s}\hat{\varphi}$ ok

$\vec{A} \to +\infty$?? $\quad \vec{A} \propto \int \frac{\vec{J}}{\mathcal{r}}d\tau$ non valida se $J \neq 0$ $|\vec{r}| \to +\infty$

$$\begin{cases} -\nabla^2\vec{A} = \mu_0\vec{J} \\ \underline{\vec{A} \to 0 \quad |\vec{r}| \to +\infty} \end{cases} \Rightarrow \vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}}{\mathcal{r}}d\tau$$

In questo caso $\vec{A}_\perp \to 0$, $A_z \neq 0$.

$$\begin{cases} -\nabla^2\vec{A} = \mu_0\vec{J} \quad \text{con } \nabla^2 \text{ 2D}, \\ \vec{A}_\perp \to 0 \; (A_z \to c \neq 0) \end{cases} \quad \begin{cases} \vec{A} = -\frac{\mu_0\vec{I}}{2\pi}\log\left(\frac{s}{s_0}\right) \\ \vec{B} = -\frac{\partial A_z}{\partial s}\hat{\varphi} = \frac{\mu_0 I}{2\pi s}\hat{\varphi} \end{cases}$$