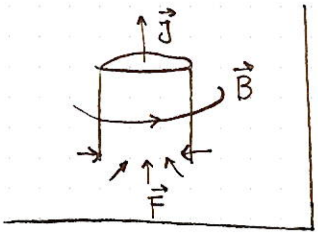
2. $J = \rho v_d = -n_e e v_d \Rightarrow v_d = -\frac{J}{e n_e} = -\frac{I_1}{\pi a^2 e n_e}$

3. $\vec F = q\vec v \times \vec B$ $\quad q = -e$, $\vec v = v_d \hat z$ $\quad \vec B = \mu_0 I_1 \frac{1}{2\pi a}\hat\varphi \Rightarrow \vec F = (-e)\left(\frac{-I_1}{\pi^2 a^2 e n_e}\right)\left(\frac{\mu_0 I_1}{2\pi a}\right)\overbrace{\hat z \times \hat\varphi}^{-\hat s} = -\frac{\mu_0 I_1^2}{2\pi^3 a^3 n_e}\hat s$



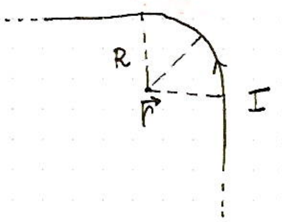
4. $u_B = \frac{1}{2\mu_0}\int_{\mathbb{R}^3} B^2(\vec r)\,dV = \frac{1}{2\mu_0}\int_{Cyl(b,L)} B^2(\vec r)\,dV$ (se $I_2 = I_1$)

$$= \frac{1}{2\mu_0}\int_0^b ds\int_0^{2\pi} s\,d\varphi\int_0^L dz \begin{cases} \left(\mu_0 I_1 \frac{s}{2\pi a^2}\right)^2 & s \le a \\ \left(\mu_0 I_1 \frac{1}{2\pi s}\right)^2 & s > a \end{cases}$$

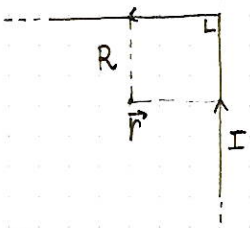
$$= \frac{1}{2\mu_0}\int_0^b ds\; 2\pi s L \left(\frac{\mu_0 I_1}{2\pi}\right)^2 \begin{cases} s^2/a^2 & s \le a \\ 1/s^2 & s > a \end{cases} \quad \begin{matrix} \to s^4/4 \\ \to \log s \end{matrix}$$

$$\Rightarrow \frac{u_B}{L} = \frac{1}{2\mu_0}\frac{2\pi(\mu_0 I_1)^2}{(2\pi)^2}\left[\int_0^a \frac{ds^4}{4a^4} + \int_a^b d\log s\right] = \frac{\mu_0 I_1^2}{4\pi}\left(\frac14 + \log\frac{b}{a}\right)$$

3. <u>sezioni di fili</u> [prob. 2 esame 2021-09-21]



a)



b)

1. Calcola $\vec B(\vec r)$ in a) e b)
2. quale è maggiore?

1. idea per a) 2 fili semi-infiniti e $\frac14$ di spira.

a) $\vec B = \frac{\mu_0 I}{4\pi s}(\sin\theta_1 + \sin\theta_2)\hat\varphi$ con $\theta_1 = \frac{\pi}{2}$, $\theta_2 = 0$ $\Rightarrow \vec B_{filo} = \frac{\mu_0 I \hat z}{4\pi R} \times 2$ fili

$\vec B(z) = \frac12 \mu_0 I \frac{a^2 \hat z}{(a^2+z^2)^{\frac32}}$ con $z = 0$ $\times \frac14 \Rightarrow \vec B_{una} = \frac{\mu_0 I}{8R}\hat z$

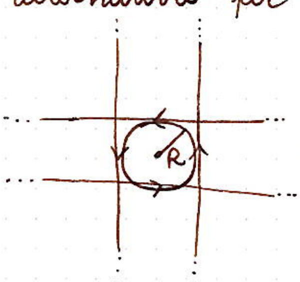
$$\boxed{\vec B_a = \frac{\mu_0 I}{2\pi R}\left(1 + \frac{\pi}{4}\right)\hat z}$$

b) di nuovo $\vec B = \frac{\mu_0 I}{4\pi s}(\sin\theta_1 + \sin\theta_2)\hat\varphi$ con $\theta_1 = \frac{\pi}{2}$ $\theta_2 = 0$ $\times 2$
$s = R$

$$\boxed{\vec B_b = \frac{\mu_0 I}{4\pi R}\left(1 + \frac{1}{\sqrt2}\right) 2\,\hat z}$$

2. $\frac{\pi}{4} \gtrless \frac{1}{\sqrt2}$? $\quad B_a$ maggiore perché filo è più vicino.

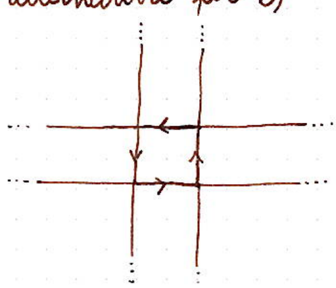
alternativa per a)



$\times \frac14$

4 fili infiniti e una spira

alternativa per b)



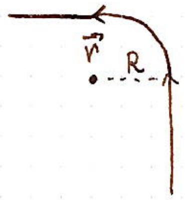
$\times \frac14$

4 fili infiniti e una spira quadrata

$$\boxed{\begin{aligned} \vec B_\circ(0) &= \frac{\mu_0 I}{2R}\hat z \\ \vec B_\square(0) &= \frac{\mu_0 I \sqrt2}{\pi R}\hat z \end{aligned}}$$

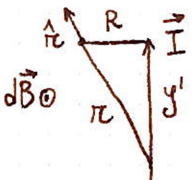
• per chi non si fida o non si ricorda le formule:

a)



• filo rettilineo:

$$\vec B(\vec r) = \frac{\mu_0}{4\pi}\int \frac{\vec I(\vec r')\times\hat{\mathscr r}(\vec r')}{\mathscr r^2(\vec r')}\, dl(r')$$



$\vec r = 0\,,\quad \vec r' = R\hat x + y'\hat y$

$\vec{\mathscr r} = \vec r - \vec r' = -R\hat x - y'\hat y$

$\hat{\mathscr r} = -R\hat x - y'\hat y$

$\vec I = I\hat y$

$\int_0^\pi \sin^3\theta\, d\theta = \int_0^\pi \overbrace{(1-\cos^2\theta)}^{\sin^2\theta}\sin\theta\, d\theta$

$= 2\int_0^{\pi/2}(1-\cos^2\theta)\sin\theta\, d\theta$ $\qquad u = \cos\theta \quad \begin{cases}\pi/2 \to 0\\ 0\to 1\end{cases}$

$= 2\int_0^1 (1-u^2)\, du$

$= 2\left[u - \frac{u^3}{3}\right]_0^1$ $\qquad \int \sin^3 = \frac43$

$= 2\left(1-\frac13\right) = \frac43$

$$\Rightarrow \vec B_{filo}(\vec r) = \frac{\mu_0 I}{4\pi}\int_{-\infty}^0 dy'\, \frac{\hat y\times(-R\hat x - y'\overbrace{\hat y}^{\to 0})}{(R^2+y'^2)^{\frac32}}$$

$\hat y\times\hat y = 0\,,\quad \hat y\times(-\hat x) = \hat z$

$$= \frac{\mu_0 I\,\hat z}{4\pi}\, R\int_{-\infty}^0 dy'\,(R^2+y'^2)^{-\frac32}$$

$y' = R\tan\theta$

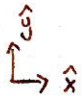
$dy' = R\,\frac{1}{\cos^2\theta} \qquad (1+\tan^2\theta)^{-1} = \cos^2\theta$

$$= \frac{\mu_0 I\,\hat z}{4\pi}\,\frac{R^2}{R^3}\int_{-\frac\pi2}^0 d\theta\, \frac{\cos^3\theta}{\cos\theta} \quad \begin{matrix}\to 1\\ \to 0\end{matrix}$$

$$= \frac{\mu_0 I}{4\pi R}\,\hat z \qquad \boxed{\vec B_{filo}(\vec r) = \frac{\mu_0 I}{4\pi R}\,\hat z}$$

• filo curvo

$\vec\gamma(\varphi) = (\cos\varphi, \sin\varphi) \qquad dl = R\,d\varphi \qquad \frac{d\gamma}{d\varphi} = (-\sin\varphi, \cos\varphi) \propto \vec I$



$$\frac{4\pi}{\mu_0}\vec B_{curvo}(\vec r) = \int_\gamma \frac{\vec I\times\hat{\mathscr r}}{\mathscr r^2}\, dl \quad \text{con } \varphi\in\left[0,\tfrac\pi2\right]$$

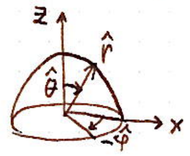
$$= \int_0^{\pi/2} R\,d\varphi\, \frac{(-\sin\varphi\,\hat x + \cos\varphi\,\hat y)\, I\times \overbrace{(-\cos\varphi\,\hat x - \sin\varphi\,\hat y)}^{-\vec\gamma(\varphi)}}{R^2}$$

$$= \int_0^{\pi/2} d\varphi\, \frac{(-\sin\varphi\,\hat x)\times(-\sin\varphi\,\hat y) + (\cos\varphi\,\hat y)\times(-\cos\varphi\,\hat x)\, I}{R}$$

$$= \int_0^{\pi/2} d\varphi\, \frac{+\sin^2\varphi\,\hat z + \cos^2\varphi\,\hat z\, I}{R}$$

$$= \frac{\hat z}{R}\int_0^{\pi/2} d\varphi = \frac\pi2\,\frac{\hat z\, I}{R} \Rightarrow \boxed{\vec B_{curvo}(\vec r) = \frac{\mu_0 I}{8R}\,\hat z}$$

coordinate sferiche

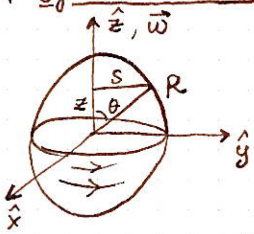


$\hat r, \hat\theta, \hat\varphi \qquad \begin{cases} x = r\sin\theta\cos\varphi\\ y = r\sin\theta\sin\varphi\\ z = r\cos\theta\end{cases}$

$\hat r\times\hat\theta = \hat\varphi$

b) In questo caso viene solo $\vec B_{filo}\times 2$

4. Sfera ruotante [probl 2, esame 2018-02-05] $\qquad \hat z\times\hat s = \hat\varphi \quad \vec s = R\sin\theta\,\hat s$



1. $\vec K = \sigma\vec v$, $\vec\omega$: $\vec v = \vec\omega\times\vec r \quad$ max K?
2. $\vec B(0) = ?$

1. $\vec K = \sigma\vec\omega\times\vec r = \sigma(\omega\hat z)\times(s\hat s)$

$= \sigma\omega R\sin\theta\,\hat\varphi$

max K = $\sigma\omega R$ a $\theta = \frac\pi2$ (equatore)

2. $\vec B(0) = \frac{\mu_0}{4\pi}\int_{S^2(R)} \frac{\vec K\times\hat{\mathscr r}}{\mathscr r^2}\, dS \quad$ con $\begin{cases}\vec K = \sigma\omega R\sin\theta\,\hat\varphi\\ \vec{\mathscr r} = -R\hat r\\ dS = R^2\sin\theta\, d\theta\, d\varphi\end{cases}$ $\Rightarrow \vec B(0) = \frac{\mu_0}{4\pi}\int_0^\pi \sin\theta\, d\theta\int_0^{2\pi} d\varphi\, R^2\, \frac{\sigma\omega R\sin\theta\,\overbrace{\hat\varphi\times(-\hat r)}^{-\hat\theta}}{R^2}$

$\vec B(0) = \frac{\mu_0}{4\pi}\,\sigma\omega R\int_0^\pi \sin^2\theta\, d\theta\int_0^{2\pi} d\varphi\,(-\hat\theta)(\theta,\varphi) \qquad \hat\theta = ?$

$-\int_0^{2\pi} d\varphi\,\hat\theta = (-2\pi)(-\sin\theta)\,\hat z \qquad \int_0^\pi \sin^3\theta\, d\theta = \frac43$

$\Rightarrow \vec B(0) = \frac{\mu_0}{4\pi}\,\sigma\omega R\, 2\pi\, \frac43\,\hat z = \frac23\mu_0\sigma R\vec\omega$

$\frac{\partial}{\partial\theta} = \underbrace{\frac{\partial}{\partial x}}_{\sim\hat x}\frac{\partial x}{\partial\theta} + \underbrace{\frac{\partial}{\partial y}}_{\sim\hat y}\frac{\partial y}{\partial\theta} + \underbrace{\frac{\partial}{\partial z}}_{\sim\hat z}\frac{\partial z}{\partial\theta} \qquad f(\overbrace{x}^{x(r,\theta,\varphi)\ \text{etc.}}, y, z)$

$= \hat x\, r\cos\theta\cos\varphi + \hat y\, r\cos\theta\sin\varphi + \hat z(-r\sin\theta)$

$\frac{\partial}{\partial\theta}\cdot\frac{\partial}{\partial\theta} = r^2\cos^2\theta\,\overbrace{(\sin^2\varphi + \cos^2\varphi)}^{1} + r^2\sin^2\theta = r^2$

$\Rightarrow \hat\theta = \frac1r\frac{\partial}{\partial\theta} = \underbrace{\cos\theta\cos\varphi\,\hat x}_{0\ \int d\varphi} + \cos\theta\sin\varphi\,\hat y - \underbrace{\sin\theta\,\hat z}_{0\ \int d\varphi}$